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Experimental Evaluation of a MIMO Adaptive Dual MPC

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Abstract: Maintaining uniformly satisfactory control performance of an MPC scheme in the face of changing operating conditions is a difficult task. An adaptive MPC scheme that directs the output towards a reference and simultaneously injects a probing signal to get more information about the system for better model identification appears to be ideally suited for achieving this goal. In this work, taking motivation from the dual control problem originally developed by Feldbaum [1960], a MIMO adaptive dual MPC (adaptive DMPC) formulation has been proposed, which does not require external probing signals to improve the model parameter estimates. The objective function of the MPC is modified to include terms that ensure that sufficient excitation is injected into the system while performing the control tasks. The efficacy of the proposed adaptive DMPC formulation is evaluated by conducting experimental studies on the benchmark heater mixer system. The experimental results demonstrate that the proposed formulation is able to inject probing inputs of small magnitude while meeting the desired servo and regulatory control objectives.

Keywords: Dual control, MPC, adaptive control, parameter estimation, system identification.

1. INTRODUCTION

Over the last three decades, model predictive control (MPC) has emerged as the most effective multivariable control scheme and has been used to control a wide variety of processes (Qin and Badgwell [2003]). The quality of the model has a large effect on the closed loop performance of a linear MPC scheme. Maintaining a high quality model so as to achieve good control performance in the face of changing operating conditions poses a difficult challenge in the process industry. This problem has been dealt with mainly by (a) incorporating robustness in the controller design (b) employing multiple model-based controller designs and (c) updating the parameters of the linear prediction model either intermittently or on-line (Morari and Lee [1999]). While (c) appears to be an attractive option, due to various constraints, such as the time required for model identification and the cost associated with the model identification exercise, the model updates are carried out infrequently. If it is desired to update model parameters online, then a variety of recursive least square algorithms are available in the system identification literature (Soderstrom and Stoica [2001], Åström and Wittenmark [2008]). These approaches, though extensively studied in the system identification and adaptive control literature, have not received the attention they deserve in the industrial applications of MPC. Qin and Badgwell [2003], in their review of industrial MPC, noticed that only two adaptive MPC algorithms had reached the marketplace by 2003 despite strong market incentive for self-tuning MPC.

The observed lack of interest in employing these AMPC formulations on industrial systems can be attributed to the reliability of on-line parameter estimation schemes, which are at the heart of any AMPC strategy. Ydstie [1997] has indicated that *instability of the parameter estimator* or the *parameter drift* is an important issue that needs to be addressed while developing an adaptive control scheme. This drift can be avoided by adding deliberate perturbations to the manipulated inputs.

In the present work we focus on maintaining high quality parameter estimates by deliberately letting manipulated inputs perturb the plant. The persistent excitation guarantees the convergence of the parameter estimates. However, in practice, the perturbation signal are often chosen through some heuristic means and this can lead to excessive excitation. Ideally, an optimal controller must direct the output towards a reference and simultaneously inject a probing signal to get more information about the system for improved model identification, so that better control can be achieved in the future. This type of controller is referred to as a dual controller. In the dual control formulation, external persistent excitation is not required because the controller itself optimally excites the process when needed. The concept of dual control was first introduced by Feldbaum [1960] as a result of an attempt to formulate optimal control problems which would give an adaptive control law. This dual character of the control law refers to the two tasks of directing the output towards specified values and investigating the plant for learning. That is, the controller finds a balance between control and excitation. A dual controller optimally probes the system when the model is poor, which generates sufficient excitation to improve the model and, in turn, the closed loop performance.

Larsson et al. [2013] developed an MPC that experiments with the plant for identification processes while simultaneously controlling the plant. The excitation is introduced through a constraint on the predicted information matrix. A similar MPC with dual features was developed by Marafioti et al. [2013]. The excitation is here guaranteed by requiring that the first element of the open-loop optimal input sequence be persistently exciting. Žáčeková et al. [2013] suggested an approach where a standard MPC problem is solved first, followed by a procedure for finding an optimal perturbation to the nominally optimal input so that the resulting control increases the minimal eigenvalue of the information matrix.

Adaptive MPC with dual control features has emerged as an attractive approach to the problem of control loop performance degradation due to model plant mismatch. Recently, Heirung et al. [2012, 2013] developed a dual control formulation based on certainty-equivalence adaptive MPC. In their most recent approach the first stage cost is reformulated from a stochastic expression into a deterministic one. The same expression is used for the next stage cost in order to ensure that excitation is rewarded by the controller. The result is a level of excitation that excites the process enough to improve the quality of the parameter estimates and thereby improves closed loop performance. However, they considered only single-input single-output (SISO) systems subjected to zero-mean white disturbances; an extension to the more general MIMO case is not obvious. Since the ability to handle multivariable systems are among the main advantages of MPC in industrial applications, we here extend the formulation proposed by Heirung et al. [2013] to deal with multiple input multiple output (MIMO) systems subjected to colored unmeasured disturbances. Our proposed extension is based on multiple MISO ARMAX models, which are better suited for colored disturbances. The efficacy of the proposed adaptive dual MPC is evaluated through experimental studies on a heater-mixer system (Thornhill et al. [2008]).

This paper is organized into four sections. In the next section, development of the proposed AMPC formulation motivated by the dual control approach is presented. The analysis of the experimental results is presented in Section 3. The major conclusions reached from the analysis are summarized in Section 4.

2. A DUAL CONTROL APPROACH TO ADAPTIVE MPC

In this section we extend the MPC-based approach to dual control proposed by Heirung et al. [2013] to handle MIMO systems subjected to unmeasured stochastic disturbances. Heirung et al. [2013] used an ARX model to formulate an algorithm for adaptive dual MPC for SISO systems. The proposed extension can in principle be carried out using MISO or MIMO versions of the ARX model, but the conventional ARX models have certain limitations when the system under consideration is subjected to colored unmeasured disturbances (Soderstrom and Stoica [2001]). For example, consider a system governed by a SISO ARMAX model of the form

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})}u(k) + \frac{C(q^{-1})}{A(q^{-1})}e(k)$$

where $e(k)$ is a zero mean white noise sequence and $C(q^{-1})$ has all roots inside the unit circle. An equivalent ARX model for this system can be obtained by rearranging the ARMAX model as

$$y(k) = \frac{B(q^{-1})/C(q^{-1})}{A(q^{-1})/C(q^{-1})}u(k) + \frac{1}{A(q^{-1})/C(q^{-1})}e(k)$$

and by truncating $\widetilde{A}(q^{-1}) = A(q^{-1})/C(q^{-1})$ and $\widetilde{B}(q^{-1}) = B(q^{-1})/C(q^{-1})$ after the coefficients of $\widetilde{A}(q^{-1})$ and $\widetilde{B}(q^{-1})$ become insignificant. The truncation order depends on the locations of the roots of $C(q^{-1})$. If $C(q^{-1})$ has root(s) close to the unit circle, then an ARX model of high order is needed to adequately capture the noise dynamics. Thus, even for a SISO system, it may be necessary to estimate a relatively large number of model parameters to adequately capture the system dynamics when an ARX structure is used to model a system subjected to colored noise. This difficulty is further compounded for a MIMO system. A model with a large number of parameters can lead to difficulties with on-line parameter estimation as a large data set is needed to keep the variance errors small (Muddu et al. [2009]). In other words, the plant needs to be perturbed for a longer time to estimate the parameters accurately. From the viewpoint of parsimony of model parameters, a better option is to employ MISO models with either ARMAX or Box-Jenkins (BJ) structure (ref. Ljung [1999]). In this work we propose to capture the system dynamics using an ARMAX model structure to keep the development simple.

2.1 ARMAX Models and On-line Parameter Estimation

Consider a MIMO system with r outputs and m manipulated inputs. The system under consideration is assumed to have a local linear approximation in the neighborhood of a desired operating point such that the approximate linear model is stably invertible. We propose to model the system as r MISO ARMAX models of the form

$$A_i(q^{-1})y_i(k) = \sum_{j=1}^{m} B_{ij}(q^{-1})u_j(k) + C_i(q^{-1})e_i(k) \quad (1)$$

where $i = 1, ..., r$. Here, $A_i(q^{-1})$, $B_{ij}(q^{-1})$ and $C_i(q^{-1})$ are polynomials in the backward shift operator q^{-1} and $\{e_i(k)\}$ represents a zero mean white noise sequence. To simplify the notation, the index i is dropped in this subsection and the i'th MISO model is represented as

$$A(q^{-1})y(k) = \sum_{j=1}^{m} B_j(q^{-1})u_j(k) + C(q^{-1})e(k) \quad (2)$$

For the purpose of online parameter estimation, this model can be expressed as

$$y(k) = \boldsymbol{\phi}^T(k-1)\boldsymbol{\theta}(k-1) + e(k) \quad (3)$$

where

$$\boldsymbol{\theta} = [a_1, ..., a_n, b_{11}, ..., b_{1n}, ..., b_{m1}, .., b_{mn}, c_1, ..., c_n]^T \quad (4)$$

is a vector containing the MISO model parameters and

$$\boldsymbol{\phi}(k-1) = [-y(k-1), ..., -y(k-n),\ u_1(k-1), \\ ..., u_m(k-n),\ e(k-1), ..., e(k-n)]^T \quad (5)$$

represents the regressor vector, which consists of inputs, outputs, and noise inputs from the past.

Remark 1. For the sake of simplifying the notation we assume that the orders of $A(q^{-1})$, $B_j(q^{-1})$, and $C(q^{-1})$ are equal (cf. equation (4)). However, the orders of the $B_j(q^{-1})$ and $C(q^{-1})$ polynomials may in general differ, and they can be of any (positive) order less than or equal to the order of $A(q^{-1})$.

A difficulty in using (5) for recursive parameter estimation is that the noise sequence $\{e(k)\}$ is unknown. This difficulty can be alleviated if we employ the extended least square (ELS) approach (also known as pseudo-linear regression or approximate ML method) for the model parameter estimation (Soderstrom and Stoica [2001], Åström and Wittenmark [2008]). In this approach, $e(k)$ is replaced by the estimated prediction error. Thus, the regressor vector is in the ELS approach modified to

$$\boldsymbol{\varphi}(k-1) = [-y(k-1), ..., -y(k-n),\ u_1(k-1), \\ ..., u_m(k-n),\ \varepsilon(k-1), ..., \varepsilon(k-n)]^T \quad (6)$$

where $e(k-i)$ has been replaced by $\varepsilon(k-i)$, which is the past innovations. Here, the innovation $\varepsilon(k)$ at instant k is

$$\varepsilon(k) = y(k) - \boldsymbol{\varphi}^T(k-1)\hat{\boldsymbol{\theta}}(k-1) \quad (7)$$

where $\hat{\boldsymbol{\theta}}(k-1)$ represents the parameter estimate obtained at instant $(k-1)$ using the ELS method. The ELS method can be summarized as

$$\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) + \mathbf{L}(k)\varepsilon(k) \quad (8a)$$

$$\mathbf{L}(k) = \mathbf{P}(k-1)\boldsymbol{\varphi}(k-1) \\ (\lambda + \boldsymbol{\varphi}^T(k-1)\mathbf{P}(k-1)\boldsymbol{\varphi}(k-1))^{-1} \quad (8b)$$

$$\mathbf{P}(k) = (I - \mathbf{L}(k)\boldsymbol{\varphi}^T(k-1))\mathbf{P}(k-1)/\lambda \quad (8c)$$

where $\mathbf{L}(k)$ represents the Kalman gain matrix, $\mathbf{P}(k)$ represents a matrix that is proportional to the covariance of the estimated parameters (referred to as the *covariance matrix* in the rest of the text) and λ with $0 < \lambda \leq 1$ represents the forgetting factor.

To carry out model identification, r MISO estimators are used in parallel. Thus, we obtain the estimates $\hat{\boldsymbol{\theta}}^{(i)}(k)$ and the corresponding covariance matrices $\mathbf{P}^{(i)}(k)$ for $i = 1, 2, ..., r$, which are then used in the proposed adaptive dual MPC (DMPC) formulation. Let $\mathcal{Y}(k)$ denote the set of inputs and outputs recorded up to time instant k; i.e.,

$$\mathcal{Y}(k) \equiv \{\mathbf{u}(k), \mathbf{u}(k-1), ..., \mathbf{y}(k), \mathbf{y}(k-1), ...\}$$

Based on the ELS parameter estimates, let the one step ahead prediction for the model be

$$\widehat{y}_i(k+1) = \mathbb{E}\{y_i(k+1)|\mathcal{Y}(k)\} = \left[\boldsymbol{\varphi}^{(i)}(k)\right]^T \hat{\boldsymbol{\theta}}^{(i)}(k)$$

Here, $\mathbb{E}(\cdot)$ represents the expectation operator. To facilitate the development of the dual controller we further assume that the ELS algorithm asymptotically generates unbiased (or consistent) estimates of the model parameters. A sufficient condition for the convergence of the pseudo-linear regression type methods for an ARMAX model (under the ideal conditions) can be found in Soderstrom and Stoica [2001]. The convergence of the ELS estimates to the true parameters implies that the innovation sequence $\{\varepsilon(k)\}$ asymptotically converges to $\{e(k)\}$. In practice, however, the model error may approach a very small value if the model order is chosen appropriately.

2.2 An Objective Function for Dual MPC

The objective for MPC-based dual control can be stated as finding the control sequence $\{\mathbf{u}(k), \mathbf{u}(k+1), ...\}$ that minimizes

$$J_\infty = \mathbb{E}\left\{\sum_{j=k+1}^{\infty}\left[\sum_{i=1}^{r} w_i E_i(j)^2 + \sum_{i=1}^{r}\mu_i u_i(j-1)^2\right]|\mathcal{Y}(k)\right\} \quad (9)$$

given data obtained up to time k, where

$$E_i(j) = r_i(j) - y_i(j) \quad (10)$$

Here, $w_i > 0$ and $\mu_i \geq 0$ are weighting parameters and $\mathbf{r}(j)$ represents the output reference or the setpoint vector at the future time instant j. Since we assume that the system is stably invertible we can set $\mu_i = 0$ for all i and work with the control objective

$$J_\infty = \mathbb{E}\left\{\sum_{j=k+1}^{\infty}\left[\sum_{i=1}^{r} w_i E_i(j)^2\right]|\mathcal{Y}(k)\right\} \quad (11)$$

The main difficulty in using this objective function is the lack of a model that can accurately predict future outputs $\mathbf{y}(k+j)$. Thus, the objective function needs to be reformulated for simultaneous probing and control. To achieve this we first rewrite the objective function as

$$J_\infty = \mathbb{E}\left\{\sum_{j=k+1}^{k+2}\left[\sum_{i=1}^{r} w_i E_i(j)^2\right]|\mathcal{Y}(k)\right\} + \\ \mathbb{E}\left\{\sum_{j=k+3}^{\infty}\left[\sum_{i=1}^{r} w_i E_i(j)^2\right]|\mathcal{Y}(k)\right\} \quad (12)$$

Now, consider the first term, $J_1 = \sum_{i=1}^{r} w_i J_{1i}$ where

$$J_{1i} = \mathbb{E}\left\{E_i(k+1)^2|\mathcal{Y}(k)\right\} \quad (13)$$

By adding and subtracting the model $\widehat{y}_i(k+1)$, we can rewrite equation (13) as

$$J_{1i} = \mathbb{E}\left\{(r_i(k+1) - \widehat{y}_i(k+1) + \delta y_i(k+1))^2|\mathcal{Y}(k)\right\} \quad (14)$$

$$\delta y_i(k+1) = \widehat{y}_i(k+1) - y_i(k+1) \quad (15)$$

Dropping the conditional expectation notation for the sake of simplicity and using the assumption that $\varepsilon_i(k) \to e_i(k)$ asymptotically, the term J_{1i} can be expressed as

$$J_{1i} = \mathbb{E}\left\{\begin{array}{l}(r_i(k+1) - \widehat{y}_i(k+1))^2 + (\delta y_i(k+1))^2 \\ -2(r_i(k+1) - \widehat{y}_i(k+1))(\delta y_i(k+1))\end{array}\right\} \quad (16)$$

where

$$\delta y_i(k+1) = \left[\boldsymbol{\varphi}^{(i)}(k)\right]^T \delta\boldsymbol{\theta}^{(i)}(k) - \varepsilon_i(k+1)$$

and $\delta\boldsymbol{\theta}^{(i)}(k) = \hat{\boldsymbol{\theta}}^{(i)}(k) - \boldsymbol{\theta}^{(i)}(k)$. The first term is deterministic and the third term is zero since the ELS algorithm is assumed to be an asymptotically unbiased estimator; i.e.,

$$\mathbb{E}\left[\delta\boldsymbol{\theta}^{(i)}(k)\right] = \overline{\mathbf{0}} \ \text{ and } \mathbb{E}\left[\varepsilon_i(k+1)\right] = 0$$

Since $\delta\boldsymbol{\theta}^{(i)}(k)$ and $\boldsymbol{\varepsilon}_i(k+1)$ are independent, using

$$\text{cov}\left[\delta\boldsymbol{\theta}^{(i)}(k)\right] = \mathbf{P}^{(i)}(k) \quad \text{and} \quad \text{cov}\left[\varepsilon_i(k)\right] = \sigma_i^2$$

we can write

$$J_{1i} = \left(r_i(k+1) - \left[\boldsymbol{\varphi}^{(i)}(k)\right]^T \hat{\boldsymbol{\theta}}^{(i)}(k)\right)^2 + \left[\boldsymbol{\varphi}^{(i)}(k)\right]^T \mathbf{P}^{(i)}(k)\boldsymbol{\varphi}^{(i)}(k) + \sigma_i^2$$

given $\mathcal{Y}(k)$. Note that σ_i is not a known quantity, but since it appears as a constant term in the objective function its value does not matter and can be treated as zero. To achieve the probing effect, the same reformulation that is used to approximate the cost function is used in the second stage; i.e.,

$$J_{2i} \simeq \left(r_i(k+2) - \left[\boldsymbol{\varphi}^{(i)}(k+1)\right]^T \boldsymbol{\theta}^{(i)}(k+1)\right)^2 + \left[\boldsymbol{\varphi}^{(i)}(k+1)\right]^T \mathbf{P}^{(i)}(k+1)\boldsymbol{\varphi}^{(i)}(k+1) + \sigma_i^2$$

We can further simplify J_∞ by truncating the infinite horizon to some finite number N. The modified approximate cost function can be expressed as

$$J_N \simeq \sum_{j=k}^{k+1}\sum_{i=1}^{r}\left[w_i\left(r_i(j+1) - \left[\boldsymbol{\varphi}^{(i)}(j)\right]^T \boldsymbol{\theta}^{(i)}(j)\right)^2 + w_i\left(\left[\boldsymbol{\varphi}^{(i)}(j)\right]^T \mathbf{P}^{(i)}(j)\boldsymbol{\varphi}^{(i)}(j) + \sigma_i^2\right)\right] + \mathbb{E}\left\{\sum_{j=k+2}^{k+N}\left[\sum_{i=1}^{r} w_i\left(E_i(j+1)\right)^2\right] |\mathcal{Y}(k)\right\}$$

Since we intend to use MPC, we further approximate the last term in J_N using the model predictions instead of expected values of the outputs, which yields a cost function

$$V_N = \sum_{j=k}^{k+1}\sum_{i=1}^{r}\left[w_i\left(r_i(j+1) - \left[\boldsymbol{\varphi}^{(i)}(j)\right]^T \boldsymbol{\theta}^{(i)}(j)\right)^2 + w_i\left(\left[\boldsymbol{\varphi}^{(i)}(j)\right]^T \mathbf{P}^{(i)}(j)\boldsymbol{\varphi}^{(i)}(j) + \sigma_i^2\right)\right] + \left\{\sum_{j=k+2}^{k+N}\left[\sum_{i=1}^{r} w_i\left(r_i(j+1) - \widehat{y}_i(j+1|k)\right)^2\right]\right\} \quad (17)$$

Note that the covariance matrices $\{\mathbf{P}^{(i)}(k+1) : i = 1, ..., r\}$ and future regressor vectors $\{\boldsymbol{\varphi}^{(i)}(k+1) : i = 1, ..., r\}$ are functions of $\mathbf{u}(k)$. As a consequence, the modified optimization objective rewards inputs that reduce the future covariance $\mathbf{P}^{(i)}(k+1)$. In other words, the controller injects inputs that improve the quality of the parameter estimates and thereby reduce the parameter uncertainty.

2.3 Output Prediction

In the proposed adaptive MPC formulation, the identified models are used for predicting future outputs. Consider a scenario at the k'th sampling instant, when given the N future inputs

$$\mathcal{U}_k \equiv \{\mathbf{u}(k|k), \mathbf{u}(k+1|k), ...\mathbf{u}(k+N-1|k)\}$$

we want to predict outputs over time window $[k+1, k+N]$. Since the future parameter vectors and future innovations are unavailable at time k we have to make further simplifying assumptions to carry out predictions using the proposed model.

- Given the information at time k, the expected value of the unknown parameters in model i is $\hat{\boldsymbol{\theta}}^{(i)}(k)$. Hence, the model outputs are predicted with
$$\hat{\boldsymbol{\theta}}^{(i)}(k+j|k) = \hat{\boldsymbol{\theta}}^{(i)}(k) \text{ for } j > 0 \text{ and for all } i \quad (18)$$
- Consistent with conventional MPC formulations, we assume the following for the future innovations for output prediction:
$$\varepsilon_i(k+j+1) = \varepsilon_i(k+j) \;\; \text{for} \;\; i = 1, 2, ..., r \quad (19)$$
where $j = 0, 1, ..., N-1$. However, a difficulty with this approach is that the sequence $\{\varepsilon_i(k)\}$ contains high frequency noise, which can lead to noisy predictions. Thus, to eliminate the effect of the high frequency noise on the predictions and limit the frequency range of the model plant mismatch, we use a unity gain innovation filter for each innovation sequence (Muddu et al. [2009]):
$$\varepsilon_{f,i}(k) = \alpha_i\varepsilon_{f,i}(k-1) + (1-\alpha_i)\varepsilon_i(k) \quad (20)$$
for $i = 1, ..., r$ and with $0 < \alpha_i < 1$ being tuning parameters. Thus, the future innovation terms in $\boldsymbol{\varphi}^{(i)}(k+j)$ are estimated as
$$\varepsilon_i(k+j|k) = \varepsilon_{f,i}(k) \text{ for } j > 0 \quad (21)$$

With the above simplifying assumptions, the predicted output for the i'th MISO ARMAX model at time $k+j$ can be expressed

$$\widehat{y}_i(k+j+1|k) = \left[\boldsymbol{\varphi}^{(i)}(k+j|k)\right]^T \widehat{\boldsymbol{\theta}}^{(i)}(k) \quad (22)$$

where the predicted regressor vector is

$$\widehat{\boldsymbol{\varphi}}^{(i)}(k+j|k) = [-\widehat{y}_i(k+j|k)... - \widehat{y}_i(k+1|k) \; - y_i(k)... - y_i(k+j-n+1) \; u_1(k+j|k).. ...u_m(k+j|k).....u_m(k+j-n) \; \varepsilon_i(k+j|k)...\varepsilon_i(k+j-n)]^T \quad (23)$$

for $j = 0, 1, ..., N-1$ and $i = 1, 2, ..., r$. Note that $\varepsilon_i(k+j)$ for $j \leq 0$ are available at instant k and are directly used in formulating $\boldsymbol{\varphi}^{(i)}(k+j|k)$.

2.4 Adaptive DMPC Formulation

Based on the modified cost function (17) and the proposed prediction model (22), an adaptive MPC scheme is proposed as follows

$$\min_{\mathcal{U}_k} V_N(k) = \sum_{j=k+1}^{k+N} E(j)^T \mathbf{W}_E E(j) + \sum_{j=k}^{k+1}\sum_{i=1}^{r} w_i \left[\boldsymbol{\varphi}^{(i)}(j|k)\right]^T \mathbf{P}^{(i)}(j)\boldsymbol{\varphi}^{(i)}(j|k) + \sum_{j=k+3}^{k+N} \boldsymbol{\Delta}\mathbf{u}^T(j) W_{\Delta u} \boldsymbol{\Delta}\mathbf{u}(j) \quad (24)$$

where $\boldsymbol{\Delta}\mathbf{u}(j) = \mathbf{u}(j) - \mathbf{u}(j-1)$, $E(j) = \mathbf{r}(j) - \widehat{\mathbf{y}}(j|k)$ and

$$\hat{y}_i(j|k) = \left[\boldsymbol{\varphi}^{(i)}(j-1|k)\right]^T \hat{\boldsymbol{\theta}}^{(i)}(k)$$

for $i = 1, ..., r$, subject to the following constraints

$$\mathbf{P}^{(i)}(k+1) = \left[\mathbf{I} - \mathbf{L}_i(k+1)\boldsymbol{\varphi}^{(i)}(k))\right]\mathbf{P}^{(i)}(k)$$

$$\mathbf{L}_i(k+1) = \mathbf{P}^{(i)}(k)\boldsymbol{\varphi}^{(i)}(k)\times \left[1+\boldsymbol{\varphi}^{(i)}(k)^T\mathbf{P}^{(i)}(k)\boldsymbol{\varphi}^{(i)}(k)\right]^{-1}$$

$$\text{for } i = 1, 2, ..., r$$

$$\Delta\mathbf{u}_{min} \leq \Delta\mathbf{u}(j) \leq \Delta\mathbf{u}_{max}$$

$$\Delta\mathbf{u}(j) = \mathbf{0}, \text{ for } j = k+N_c, .., k+N-1$$

Here, N_c is the control horizon and $\mathbf{W}_E = diag[w_1, ..., w_r]$ is the tracking error weighting matrix. Note that we introduced an input-move suppression term with a corresponding tuning matrix $\mathbf{W}_{\Delta u} \geq 0$, and that this can be used to adjust the intensity of the probing effect. That is, the weighting matrix $\mathbf{W}_{\Delta u}$ can be used to counteract excessively large input changes that can otherwise occur when $\mathbf{P}^{(i)}(k)$ is large. Also note that the proposed adaptive DMPC formulation results in a constrained non-convex optimization problem that has to be solved with a nonlinear programming (NLP) solver.

3. EXPERIMENTAL EVALUATION

We now demonstrate an experimental evaluation of the proposed adaptive DMPC algorithm carried out using the benchmark Continuous Stirred Tank Heater (CSTH) system (Thornhill et al. [2008]) at the Automation Lab in the Chemical Engineering Department at I.I.T. Bombay.

3.1 Plant Description

The CSTH setup consists of two tanks in series as shown in Fig. 1. The cold water flow (F_1) from the reservoir is heated using a 4 kWH heating coil in Tank 1. The water level in Tank 1 remains constant and the hot water overflows to Tank 2 where it is mixed with cold water flow F_2. The water in Tank 2 can be heated using another 3.5 kHW heating coil. To make the system more complex and interactive, a recycle flow (F_R) is set up from the bottom of Tank 2 to Tank 1 using a metering pump. Cold water inflows to both the tanks can be manipulated using pneumatic control valves CV-1 and CV-2. Also, the heat input to both heaters can be manipulated using two thyristor power controller (TPC) systems, which are driven by 4-20 mA current inputs. From a control viewpoint the CSTH is a MIMO system with three manipulated inputs (4 to 20 mA current inputs to TPC 1 (u_4), TPC 2 (u_5), and to CV-2 (u_2)), and three controlled outputs (temperature in Tank 1 (T_1), temperature in Tank 2 (T_2) and water level in Tank 2 (h_2)). The current input to CV-1, which can be used to manipulate the cold water flow to Tank 1, and the temperature of the cold water inflows both act as unmeasured disturbances. This setup is controlled with a PC (with an Intel core i5 processor and 8 GB RAM) using a combination of LabView version 2012 and MATLAB. A sampling interval of 5 seconds is used in this work for carrying out identification and control studies.

In the experimental study, the level in the second tank (h_2) is maintained at 50 % (i.e., 20 cm) using a PI controller (k_c = 1.723, τ_I = 2 min), which manipulates current input to control valve (CV-2). The inputs to CV-1 and

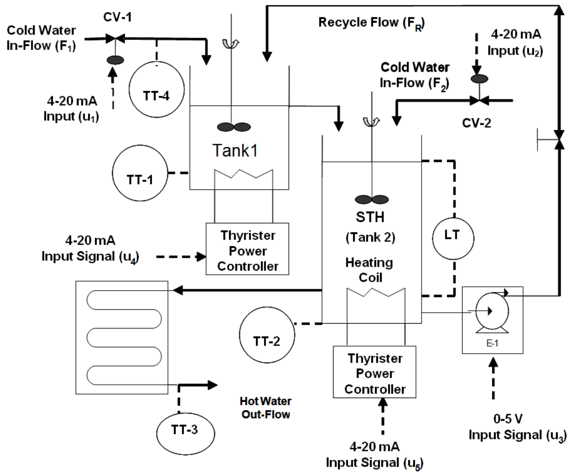

Fig. 1. Schematic diagram of CSTH.

the recycle flow metering pump are kept constant at 50 % levels. Thus, for the evaluation of the adaptive DMPC, the system is reduced to a 2×2 configuration with u_4 and u_5 as manipulated inputs and the tank temperatures (T_1 and T_2) as the controlled outputs.

3.2 Closed Loop Studies

Before implementing the proposed adaptive DMPC it is necessary to decide a suitable ARMAX model structure. We perturbed the CSTH system in open loop by simultaneously introducing low frequency pseudorandom binary sequences (PRBS) in the heating inputs to the tanks. We used the resulting data to identify an ARMAX model using the System Identification Toolbox in MATLAB. Using first order MISO ARMAX models were sufficient for ensuring that the innovation sequences $\{e_i(k)\}$ are white noise for each output. However, a minimum of 9'th and 12'th order MISO ARX models were needed to obtain white noise innovation sequences for T_1 and T_2, respectively. This may be attributed to fact that the C polynomials in the identified MISO ARMAX models have a pole close to 0.88.

We developed and implemented the adaptive DMPC on the CSTH system using two MISO second order ARMAX models. Each of these ARMAX models is of the form

$$(1+\sum_{i=1}^{2} a_i q^{-i})y(k) = \sum_{j=1}^{2}\left(\sum_{i=1}^{2} b_{ji}q^{-i}\right)u_j(k) + (1+\sum_{i=1}^{2} c_i q^{-i})e(k) \quad (26)$$

The tuning parameters used for the adaptive DMPC formulation are set to $N = 60$, $N_c = 6, \alpha_i = 0.9$ for all $i, \mathbf{W}_E = \mathbf{I}$ and $\mathbf{W}_{\Delta U} = diag\,[\,2\;\,1\,]$ and

$$\mathbf{u}_{\min} = [4\;\,4]^T,\; \mathbf{u}_{\max}\,[20\;\,20]^T$$

The initial model identified from the open loop data was used to initialize the parameter estimators and the initial covariance matrices were selected as $\mathbf{P}^{(1)}(0) = \mathbf{P}^{(2)}(0) = 10^4\mathbf{I}$. We deliberately set the initial covariances to high numbers and the adaptive DMPC was started when the parameter estimates stabilized and the covariances reduced significantly. The system was controlled using the

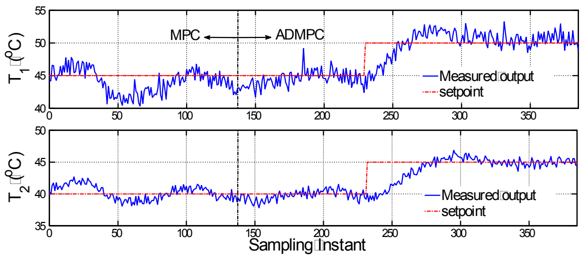

Fig. 2. CSTH Experiment : Setpoint Tracking.

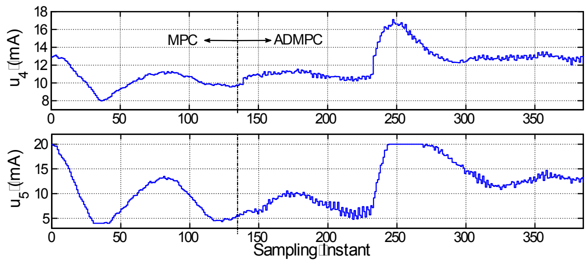

Fig. 3. CSTH Experiment : Manipulated Inputs.

conventional (non-adaptive) MPC that the initial model employed for predictions prior to starting the adaptive DMPC. The adaptive DMPC was implemented using the constrained NLP solver '*fmincon*' from the MATLAB Optimization Toolbox. Average computation time for the adaptive DMPC computations at each sampling instant was found to be 0.5482 seconds.

The closed loop experiments consist of (a) a sequence of positive and negative setpoint changes in both tank temperatures (a servo problem) and (b) a large magnitude step change in the cold water inflow to Tank 1 (a regulatory problem). Performance of the adaptive DMPC for the servo problem is presented in Fig.2 and the corresponding profiles of the manipulated inputs are presented in Fig.3. As shown in Fig.2, the controller is able to achieve quick transitions to the desired setpoint and settle the reference temperatures without any offset. The probing effect of the proposed adaptive DMPC formulation is visible in Fig.3, where time-varying low-amplitude perturbations are introduced after switching to adaptive DMPC from conventional MPC. These perturbations of varying intensity are continuously introduced throughout the experiment with adaptive DMPC. Since the high-frequency excitation may increase actuator wear, an operator may consider turning off the dual feature if it is deemed unnecessary based on some performance criterion. Since the high-frequency excitation may increase actuator wear, an operator may consider turning off the dual feature if it is deemed unnecessary based on some performance criterion. Note that the manipulated input profiles generated by the conventional MPC are smoother and without any such excitation.

4. CONCLUSION

In this work, we develop a MIMO adaptive DMPC using ARMAX models. The efficacy of the proposed control scheme is evaluated by conducting experimental studies on the benchmark heater-mixer setup. We show that despite the complexity of the algorithm, we are able to implement the controller for real-time control with a fairly standard implementation and achieve fast control input computation. Analysis of the experimental results reveals that if the tuning parameters are selected carefully, the proposed adaptive DMPC is able to inject input perturbations that are sufficient for maintaining the health of the on-line parameter estimators. When the system is operating at a fixed setpoint, these fluctuations are found to be of variable and low amplitudes, thereby introducing minimal disturbance in the plant operation. Though initial experimental studies have shown promising results, a number of issues remain to be resolved. The ARMAX structure leads to nonlinear parameter models and the ELS algorithm is a nonlinear estimator. Thus, alternate model structures that are parsimonious in parameters are currently being examined for cases in which the system is subjected to correlated unmeasured disturbances.

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