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State and Input Estimation of an Anaerobic Digestion Reactor using a Continuous-discrete Unknown Input Observer ⋆

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Abstract: The objective of this paper is to address the problem of state estimation in an Anaerobic Digestion Reactor (ADR) with unknown inputs - typically some influent concentrations, and to compare two different state and input estimation schemes. The first one is the classical Extended Kalman Filter (EKF) based on an augmented system that considers a slowly varying input (approach that has been already applied to this system and reported in the literature), whereas the second one is a recently proposed Unknown Input Observer (UIO), formulated in the spirit of a Kalman Filter, for continuous estimation with discrete measurements. The two filters are evaluated in simulation, demonstrating the superiority of the UIO.

Keywords: State estimation; Input estimation; Robust estimation; Biotechnology.

1. INTRODUCTION

Anaerobic digestion (AD) of organic waste and wastewater is increasingly applied as it is an important source of renewable energy in the form of biogas (a mixture of mostly carbon dioxide and methane), and can be used combined to other process units in biorefineries. However, the AD process has complex dynamics, is quite sensitive to input fluctuations, and requires tight control. Unfortunately, the development of efficient controllers is hampered by the lack of on-line measurements of some key component concentrations. The missing information has therefore to be reconstructed by means of state estimation schemes, the so-called software sensors, which blend the information of a process model and of some available on-line probes. The state estimation problem has usually to be formulated in the presence of unknown inputs, i.e. unknown component concentrations in the process influent.

The problem of state estimation in AD process has usually been dealt with in two ways [1]: (a) the asymptotic observer which allows to reconstruct the process state despite the lack of knowledge of the kinetics - however, the asymptotic observer is very sensitive to unknown process inputs, (b) interval observers, which allows to predict intervals of variations for the state variables based on intervals for the process parameters and inputs - however these intervals can be delicate to exploit for control.

In this study, attention is focused on the design of unknown input observers for monitoring the AD process. Unknown Input Observers (UIO) are dynamical systems that estimate the state variables of a system robustly with respect to the disturbances or unknown inputs that affect the system. For example, in a recent paper [9], the authors have proposed the design of an UIO consisting of three parts, i.e., two supertwisting observers and an asymptotic observer for estimating two biomass and two inlet substrate concentrations in a AD process described by a two-step reaction model.

In the present study, attention is focused on filters, optimal in a minimum-variance unbiased sense. On the one hand, we implement the classical Extended Kalman Filter (EKF) applied to a two-step reaction model supplemented by an exosystem assuming that the unknown input concentration varies slowly. This approach is similar to the proposal of [5], where an Unscented Kalman Filter (UKF) is designed instead. This latter study shows satisfactory results for some of the state variables, with the exception of the biomass concentrations which cannot be measured and therefore do not allow a full validation of the estimation approach. This observation motivates the use of alternative techniques, and we consider an UIO filter proposed in [3], and extended to nonlinear continuous-time models associated to discrete-time (and often rare) measurements in [8].

This paper is organized as follows. The next section presents the AD process model under consideration and the state estimation problem. A global observability assessment is given in section

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3 in accordance with the available measurements. In section 4, some background material about state and input estimation is discussed, while the main results about the evaluation of the performance of the two filters are presented in section 5. Finally, some conclusions are drawn and future work commented in section 6.

2. ANAEROBIC DIGESTION MODEL

Several two-step reaction models have been proposed in the last decades, including the model of Hill [6], and the nowadays very popular AM2 model developed in [2].

In the present study, inspired by the work of Haugen, et al. [4] where the model of Hill is prefered, we also adopt the latter. This model consists of four mass balance ordinary differential equations

$$\begin{aligned}\dot{S}_{bvs} &= \left(S_{bvs_{in}} - S_{bvs}\right)\frac{F_{feed}}{V} - \mu\left(S_{bvs}\right)k_1 X_{acid}\\ \dot{S}_{vfa} &= \left(S_{vfa_{in}} - S_{vfa}\right)\frac{F_{feed}}{V} + \mu\left(S_{bvs}\right)k_2 X_{acid} -\\ &\quad -\mu_c\left(S_{vfa}\right)k_3 X_{meth}\\ \dot{X}_{acid} &= \left(\mu\left(S_{bvs}\right) - K_d - \frac{F_{feed}/b}{V}\right)X_{acid}\\ \dot{X}_{meth} &= \left(\mu_c\left(S_{vfa}\right) - K_{dc} - \frac{F_{feed}/b}{V}\right)X_{meth}\end{aligned} \tag{1}$$

where S_{bvs} is the concentration of organic substrate (biodegradable volatile solids) in $[gBVS/L]$, S_{vfa} is the concentration of volatile fatty acids in $[gVFA/L]$, X_{acid} represents the acidogenic bacteria in $[g\ acidogens/L]$, and X_{meth} the methanogenic bacteria in $[g\ methanogens/L]$. The factor $\frac{F_{feed}}{V}$ represents the dilution rate in $[(LCH_4/d)/L]$, and k_1, k_2, k_3 are the stoichiometric coefficients.

As in [4], it is considered that

$$\begin{aligned}S_{bvs_{in}} &= B_0 S_{vs_{in}}\\ S_{vfa_{in}} &= A_f S_{bvs_{in}} = A_f B_0 S_{vs_{in}}\end{aligned}$$

The first equation defines the portion of the raw waste which can serve as substrate (biodegradable part) and the second one defines the portion of that biodegradable material which is initially in the acid form. Parameters B_0 and A_f are considered in the original Hill model [6] and obtained from laboratory test in [4]. In this way, both concentration inputs depend on $S_{vs_{in}}$, which is then considered as the unknown input.

The measurable output is the methane gas flow rate (gas production) in $[LCH_4/d]$ given by

$$F_{meth} = V\mu_c\left(S_{vfa}\right)k_5 X_{meth} \tag{2}$$

The reaction rate functions are of Monod type

$$\begin{aligned}\mu\left(S_{bvs}\right) &= \mu_m\frac{S_{bvs}}{K_s + S_{bvs}}\\ \mu_c\left(S_{vfa}\right) &= \mu_{mc}\frac{S_{vfa}}{K_{sc} + S_{vfa}}\end{aligned} \tag{3}$$

The maximum reaction rates μ_m, μ_{mc} are functions of the reactor temperature as in the original Hill model [6]

$$\mu_m\left(T_{reac}\right) = \mu_{mc}\left(T_{reac}\right) = 0.013T_{reac} - 0.129 \tag{4}$$

for $20[^\circ C] < T_{reac} < 60[^\circ C]$.

The considered values of the model parameters are the same as in [4], and correspond to a real-life pilot plant.

A_f	$=$	0.69	$[(gVFA/L)/(gBVS/L)]$
B_0	$=$	0.25	$[(gBVS/L)/(gVS/L)]$
b	$=$	2.90	$[d/d]$
k_1	$=$	3.89	$[gBVS/(g\ acidogens/L)]$
k_2	$=$	1.76	$[gVFA/(g\ acidogens/L)]$
k_3	$=$	31.7	$[gVFA/(g\ methanogens/L)]$
k_5	$=$	26.3	$[L/g\ methanogens]$
K_d	$=$	0.02	$[d^{-1}]$
K_{dc}	$=$	0.02	$[d^{-1}]$
K_s	$=$	15.5	$[gBVS/L]$
K_{sc}	$=$	3	$[gVFA/L]$
V	$=$	250	$[L]$
T_{reac}	$=$	35	$[^\circ C]$

3. OBSERVABILITY ANALYSIS

In the classical observability concept, all inputs are assumed to be known. When some inputs are unknown, a stricter property must be proved, that is a robust observability or observability with unknown inputs. Such a property can be tested using the method described in [7], which is based on the analysis of solutions of an error dynamics. This method can also be understood as an analysis of distinguishability of states under the assumption that the output y and the control inputs u are perfectly known.

In this case, one can define a copy of system (1) that renames the state vector $s = [S_{bvs}, S_{vfa}, X_{acid}, X_{meth}]$, the unknown input $w = S_{vs_{in}}$ and the output $y = F_{meth}$ as $x = [x_1, x_2, x_3, x_4]$, $\bar{w}$ and y_x, respectively. That is

$$\begin{aligned}\dot{x}_1 &= \left(B_0\bar{w} - x_1\right)u - \mu\left(x_1\right)k_1 x_3\\ \dot{x}_2 &= \left(A_f B_0\bar{w} - x_2\right)u + \mu\left(x_1\right)k_2 x_3 - \mu_c\left(x_2\right)k_3 x_4\\ \dot{x}_3 &= \left(\mu\left(x_1\right) - K_d - \frac{u}{b}\right)x_3\\ \dot{x}_4 &= \left(\mu_c\left(x_2\right) - K_{dc} - \frac{u}{b}\right)x_4\\ y_x &= V\mu_c\left(x_2\right)k_5 x_4\end{aligned} \tag{5}$$

where $u = D = \frac{F_{feed}}{V}$.

An error between the states of the original system and those of the copy can be defined as $\epsilon = x - s$. Then the state error dynamics with the same known input $u = \frac{F_{feed}}{V}$ is

$$\begin{aligned}\dot{\epsilon}_1 &= B_0 u\left(w - \bar{w}\right) - u\epsilon_1 - \mu_m k_1\phi_1\left(x_1, x_3, \epsilon_1, \epsilon_3\right)\\ \dot{\epsilon}_2 &= A_f B_0 u\left(w - \bar{w}\right) - u\epsilon_2 - \mu_m k_2\phi_1\left(x_1, x_3, \epsilon_1, \epsilon_3\right) -\\ &\quad -\mu_{mc}k_3\phi_2\left(x_2, x_4, \epsilon_2, \epsilon_4\right)\\ \dot{\epsilon}_3 &= -\left(K_d + \frac{u}{b}\right)\epsilon_3 - \mu_m\phi_1\left(x_1, x_3, \epsilon_1, \epsilon_3\right)\\ \dot{\epsilon}_4 &= -\left(K_{dc} + \frac{u}{b}\right)\epsilon_4 + \mu_{mc}\phi_2\left(x_2, x_4, \epsilon_2, \epsilon_4\right)\end{aligned}$$

where

$$\phi_1(x_1,x_3,\epsilon_1,\epsilon_3)=\frac{x_1x_3}{K_s+x_1}-\frac{(x_1-\epsilon_1)(x_3-\epsilon_3)}{K_s+(x_1-\epsilon_1)}$$
$$\phi_2(x_2,x_4,\epsilon_2,\epsilon_4)=\frac{x_2x_4}{K_{sc}+x_2}-\frac{(x_2-\epsilon_2)(x_4-\epsilon_4)}{K_{sc}+(x_2-\epsilon_2)}$$

The output error $e=y_x-y$ is

$$e=Vk_5\mu_{mc}\phi_2(x_2,x_4,\epsilon_2,\epsilon_4)$$

The test of insdistinguishability is based on this error system: if for each known pair u, y (which implies $e=0$, i.e. $y=y_x$) the only solution is $\epsilon=0$, during some interval of time, there exists distinguishability of the states and the system is observable. If there are solutions different from zero, it should occur that $\epsilon\to 0$ in order to have detectability with unknown inputs. If in addition, the only solution for the inputs is that $w=\bar{w}$ there exists distinguishability of the inputs, so the unknown input of the system can be estimated.

In the case of ADR, if $e=0$, then $\phi_2(\cdot)=0$ since $V,\ k_5,\ \mu_{mc}>0$ and $\dot{\epsilon}_4$ reduces to $-\left(K_{dc}+\frac{u}{b}\right)\epsilon_4$. It is evident that $\epsilon_4\to 0$ since $K_{dc}>0$. At this point it is just required that $u\geq 0$. From the definition of $\phi_2(\cdot)$ one can prove that if $\epsilon_4\to 0$ then $\epsilon_2\to 0$.

In addition, one can define $\epsilon_A=\epsilon_2-A_f\epsilon_1$, and the rest of the system can be written as

$$\begin{aligned}\dot{\epsilon}_A&=-u\epsilon_A+(\mu_m k_2+\mu_m k_1 A_f)\,\phi_1(\epsilon_A,\epsilon_2,\epsilon_3)\\ \dot{\epsilon}_3&=-\left(K_d+\frac{u}{b}\right)\epsilon_3+\mu_m\phi_1(\epsilon_A,\epsilon_2,\epsilon_3)\end{aligned}\qquad(6)$$

As can be seen from the first equation of (6), one necessary condition for the convergence is that $u>0$.

This system can be solved numerically using the model parameter values listed in section 2 and $F_{feed}=55$ in order to assess the convergence to zero. Four trajectories (one for each quadrant) are presented in Fig. 1 .

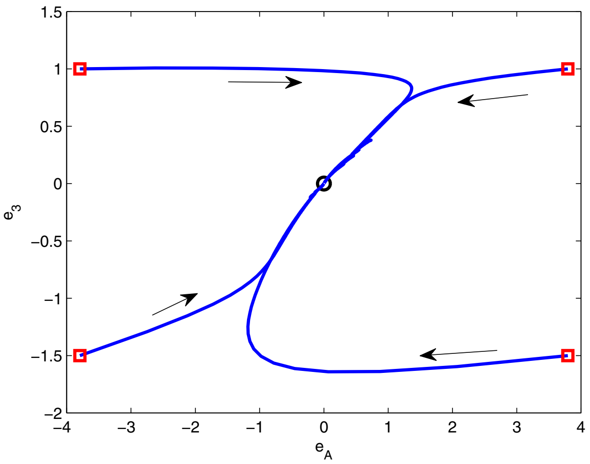

Fig. 1. Phase portrait for four initial conditions (red squares). The origin is marked as a black circle.

Since all errors converge to zero, one can conclude that this system is locally detectable with unknown inputs. Due to space constraints, the complete and detailed proof is not presented here.

4. STATE AND INPUT ESTIMATION

We consider the nonlinear continuous-time model with discrete-time measurements (7a)-(7b)

$$\dot{x}=f(x,u,w)\qquad(7a)$$
$$y_{[k]}=Cx_{[k]}+v_{[k]}\qquad(7b)$$

where $x\in\mathbb{R}^n$ is the state vector, $u\in\mathbb{R}^p$ is the nonmeasurable input vector, $y\in\mathbb{R}^m$ is the output (measurement) vector. We assume that w and $v_{[k]}$ are stationary zero-mean white noise processes with covariance matrices $\mathbb{Q}$ and R. In addition, we assume that x_0, w and $v_{[k]}$ are uncorrelated.

Auxiliar computations are the linearized model

$$\delta\dot{x}=\mathbb{A}(t)\,\delta x+\mathbb{B}(t)\,\delta u,$$

where $\mathbb{A}(t)=\frac{\partial f}{\partial x}\left(x(t),u_{[k|k]}\right)$ and $\mathbb{B}(t)=\frac{\partial f}{\partial u}\left(x(t),u_{[k|k]}\right)$, and its discretized version

$$\delta x_{[k+1]}=A_{[k]}\delta x_{[k]}+B_{[k]}\delta u_{[k]}$$

4.1 EKF for augmented system

A traditional formulation of an Extended Kalman Filter (EKF) [10] is the following

- Time Update (TU)
$$\dot{\hat{x}}=f(\hat{x},u)$$
$$\dot{P}=\mathbb{A}(t)\,P+P\mathbb{A}^T(t)+\mathbb{Q}$$
- Measurement Update (MU)
$$\hat{x}_{[k+1|k+1]}=\hat{x}_{[k+1|k]}+L_{[k+1]}\left(y_{[k+1]}-C\hat{x}_{[k+1|k]}\right)$$
$$L_{[k+1]}=P_{[k+1|k]}C^T\mathcal{R}$$
$$P_{[k+1|k+1]}=P_{[k+1|k]}-P_{[k+1|k]}C^T\mathcal{R}CP_{[k+1|k]}$$
where $\mathcal{R}=\left(CP_{[k+1|k]}C^T+R\right)^{-1}$

When it is not possible to measure the input u, a conventional KF (or EKF) can be proposed to an augmented system where a new state is defined $x_{n+1}=u$, where slow variant inputs are assumed and affected by a random disturbance so that $\dot{x}_{n+1}=w_{n+1}$ [5].

4.2 Continuous-discrete UIO

An UIO algorithm for this system has been proposed in [8], where the prediction is made using the original nonlinear model, starting from the last corrected state and, since u is unknown on the prediction interval, the last estimated value of this input is used instead. The propagation of the covariance matrix P, is the same as in the standard EKF.

The extended continuous-discrete UIO is given by the following equations

- Time Update
$$\dot{\hat{x}}=f\left(\hat{x},u_{[k|k]}\right)$$
$$\dot{P}=\mathbb{A}(t)\,P+P\mathbb{A}^T(t)+\mathbb{Q}$$
with $\mathbb{A}(t)=\frac{\partial f}{\partial x}\left(\hat{x}(t),u_{[k|k]}\right)$.
$$\hat{x}_{[k+1|k]}=\hat{x}((k+1)\,T)$$
$$P_{[k+1|k]}=P((k+1)\,T)$$

- Estimation of the Unknown Input

$$\bar{R}_{[k+1]} = C\bar{P}_{[k+1|k]}C^T + R$$
$$\bar{\mathcal{M}}_{[k+1]} = \left(F_{[k]}^T \bar{R}_{[k+1]}^{-1} F_{[k]}\right)^{-1} F_{[k]}^T \bar{R}_{[k+1]}^{-1}$$
$$\delta\hat{u}_{[k|k+1]} = \bar{\mathcal{M}}_{[k+1]}\left(y_{[k+1]} - C\hat{\bar{x}}_{[k+1|k]}\right)$$
$$P_{u[k|k+1]} = \left(F_{[k]}^T \bar{R}_{[k+1]}^{-1} F_{[k]}\right)^{-1}$$

- Measurement update

$$\begin{aligned}
\hat{\bar{x}}_{[k+1|k+1]} &= \hat{\bar{x}}_{[k+1|k]} + B_{[k]}\delta\hat{u}_{[k|k+1]} \\
\bar{L}_{[k+1]} &= \bar{P}_{[k+1|k]}C^T\bar{R}_{[k+1]}^{-1} \\
\bar{P}_{[k+1|k+1]} &= \bar{P}_{[k+1|k]} + B_{[k]}P_{u[k|k+1]}B_{[k]}^T - \\
&\quad -B_{[k]}P_{u[k|k+1]}F_{[k]}^T\bar{L}_{[k+1]}^T - \bar{L}_{[k+1]}F_{[k]}P_{u[k|k+1]}B_{[k]}^T \\
\hat{x}_{[k+1|k+1]} &= \hat{\bar{x}}_{[k+1|k+1]} + \bar{L}_{[k+1]}\left(y_{[k+1]} - C\hat{\bar{x}}_{[k+1|k+1]}\right) \\
P_{[k+1|k+1]} &= \bar{P}_{[k+1|k+1]} - \\
&\quad -\bar{L}_{[k+1]}\left(\bar{R}_{[k+1]} - F_{[k]}P_{u[k|k+1]}F_{[k]}^T\right)\bar{L}_{[k+1]}^T \\
\hat{u}_{[k+1|k+1]} &= \hat{u}_{[k|k]} + \delta\hat{u}_{[k|k+1]}
\end{aligned} \tag{8}$$

where $F_{[k]} = CB_{[k]}$.

In this application, the measurement F_{meth} is a nonlinear combination of states S_{vfa} and X_{meth}. Therefore, in order to perform the Estimation of the Unknown Input and the Measurement Update, a linearization must be achieved in order to express it as in (7b). Previous applications of this algorithm considered just the case where some states were measured.

5. COMPARISON OF ESTIMATION STRATEGIES FOR THE ADR

Nonlinear model (1) is considered with the numerical values of the parameters reported in [4].

Output F_{meth} (methane flow rate) is assumed to be measured once a day ($T = 1$) with a measurement error of $1.2\ [LCH_4/d]$.

True initial conditions are defined as $S_{bvs}(0) = 5.2155$, $S_{vfa}(0) = 1.0094$, $X_{acid}(0) = 1.3128$, $X_{meth}(0) = 0.3635$.

Known input F_{feed} is also defined as a constant of 55.

Unknown input $S_{vs_{in}}$ is defined as a step-wise function where

$$S_{vs_{in}} = \begin{cases} 30.2 & 0 \leq t \leq 70 \\ 40 & 70 < t \leq 110 \\ 50 & 110 < t \leq 200 \end{cases}$$

5.1 *EKF for augmented system*

The design parameters of the estimator are those of [5]:

- Matrix P_0 is defined such that $P_{0_{i,i}} = [0.01\hat{x}_i(0|0)]^2$
- Matrix R is defined as the measurement variance $R = \text{var}\left(F_{meth}\right) = 1.44$
- Matrix Q is defined such that $\mathbb{Q}_{i,i} = [0.0005 m_i \hat{x}_i(0|0)]^2$, with $\{m_i\} = \{10,\ 1,\ 1,\ 1,\ 10\}$.

The initial estimated state vector is

$$\hat{x}_i(0|0) = \left[\hat{S}_{bvs}(0), \hat{S}_{vfa}(0), \hat{X}_{acid}(0), \hat{X}_{meth}(0), \hat{S}_{vs_{in}}(0)\right]$$

since for this case, the unknown input is part of the estimated states.

Initial values are then $\hat{S}_{bvs}(0) = 5.9978$, $\hat{S}_{vfa}(0) = 1.1608$, $\hat{X}_{acid}(0) = 1.5097$, $\hat{X}_{meth}(0) = 0.4180$, $\hat{S}_{vs_{in}}(0) = 34.73$, i.e. 15% error with respect to the true initial values.

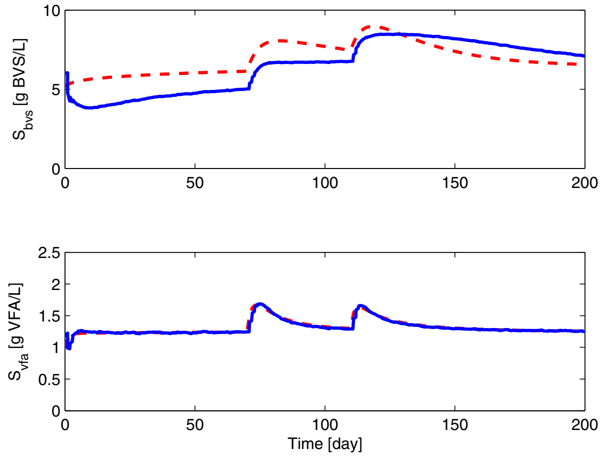

Fig. 2. Evolution of the real and estimated states S_{bvs}, S_{vfa}. Red dashed line: true values. Blue line: estimation.

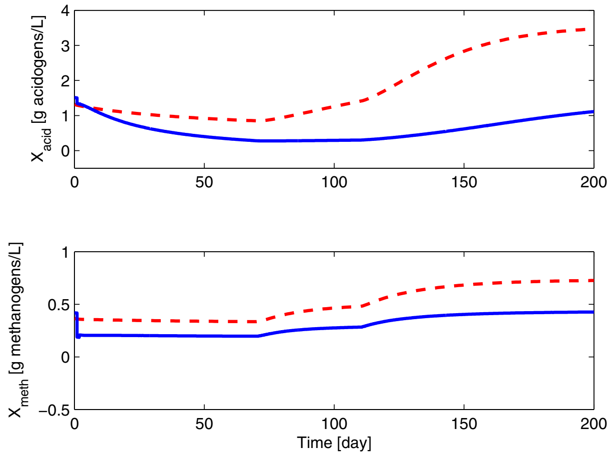

Fig. 3. Evolution of the real and estimated states X_{acid}, X_{meth}. Red dashed line: true values. Blue line: estimation.

Results are shown in Figures 2, 3 and 4, where the estimates are presented as blue solid lines and compared with the real value (red dashed lines).

As can be seen, the estimation error is relatively small for state S_{bvs}; the estimation of state S_{vfa} is almost perfect, but for states X_{acid} and X_{meth} the estimation error is large.

For the unknown input $S_{vs_{in}}$, the estimate follows the evolution of the process input, but never reaches it. In fact, the relative estimation error is around 30%.

5.2 *Continuous-discrete UIO*

In this case the initial estimated state vector is

$$\hat{x}_i(0|0) = \left[\hat{S}_{bvs}(0), \hat{S}_{vfa}(0), \hat{X}_{acid}(0), \hat{X}_{meth}(0)\right]$$

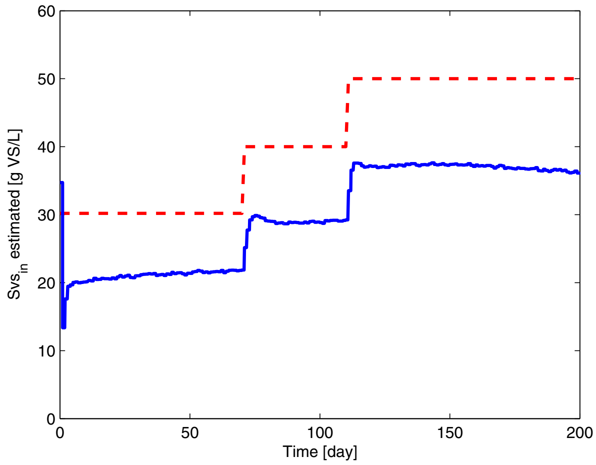

Fig. 4. Evolution of the real and estimated input $S_{vs_{in}}$. Red dashed line: true values. Blue line: estimation.

where initial conditions are set with 50% error in all the states, that is $\hat{S}_{bvs}(0) = 7.8233$, $\hat{S}_{vfa}(0) = 1.5141$, $\hat{X}_{acid}(0) = 1.9692$, $\hat{X}_{meth}(0) = 0.5453$.

The initial condition for the estimation of the input is $\hat{S}_{vs_{in}}(0) = 45.3$, as well 50% error with respect to the true value.

The design parameters of the estimator are $P_0 = \mathbb{I}_4$, where $\mathbb{I}_4$ represents an identity matrix of dimension 4×4, $\mathbb{Q} = 1 \times 10^{-4}\mathbb{I}_3$, matrix R is defined as $R = 1.44$.

In Figures 5 and 6, the state estimates are shown as blue solid lines and are compared with the real values of each variable (red dashed lines).

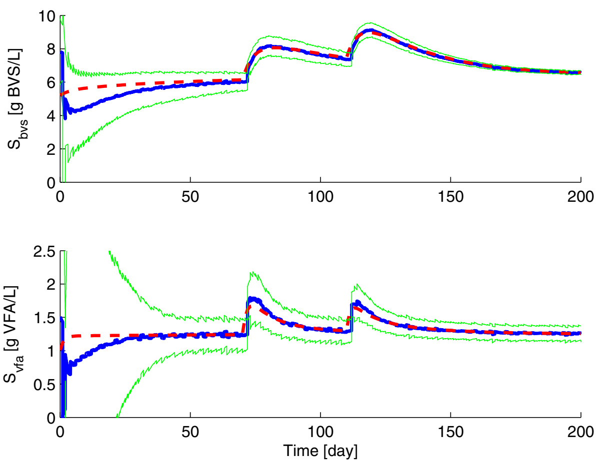

Fig. 5. Evolution of the real and estimated states S_{bvs}, S_{vfa}. Red dashed lines: true values. Blue line: estimation. Green lines: confidence intervals

In Figure 7, the input estimate is shown as a solid blue line and is compared with the real value (red dashed line). The estimation error for the unknown input is shown in Figure 8.

The filter convergence is fast and the general performance is quite satisfactory. 95 % confidence intervals are also drawn

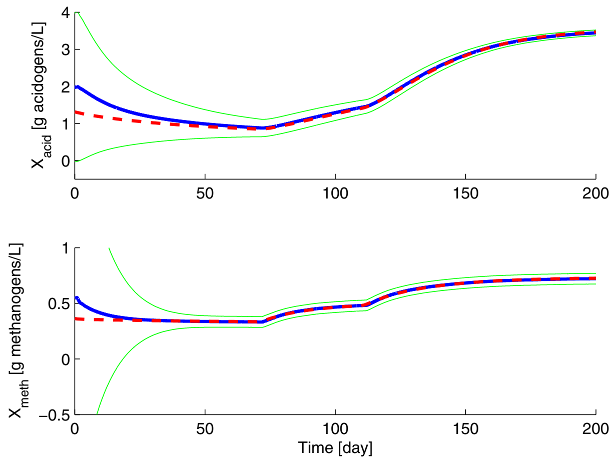

Fig. 6. Evolution of the real and estimated states X_{acid}, X_{meth}. Red dashed lines: true values. Blue line: estimation. Green lines: confidence intervals

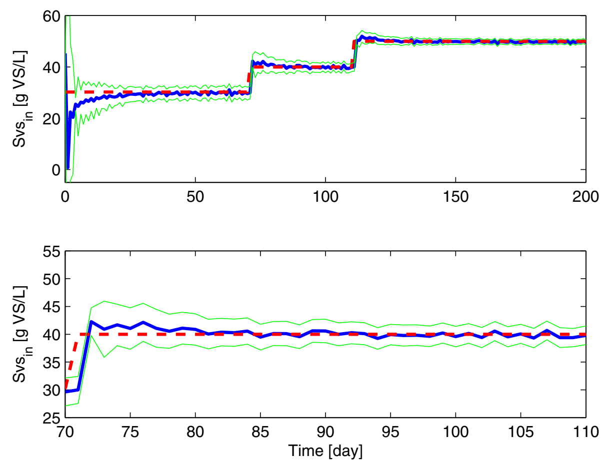

Fig. 7. Evolution of the real and estimated input $S_{vs_{in}}$. Red dashed lines: true values. Blue line: estimation. Green lines: confidence intervals

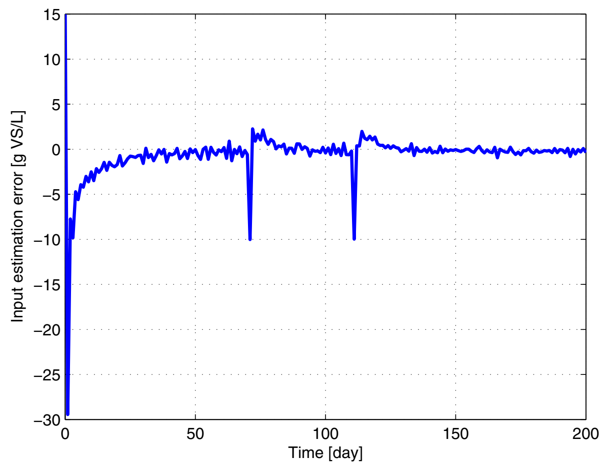

Fig. 8. Evolution of the estimation error of input $S_{vs_{in}}$

(green lines) using the information provided by the matrices P and P_u showing the reliability of the estimates.

Even tough the UIO test is more severe (the initial conditions of the UIO are further away from the true values, as compared to the test of the EKF), the estimation of all states is more satisfactory than in the case of the EKF estimator. The most valuable result is that states X_{acid} and X_{meth} are clearly better estimated.

For the unknown input, one can see that after 30 days, the estimation error is less than ± 1.5 $[gVS/L]$. Only when the value of $S_{vs_{in}}$ changes the estimation error becomes transiently larger. In Figure 7 ,one can see a detail of the input estimation from 70 to 110 days; a small overshoot is observed in the transition between $S_{vs_{in}}$ values, but after 10 days the estimation has almost reached the true value.

The average relative estimation error is 3.14%, which is significantly less than the 30% obtained with the EKF.

The general performance is better in the case of the UIO without great changes in the programming, that is, both algorithms are Kalman-filter like. By including a part of the algorithm specialized in the estimation of the input and then using its update value in the update estimation of the states, the results become clearly better. In the case of the EKF the unknown input receives no special treatment, one just imposes null dynamics for it. That is why one can then expect that for other kind of inputs (that varies faster), the performance of the UIO will be even better. An open question is how fast can be the unknown inputs, since there are linearization procedures throughout the whole algorithm that can affect the estimation.

6. CONCLUSIONS AND FUTURE WORK

In this study, attention is focused on the design of unknown input observers for monitoring the AD process, and in particular filters optimal in a minimum-variance unbiased sense. In the AD process, the measurement is a nonlinear combination of two states, which was not the case of previous applications of this algorithm and therefore considered as a challenge to test it.

The inherent robustness properties of the unknown input filter is clearly apparent in our numerical tests, which demonstrate the performance superiority over the classical EKF implemented for an augmented system, without making the algorithm too complex.

As future work, both algorithms will be tested with experimental data in order to have a complete comparison between the two estimation schemes.

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