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Identification of Time-Delay Systems: a State-Space Realization Approach

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Abstract: A state-space realization approach is presented for the identification of time-delay dynamic systems. It is proposed an experiment with low complexity input signals such as the double pulse. The proposed identification method is a generalization of the Ho-Kalman-Kung technique for pulsed input signals. Due to its state-space formulation, it is essentially suitable to both SISO and MIMO systems.

Keywords: System identification, state-space realization, time-delay, pulse signals, simple experiment.

1. INTRODUCTION

Time-delay is present in many industrial processes. Together with the increasing expectations of dynamic performance, the time-delay estimation is essential in multiple control techniques. A review of time-delay estimation methods can be found in Bjorklund and Ljung (2003), Richard (2003), and Shalchian et al. (2010).

The classical realization problem consists of finding a state-space representation using information of the system impulse response, Katayama (2005). This problem has been studied over the years resulting in some identification methods like Ho and Kalman (1966). Such methods are not easily applied in practice because they require knowledge of the system impulse response. Papers such as van Helmont et al. (1990), Miller and de Callafon (2012), and Miller and de Callafon (2009) present an evolution of the technique using the system step response. The state-space formulation makes those identification methods intrinsically suitable to SISO and MIMO systems.

In this paper, it is presented a state-space realization approach to obtain models with explicit time-delay, using a combination of input pulse signals. Pulse signals are suitable for modeling a large set of industrial processes. Simple and fast experiments are essential to avoid production losses, as seen in de la Barra et al. (2008) and references.

A review of the classical Ho-Kalman-Kung method is shown in Section 3, then a time-delay estimation technique is presented in Section 4, followed by the proposed identification algorithm in Section 5. Finally some examples are shown in Section 6.

2. PRELIMINARIES

Assume a linear, time-invariant, discrete-time system, presented in a state-space form as

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t-t_d) \\ y(t) &= Cx(t) + Du(t-t_d) + v(t) \end{aligned} \tag{1}$$

in which $x(t) \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^n$, $C \in \mathbb{R}^n$, $D \in \mathbb{R}$, $v(t)$ is a measurement noise and t_d is a time-delay.

The system has also an alternative representation as a convolution sum

$$y(t) = \sum_{k=0}^{\infty} G(k)u(t-k-t_d) + v(t) \tag{2}$$

in which $G(k)$ is the impulse response (Markov Parameters) of the non-delayed system

$$G(k) = \begin{cases} D, & k = 0 \\ CA^{k-1}B, & k > 0 \end{cases}. \tag{3}$$

The extended observability (Γ) and controllability (Ω) matrices are defined as

$$\Gamma = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^r \end{bmatrix} \tag{4}$$

$$\Omega = \begin{bmatrix} B & AB & A^2B & \dots & A^rB \end{bmatrix} \tag{5}$$

in which $r > n$.

Problem statement: Given the input and output data from simple experiments, with input pulses, the goal is to extract the time-delay and construct a state-space representation such as 1.

3. THE HO-KALMAN-KUNG REALIZATION ALGORITHM (HKK)

The realization problem was proposed by Kalman as the construction of a state-space model from experimental impulse response data, Kalman (1963). The Ho-Kalman algorithm proposes a solution to the realization problem through the decomposition of the Hankel matrices of Markov parameters. The problem was later refined by Kung (1978) using the SVD (Singular Value Decomposition) to generate a realization of minimum rank.

For $t_d = 0$, the past (u_p) and future (u_f) inputs are related to the future outputs (y_f) as

$$y_f = Hu_p + Tu_f + v \tag{6}$$

in which,

$$\begin{aligned} u_p &= [u(0)\ u(-1)\ u(-2)\ ...]^T \\ u_f &= [u(1)\ u(2)\ u(3)\ ...]^T \\ y_f &= [y(1)\ y(2)\ y(3)\ ...]^T \\ v &= [v(1)\ v(2)\ v(3)\ ...]^T, \end{aligned} \tag{7}$$

T is a block-Toeplitz matrix and H a block-Hankel matrix of Markov parameters. In other words, (6) is the equivalent matrix form of (2)

$$H = \begin{bmatrix} G(1) & G(2) & .. \\ G(2) & G(3) & .. \\ G(3) & G(4) & .. \\ \vdots & \vdots & \end{bmatrix} = \begin{bmatrix} CB & CAB & .. \\ CAB & CA^2B & .. \\ CA^2B & CA^3B & .. \\ \vdots & \vdots & \end{bmatrix} \tag{8}$$

$$T = \begin{bmatrix} G(0) & 0 & 0 & ... \\ G(1) & G(0) & 0 & ... \\ G(2) & G(1) & G(0) & ... \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{9}$$

Properties of Markov parameters matrix H:

- H is the product of extended observability matrix (Γ) and extended controllability matrix (Ω)

$$H = \Gamma\Omega = \begin{bmatrix} C \\ CA \\ CA^2 \\ .. \end{bmatrix} \begin{bmatrix} B & AB & A^2B & .. \end{bmatrix} \tag{10}$$

- $\overrightarrow{H}$ is defined shifting one row up or one column to the left of H

$$\overrightarrow{H} = \Gamma A \Omega. \tag{11}$$

Assume the system is controllable and observable

$$rank(\Gamma) = rank(\Omega) = rank(H) = n. \tag{12}$$

Suppose $r + l$ Markov parameters are known and $r + l \geq 2n + 1$, then the Hankel matrices H and $\overrightarrow{H}$ are defined as

$$H = \begin{bmatrix} G(1) & G(2) & ... & G(l) \\ G(2) & G(3) & ... & G(l+1) \\ \vdots & \vdots & & \vdots \\ G(r) & G(r+1) & ... & G(r+l-1) \end{bmatrix} \tag{13}$$

$$\overrightarrow{H} = \begin{bmatrix} G(2) & G(3) & ... & G(l+1) \\ G(3) & G(4) & ... & G(l+2) \\ \vdots & \vdots & & \vdots \\ G(r+1) & G(r+2) & ... & G(r+l) \end{bmatrix}. \tag{14}$$

The high rank matrix H can be approximated by a *rank n* matrix $\hat{H}$ defined from the SVD (Singular Value Decomposition) of H as

$$H = [U_n\ U_s] \begin{bmatrix} \Sigma_n & 0 \\ 0 & \Sigma_s \end{bmatrix} \begin{bmatrix} V_n^T \\ V_s^T \end{bmatrix}. \tag{15}$$

$\hat{H}$ is by definition as close as possible to H in a 2-norm sense, Eckart and Young (1936), in which

$$\begin{aligned} \hat{H} &= \underset{rank(\hat{H})=n}{\arg\min} \left\| \hat{H} - H \right\|_2 \\ \hat{H} &= U_n \Sigma_n V_n^T. \end{aligned} \tag{16}$$

If the model order n is unknown, it can be determined from searching for a significant drop-off in the singular values of H, listed on the main diagonal of Σ.

Estimates of the observability and controllability matrices are given by

$$\hat{\Gamma} = U_n \Sigma_n^{1/2}\ \hat{\Omega} = \Sigma_n^{1/2} V_n^T \tag{17}$$

finally, the state-space realization is calculated using the relations in table 1.

Table 1. State-Space matrices computation

$\hat{A} = (\hat{\Gamma})^\dagger \overrightarrow{H} (\hat{\Omega})^\dagger = \Sigma_n^{-1/2} U_n^T \overrightarrow{H} V_n \Sigma_n^{-1/2}$
$\hat{B} = \hat{\Omega}_{(:,1)}$
$\hat{C} = \hat{\Gamma}_{(1,:)}$
$\hat{D} = G(0)$

The operator $(\cdot)^\dagger$ represents the pseudoinverse to the left and the subscripts in $\hat{\Gamma}$ and $\hat{\Omega}$, are the Matlab™ style of indexing. Further discussion can be found in Kung (1978) and Juang and Pappa (1985).

4. TIME-DELAY ESTIMATION (HKK-TD)

A time-delay estimate will be extracted from the impulse response information, embedded in H matrix. The first column of H, by definition, contains the first r elements of the system impulse response. If r is chosen as $r > t_d$, then the number of elements close to zero in $H_{(1,:)}$ represents the system time-delay

$$H_{(:,1)} = \begin{bmatrix} G(1) \\ .. \\ G(t_d) \\ --- \\ G(t_{d+1}) \\ .. \\ G(r) \end{bmatrix} = \begin{bmatrix} 0 \\ .. \\ 0 \\ --- \\ G(t_{d+1}) \\ .. \\ G(r) \end{bmatrix}. \tag{18}$$

If a time-delayed system ($G_d(k)$) is represented by its Markov parameters in a matrix form H_d, then the equivalent system without the time-delay ($G_0(k)$) can be represented by $H_0 = H_{d(t_d+1:r,:)}$.

Based on $G_d(k + t_d) = G_0(k)$, H_0 is constructed by the elimination of the t_d first rows of H_d. Equation (19) represents the dynamics of the original system without time-delay

$$y_{f(t_d+1:r)} = H_{(t_d+1:r,:)} u_p + T_{(t_d+1:r,:)} u_f + v_{(t_d+1:r)}. \tag{19}$$

The following procedure can be used to generate the state-space realization plus time-delay model

1. Assume H_d is known or estimated in a previous step;
2. Estimate the time-delay from the first column of H_d and reject t_d initial rows of H_d to generate H_0;

(3) Use the Ho-Kalman-Kung procedure, on H_0, to estimate the state-space representation of the system without time-delay

$$H_0 \Rightarrow \begin{array}{c} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) + v(t) \end{array};$$

(4) Shift the input of the model obtained in step 3 to construct the HKK-TD estimate

$$H_d \Rightarrow \begin{array}{c} x(t+1) = Ax(t) + Bu(t-t_d) \\ y(t) = Cx(t) + Du(t-t_d) + v(t) \end{array}.$$

In order to have a more robust time-delay estimate, the columns of H_d can be averaged if the estimated Markov parameters are corrupted by noise.

5. PULSE BASED REALIZATION ALGORITHM WITH TIME-DELAY

The system impulse response, in many cases, is difficult to be obtained by a direct experiment. The generalization of the Ho-Kalman-Kung method for a step signal and correlation functions are found in Miller and de Callafon (2009). In this paper, it is proposed an extension to a sequence of pulsed signals. Pulses of different width and amplitude can excite the process at desired frequencies which are relevant for system identification.

5.1 Single pulse based realization algorithm (SPBR-TD)

All pulse based input signals used in this paper are constructed from a *base-pulse* signal defined in (20). See Fig. 1.

$$u_B = \begin{cases} u(t) = 0,\ t < \gamma \\ u(t) = \beta,\ \gamma \le t < \alpha + \gamma \\ u(t) = 0,\ t \ge \alpha + \gamma \end{cases} \tag{20}$$

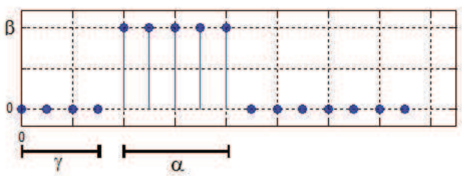

Fig. 1. base-pulse definition

Assume the system without time-delay ($t_d = 0$). The Hankel matrices of the output are

$$Y = \begin{bmatrix} y(1) & y(2) & \dots & y(l) \\ y(2) & y(3) & \dots & y(l+1) \\ \vdots & \vdots & & \vdots \\ y(r) & y(r+1) & \dots & y(N-1) \end{bmatrix} \tag{21}$$

$$\overrightarrow{Y} = \begin{bmatrix} y(2) & y(3) & \dots & y(l+1) \\ y(3) & y(4) & \dots & y(l+2) \\ \vdots & \vdots & & \vdots \\ y(r+1) & y(r+2) & \dots & y(N) \end{bmatrix}.$$

The equation (6) is directly extended to a block-Hankel form as

$$Y = HU_p + TU + V \tag{22}$$

$$\overrightarrow{Y} = \overrightarrow{H}U_p + \overrightarrow{T}U + \overrightarrow{V} \tag{23}$$

in which T and $\overrightarrow{T}$ are block-Toeplitz matrices, $U \in R^{(r)\times(l)}$ is the Hankel matrix of the input, $Y \in R^{(r)\times(l)}$ is the Hankel matrix of the output and $U_p \in R^{(l)\times(l)}$ is a upper triangular matrix.

$$T = \begin{bmatrix} G(0) & & \\ G(1) & G(0) & \\ \vdots & \vdots & \ddots \\ G(r-1) & G(r-2) & \dots & G(0) \end{bmatrix} \tag{24}$$

$$\overrightarrow{T} = \begin{bmatrix} G(1) & G(0) & & \\ G(2) & G(1) & \ddots & \\ \vdots & \vdots & \ddots & G(0) \\ G(r) & G(r-1) & \dots & G(1) \end{bmatrix} \tag{25}$$

$$U_p = \begin{bmatrix} u(0) & u(1) & \dots & u(l-1) \\ 0 & u(0) & \dots & u(l-2) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & u(0) \end{bmatrix} \tag{26}$$

The goal here is to find an estimate of H and apply the Ho-Kalman-Kung algorithm. The single pulse input is not persistent, $rank(U) \neq n$, so it can not be used to create a projector that separates H and T, as is done in traditional subspaces identification mechanisms.

Consider the input as a base-pulse signal. It is possible to explicitly calculate an approximation of the term TU on (22), then compute an estimate of H. If $r < \alpha$ and $r < \gamma$, then the Hankel input matrix U_B can be written in 3 parts

$$U_B = [U_{B\Delta} \mid U_{B\Phi} \mid U_{B\Psi}] \tag{27}$$

$$U_{B\Delta} = \begin{bmatrix} 0 & .. & 0 & 0 & 0 \\ 0 & .. & 0 & 0 & \beta \\ 0 & .. & 0 & \beta & \beta \\ .. & .. & .. & .. & .. \\ 0 & .. & \beta & \beta & \beta \end{bmatrix} U_{B\Phi} = \begin{bmatrix} \beta & .. & \beta \\ \beta & .. & \beta \\ \beta & .. & \beta \\ .. & .. & .. \\ \beta & .. & \beta \end{bmatrix} U_{B\Psi} = \begin{bmatrix} \beta & \beta & \beta \\ \beta & \beta & 0 \\ \beta & 0 & 0 \\ .. & .. & .. \\ 0 & 0 & 0 \end{bmatrix}$$

in which $U_{B\Delta} \in R^{(r)\times(\gamma)}$, $U_{B\Phi} \in R^{(r)\times(\alpha-r)}$ and $U_{B\Psi} \in R^{(r)\times(r-1)}$.

By ignoring the noise v and replacing (25) and (27),

$$TU_B = [\Delta\ \Phi\ \Psi]$$

$$\Delta = \beta \begin{bmatrix} 0 & 0 & 0 & .. & 0 \\ 0 & 0 & 0 & .. & G_0 \\ 0 & 0 & 0 & .. & G_0 + G_1 \\ .. & .. & .. & .. & .. \\ 0 & 0 & G_0 & .. & G_0 + G_1 .. + G_{r-3} \\ 0 & G_0 & G_0 + G_1 & .. & G_0 + G_1 .. + G_{r-2} \end{bmatrix}$$

$$\Phi = \beta \begin{bmatrix} G_0 & .. & G_0 \\ G_0 + G_1 & .. & G_0 + G_1 \\ G_0 + G_1 + G_2 & .. & G_0 + G_1 + G_2 \\ .. & .. & .. \\ G_0 + G_1 .. + G_{r-2} & .. & G_0 + G_1 .. + G_{r-2} \\ G_0 + G_1 .. + G_{r-1} & .. & G_0 + G_1 .. + G_{r-1} \end{bmatrix}$$

$$\Psi = \beta \begin{bmatrix} G_0 & G_0 & .. & G_0 \\ G_0 + G_1 & G_0 + G_1 & .. & G_1 \\ G_0 + G_1 + G_2 & G_0 + G_1 + G_2 & .. & G_2 \\ .. & .. & .. & .. \\ G_0 + G_1 .. + G_{r-2} & G_1 + G_2 .. + G_{r-2} & .. & G_{r-2} \\ G_1 + G_2 .. + G_{r-1} & G_2 + G_3 .. + G_{r-1} & .. & G_{r-1} \end{bmatrix}$$

It's known that

$$y(t) = \sum_{k=0}^{\infty} G(k)u(t-k)$$

the matrix TU_B can be rewritten only as a function of the output signal y

$$\Delta = \begin{bmatrix} 0 & 0 & 0 & .. & 0 \\ 0 & 0 & 0 & .. & y_\gamma \\ 0 & 0 & 0 & .. & y_{\gamma+1} \\ .. & .. & .. & .. & .. \\ 0 & 0 & y_\gamma & .. & y_{\gamma+r-2} \\ 0 & y_\gamma & y_{\gamma+1} & .. & y_{\gamma+r-3} \end{bmatrix}$$

$$\Phi = \begin{bmatrix} y_\gamma & .. & y_\gamma \\ y_{\gamma+1} & .. & y_{\gamma+1} \\ y_{\gamma+2} & .. & y_{\gamma+2} \\ .. & .. & .. \\ y_{\gamma+r-2} & .. & y_{\gamma+r-2} \\ y_{\gamma+r-1} & .. & y_{\gamma+r-1} \end{bmatrix}$$

$$\Psi = \begin{bmatrix} y_\gamma & y_\gamma & .. & y_\gamma \\ y_{\gamma+1} & y_{\gamma+1} & .. & y_{\gamma+1} - y_\gamma \\ y_{\gamma+2} & y_{\gamma+2} & .. & y_{\gamma+2} - y_{\gamma+1} \\ .. & .. & .. & .. \\ y_{\gamma+r-2} & y_{\gamma+r-2} - y_\gamma & .. & y_{\gamma+r-2} - y_{\gamma+r-3} \\ y_{\gamma+r-1} - y_\gamma & y_{\gamma+r-1} - y_{\gamma+1} & .. & y_{\gamma+r-1} - y_{\gamma+r-2} \end{bmatrix}$$

in which $\Delta \in R^{(r)\times(\gamma)}$, $\Phi \in R^{(r)\times(\alpha-r)}$ and $\Psi \in R^{(r)\times(r-1)}$.

So H and $\overrightarrow{H}$ are computed as

$$H \approx (Y - TU_B)U_p^\dagger \quad (28)$$
$$\overrightarrow{H} \approx (\overrightarrow{Y} - \overrightarrow{TU_B})U_p^\dagger.$$

Once an approximation of H is calculated, the concepts of Section 4 are used to extract the time-delay, then the state-space realization is computed as in Section 3.

5.2 Multiple pulse based realization algorithm (MPBR-TD)

Richer excitation signals can be generated by the combination of different base-pulse signals. The double pulse is the simplest one. It is an easy to generate signal, suitable for many industrial processes, that gives information of both rising and falling dynamics. Two configurations are illustrated in Fig. 2 and 3.

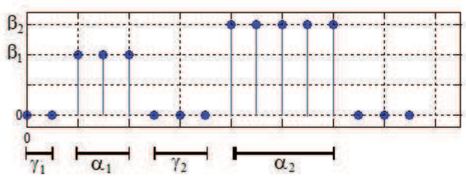

Fig. 2. Double Pulse (case 1)

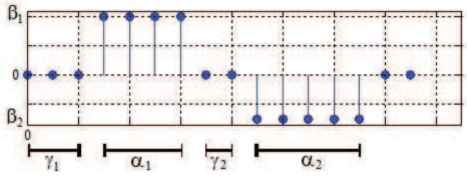

Fig. 3. Double Pulse (case 2)

The MPBR-TD method is a direct extension of SPBR-TD. Defining a multi-pulse signal as a concatenation of n base-pulse signals, then

$$U = [U_{B1}\ U_{B2}\ ..\ U_{Bn}].$$

TU_{B1}, TU_{B2} and TU_{Bn} are computed individually as in the Section 5.1

$$TU = [TU_{B1}\ TU_{B2}\ ..\ TU_{Bn}]. \quad (29)$$

Once TU is defined, H is calculated using (28) and finally the state-space realization is generated by HKK-TD algorithm. Note the following restriction have to be made to both base-pulse signals $r < \alpha_n$ and $r < \gamma_n$.

6. RESULTS

To evaluate the proposed method, one simulated example and one pilot-scale application are shown.

6.1 Simulation example

Assume the true system is given by

$$y(t) = G(q)u(t) + e(t) \quad (30)$$

$$G(q) = \frac{0,4q^{-8}}{(q-0,6)(q-0,3)} \quad (31)$$

in which $e(t)$ is a white noise, $E\{e(t)\} = 0$, $\sigma = 0,05$ and sample-time $T_s = 0.001s$.

It was applied a double pulse input with different width and amplitude. The identification data is illustrated on Fig. 4.

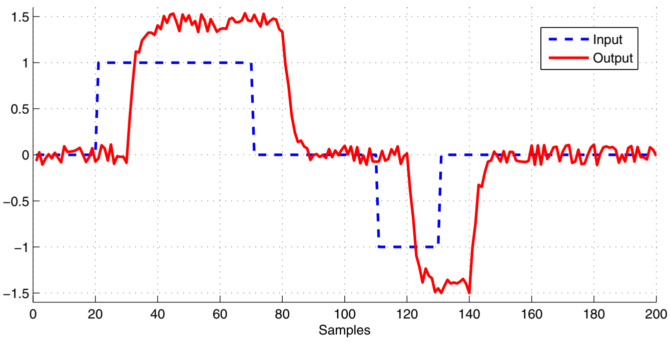

Fig. 4. Input and output signals

Three identification methods were used

MPBR-TD model

$$A = \begin{bmatrix} 0.860 & 0.248 & 0.040 \\ -0.269 & 0.291 & 0.562 \\ -0.163 & -0.398 & -0.509 \end{bmatrix} \quad B = \begin{bmatrix} -0.481 \\ -0.168 \\ 0.089 \end{bmatrix}$$
$$C = [-0.518\ 0.470\ -0.052] \quad D = [-0.023]$$
$$t_d = 8$$

ARX model

$$G(k)_{ARX} = \frac{-0.005q^{-8} - 0.009q^{-9} + 0.444q^{-10}}{1 - 0.724q^{-1} - 0.012q^{-2} + 0.044q^{-3}}$$

N4sid model

$$A = \begin{bmatrix} 0.93 & -0.10 & -0.00 & 0.02 \\ 0.17 & 0.91 & -0.30 & -0.14 \\ -0.03 & 0.29 & 0.61 & 0.85 \\ -0.03 & -0.08 & -0.58 & 0.61 \end{bmatrix} \quad B = \begin{bmatrix} 0.01 \\ -0.01 \\ -0.10 \\ 0.07 \end{bmatrix}$$
$$C = [8.10\ 2.11\ 0.65\ -0.24] \quad D = [0]$$

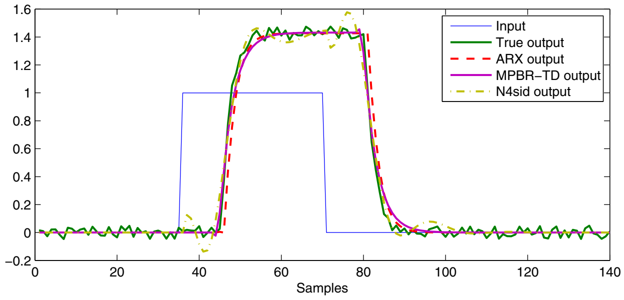

Fig. 5. Validation input-output data

The ARX and N4sid models were identified using the *System Identification Toolbox* of Matlab. The model order and the delay estimation, of those two models, were chosen by the toolbox algorithm. Note the ARX and MPBR-TD models have lower order because of the time-delay estimation. On the other hand, the N4sid model has a higher order and a more oscillatory response.

Table 2. Simulation results

Method	Model order	Time-delay (samples)	RMSD
ARX	3	8	$62 * 10^{-4}$
N4sid	4	0	$38 * 10^{-4}$
MPBR-TD	3	8	$19 * 10^{-4}$

It was applied a single pulse to the system, as a validation signal. The output of the identified models are displayed on Fig. 5. It is shown in table 2 that the proposed method has the smallest RMSD (Root Mean Square Deviation)

6.2 Pilot-Scale Application

The pilot-scale flow plant represented by Fig. 6 and 7 was used in the experimental application. The plant input was chosen as the pump frequency $(0 - 3200RPM)$ and the output is the water flow $(0 - 10l/min)$. Both ranges are normalized to $0 - 100\%$. The operating point is $input = 60\%$ and $output = 40\%$ with sampling period $T_s = 0.2s$.

Fig. 6. Plant photo

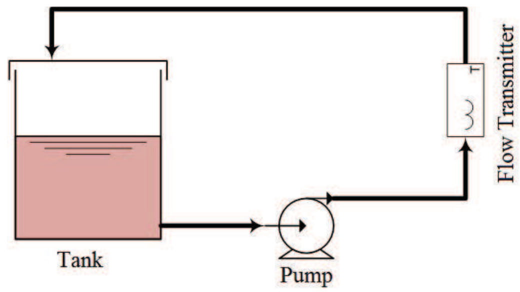

Fig. 7. Plant layout

A double pulse signal was applied to the system, followed by a single pulse used at the validation of the identified models. It is shown in Fig. 8 and table 3, that the proposed model has lower order and lower RMSD compared to the ARX estimate.

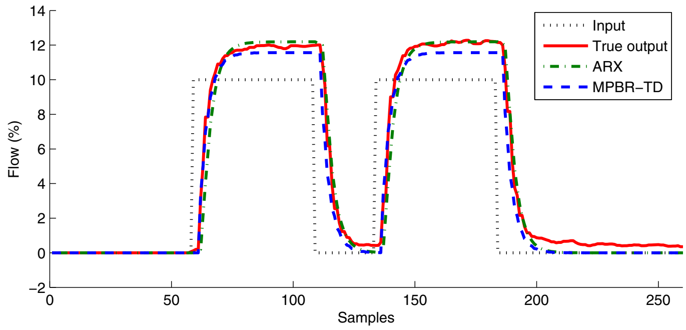

Fig. 8. Identification input-output data

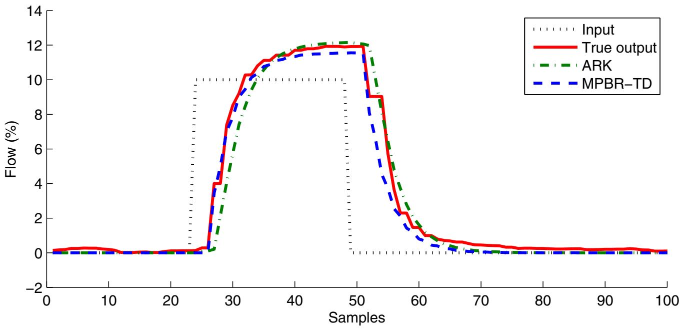

Fig. 9. Validation input-output data

Table 3. Pilot scale results

Method	M. order	T. Delay (s)	RMSD
ARX	5	1	0.63
MPBR-TD	4	2	0.43

Two models were obtained

MPBR-TD model

$$A = \begin{bmatrix} -0.96 & 0.03 & -0.34 & -0.02 \\ 0.02 & 0.69 & 0.02 & -0.28 \\ 0.34 & 0.01 & -0.92 & -0.10 \\ -0.03 & -0.28 & 0.10 & -0.74 \end{bmatrix} \quad B = \begin{bmatrix} -0.01 \\ -0.56 \\ -0.00 \\ -0.15 \end{bmatrix}$$
$$C = [-0.01 \ -0.56 \ -0.01 \ -0.15] \qquad D = [0]$$
$$t_d = 2$$

ARX model

$$G(k)_{ARX} = \frac{0.01q^{-1} + 0.01q^{-2} + 0.18q^{-3} + 0.07q^{-4} + 0.12q^{-5}}{1 - 0.35q^{-1} - 0.30q^{-2} - 0.14q^{-3} + 0.19q^{-4} - 0.07q^{-5}}$$

7. CONCLUSIONS

It was shown an algorithm to generate state-space realizations of a time-delay system, based on the Ho-Kalman-Kung method. This technique is mostly suitable to industrial processes due the fact it uses simple excitation signals as the double pulse, in contrast to a richer signal of a subspace approach. The proposed method, including the time-delay estimation step, is also applicable to SISO or MIMO systems. In the example section the algorithm was successfully applied to both simulated and experimental plants.

REFERENCES

Bjorklund, S. and Ljung, L. (2003). A review of time-delay estimation techniques. *Control and Communication group in Linkoping.*

de la Barra, B., Jin, L., Kim, Y., and Mossberg, M. (2008). Identification of first-order time-delay systems using two different pulse inputs. *17th IFAC World Congress*, 366/371.

Eckart, C. and Young, G. (1936). The approximation of one matrix by another of lower rank. *Psychometrika*, (vol1 n3), 1/10.

Ho, B.L. and Kalman, R.E. (1966). Effective construction of linear state-variable models from input/output functions. *Regelungstechnik vol. 14*, (no. 12), 545/548.

Juang, J.N. and Pappa, R.S. (1985). An eigensystem realization algorithm (era) for modal parameter identification and model reduction. *Workshop on Identification and Control of Flexible Space Structures*, (3), 299/318.

Kalman, R.E. (1963). Mathematical description of linear dynamical systems. *Journal of the Society for Industrial and Applied Mathematics*, 152/155.

Katayama, T. (2005). *Subspace methods for system identification : a realization approach.* Springer.

Kung, S. (1978). A new identification and model reduction algorithm via singular value decomposition. *12th Asilomar Conference on Circuits, Systems, and Computers*, 705/714.

Miller, D.N. and de Callafon, R.A. (2009). Subspace identification from classical realization methods. *15th IFAC Symposium on System Identification.*

Miller, D.N. and de Callafon, R.A. (2012). Identification of linear time-invariant systems via constrained step-based realization. *16th IFAC Symposium on System Identification*, 1155/1160.

Richard, J.P. (2003). Time-delay systems: an overview of some recent advances and open problems. *Automatica*, (no. 39), 1667/1694.

Shalchian, J., Khaki-Sedigh, A., and Fatehi, A. (2010). A subspace based method for time delay estimation. *International Symposium on Communications, Control and Signal Processing ISCCSP.*

van Helmont, J., van der Weiden, A., and Anneveld, H. (1990). Design of optimal controller for a coal fired benson boiler based on a modified approximate realisation algorithm. *Applications of Multivariable System Techniques - Elsevier Applied Science*, 313/320.

