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Data-Driven Two-Dimensional LQG Benchmark Based Performance Assessment for Batch Processes under ILC [⋆]

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Abstract: A novel data-driven control performance assessment (CPA) method is proposed for batch processes controlled by iterative learning control (ILC) based on two-dimensional linear quadratic Gaussian (LQG) benchmark. Previous studies on CPA for ILC are based on an assumption that the model of the controlled batch process is known, whereas this study proposes a model-free CPA method. Based on the two-dimensional system theory, the closed-loop batch process under ILC can be converted into a two-dimensional Roesser model. This study proposes a novel closed-loop two-dimensional subspace identification method for the converted parameters unknown two-dimensional Roesser model. Using the identified model, the two-dimensional LQG tradeoff performance assessment surface can be obtained. The proposed method is verified by performing some simulations.

Keywords: batch process, iterative learning control (ILC), control performance assessment (CPA), LQG benchmark, two-dimensional subspace identification, Roesser model.

1. INTRODUCTION

As batch processes are widely applied in industrial processes, iterative learning control (ILC) has been developed rapidly and has been proved to be an effective control strategy for batch processes since it was first presented (Uchiyama (1978), Arimoto et al. (1984a)). With the improvement of ILC, various learning laws have been developed and the control performance has been improved e.g., P-, PI-, PD-, PID-type ILC (Saab (1994), Arimoto et al. (1984b), Wang et al. (2013), Madady (2013), Wang et al. (2009)). In recent years, it has been combined with other control algorithms to improve control performance, such as AILC (Tayebi (2004)) and L-MPC (Wang et al. (2010)). Batch processes under ILC inherently have two dynamic update directions, the iterative axis and the time axis, which shows that ILC systems have a two-dimensional (2-D) structure. Since 1990s, some scholars begun analyzing and designing the ILC based on 2-D system theory (Geng and Jamshidi (1990)); many similar studies have been conducted since then (Shi et al. (2005), Dabkowski et al. (2013)).

To ensure high efficiency of control systems, performance assessment techniques are applied to monitor performance degradation and identify potential improvements. To assess the performance of a control system, first, a benchmark should be chosen as a reference. There are many studies on control performance assessment based on different types of benchmarks, such as the minimum variance control (MVC) benchmark (Harris (1989)), linear quadratic Gaussian (LQG) benchmark (Huang and Shah (1999)) and other benchmarks (Yuan et al. (2009)). However, only a few studies have reported concerning CPA for ILC. Chen and Kong (Chen and Kong (2009)) used the MVC-based optimal ILC as the benchmark to assess the control performance of batch processes. This method estimated the minimum variance bounds and achievable bounds under the assumption that each ILC controller influenced either stochastic or deterministic control performance. Also based on the MVC law, Farasat and Huang (Farasat and Huang (2013)) suggested a new method for assessing the control performance. This method introduced a tradeoff between deterministic and stochastic control performance, as described by a tradeoff curve. However, because MVC is characterized by inordinate control moves and has poor robustness, the MVC benchmark is not desirable or achievable in many practical applications. Wei and Wang (Wei and Wang (2014)) proposed a method to assess the ILC performance based on a 2-D model transferred from an ILC-controlled batch process and designed a 2-D LQG benchmark for the transferred 2-D system, which extended the conventional performance assessment tradeoff curve to a novel tradeoff surface. All previous studies are based on the known model, but in practice, the process model is usually unknown. If the model of the batch process is unknown, a system identification algorithm should be used first for CPA.

Recently, identification of 2-D systems has attracted increasing interests.Due to the coupled structure of the two direction states in 2-D systems (Kaczorek (1985)), the research of 2-D systems is much more complex than that of 1-D systems, even the adaptability of standard identifica-

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tion methods from 1-D extension to 2-D systems has been very limited. In order to analysis conveniently and solve the general 2-D state-space system identification problem, a special format, the causal, recursive, and separable-in-denominator (CRSD) system in the Roesser form has been increasingly explored in the past decade. Fortunately, the ILC controlled batch process model can be converted into a CRSD Roesser form. It is Ramos (Ramos (1994)) who first applied the subspace identification approach to the problem of 2-D CRSD system identification directly from given 2-D input-output data. Four standard subspace-based system identification algorithms which utilize the two-dimensional data have been developed for the open-loop 2-D CRSD model by J.A. Ramos et al. (Ramos et al. (2011), Ramos and dos Santos (2011)).

The field of closed-loop 2-D CRSD system identification continues to require feedback control, and efficiently identifying a closed-loop 2-D CRSD model remains a challenging problem on its own. Contrary to the standard open-loop subspace-based algorithms (e.g., N4SID, MOESP, and CVA) (Qin (2006)) available in the literature until now, the fundamental assumption (Van Overschee and Moor (1996)) that there is no correlation between the unknown noise and the input no longer holds under the closed-loop condition. That is to say, the existing subspace methods yield biased solutions in closed-loop 2-D CRSD system identification, which requires special treatment.

This study proposed a novel data-driven CPA method for the ILC controlled batch processes. When the process model parameters are unknown, a novel closed-loop two-dimensional subspace identification method is proposed. Based on the identified model, a 2-D LQG benchmark can be used and a tradeoff surface can be obtained to assess the control performance of ILC controlled batch processes.

The remainder of this paper is arranged as follows. Section 2 describes the transferred model unknown 2-D ILC system. Section 3 introduces the modified subspace identification scheme. Section 4 discusses performance assessment of the model unknown ILC system. Sections 5 and 6 present the simulation results and the concluding remarks, respectively.

2. SYSTEM DESCRIPTION

Consider the following batch process described as a time-invariant state space model, which is unknown:

$$\begin{aligned} x_{r,s+1} &= A_0 x_{r,s} + B_0 u_{r,s} + w_{r,s}, \\ y_{r,s} &= C_0 x_{r,s} + v_{r,s}, \end{aligned} \tag{1}$$

where $s = 0, 1, 2, ..., T$, $r = 0, 1, 2, ...$; $x_{r,s} \in \mathbb{R}^n$ is the state vector, $u_{r,s} \in \mathbb{R}^m$ is the input vector, and $y_{r,s} \in \mathbb{R}^p$ is the output vector; A_0, B_0, and C_0 are real matrixes with unknown elements and appropriate dimensions. $w_{r,s}$ and $v_{r,s}$ are Gaussian white noise.

A general ILC updating law can be given as follows:

$$u_{r,s} = u_{r-1,s} + \varphi_{r,s}. \tag{2}$$

Define the tracking error:

$$e_{r,s} \triangleq y_s^{ref} - y_{r,s}, \tag{3}$$

and the notation:

$$\delta_R \xi_{r,s} \triangleq \xi_{r,s} - \xi_{r-1,s}, \ (\xi = x, u, y). \tag{4}$$

Based on (1)-(4), one can get the following equations:

$$\delta_R x_{r,s+1} = A_0 \delta_R x_{r,s} + B_0 \varphi_{r,s} + \delta_R w_{r,s}, \tag{5}$$

$$\begin{aligned} e_{r,s+1} = e_{r-1,s+1} &- C_0 A_0 \delta_R x_{r,s} - C_0 B_0 \varphi_{r,s} \\ &- C_0 \delta_R w_{r,s} - \delta_R v_{r,s}. \end{aligned} \tag{6}$$

Combining (5) and (6), a transferred 2-D system can be derived:

$$\begin{aligned} \begin{bmatrix} e_{r,s+1} \\ \delta_R x_{r,s+1} \end{bmatrix} &= \begin{bmatrix} \mathbf{I} & -C_0 A_0 \\ \mathbf{0} & A_0 \end{bmatrix} \begin{bmatrix} e_{r-1,s+1} \\ \delta_R x_{r,s} \end{bmatrix} \\ &+ \begin{bmatrix} -C_0 B_0 \\ B_0 \end{bmatrix} \varphi_{r,s} + \begin{bmatrix} -C_0 \\ \mathbf{I} \end{bmatrix} \delta_R w_{r,s} + \begin{bmatrix} -\mathbf{I} \\ \mathbf{0} \end{bmatrix} \delta_R v_{r,s+1}. \end{aligned} \tag{7}$$

The output equation can be described as follows:

$$y_{r,s}^{2d} = \begin{bmatrix} C_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \delta_R x_{r,s} \\ e_{r-1,s+1} \end{bmatrix} + \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \delta_R v_{r-1,s}. \tag{8}$$

By introducing the following notations:

$$x_{r,s}^h \triangleq e_{r-1,s+1}, \quad x_{r,s}^v \triangleq \delta_R x_{r,s}, \tag{9}$$

then (7) can be represented as a Roesser model.

$$\begin{aligned} \begin{bmatrix} x_{r+1,s}^h \\ x_{r,s+1}^v \end{bmatrix} &= A \begin{bmatrix} x_{r,s}^h \\ x_{r,s}^v \end{bmatrix} + B \varphi_{r,s} \\ &+ W \delta_R w_{r,s} + H \delta_R v_{r,s+1}. \end{aligned} \tag{10}$$

Notice that the lower left part of the matrix A is zero, a notable feature of the transferred model. In the 2-D Roesser system (10), designing the input signal $\varphi_{r,s}$ is equivalent to designing the updating law for the original ILC system, and performance assessment of the ILC system is transferred to assess the performance of the transferred 2-D system (10). Because the original ILC model parameters are unknown, the transferred 2-D model is unknown as well. Therefore, it should be identified before performance assessment.

3. CLOSED-LOOP SUBSPACE IDENTIFICATION FOR 2-D CRSD SYSTEM

3.1 Problem formulation and subspace equations

If the 2-D CRSD system is observable, one can turn the 2-D CRSD Roesser model to its process form described as following equivalent innovation representation:

$$\begin{aligned} \begin{bmatrix} x_{r+1,s}^h \\ x_{r,s+1}^v \end{bmatrix} &= \begin{bmatrix} A_1 & A_2 \\ 0 & A_4 \end{bmatrix} \begin{bmatrix} x_{r,s}^h \\ x_{r,s}^v \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u_{r,s} \\ &+ \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \theta_{r,s}, \end{aligned} \tag{11}$$

$$y_{r,s} = [\, C_1 \ \ C_2 \,] \begin{bmatrix} x_{r,s}^h \\ x_{r,s}^v \end{bmatrix} + D u_{r,s} + \theta_{r,s}. \tag{12}$$

The white noise vectors $\theta_{r,s}$ is the innovation sequence of the Kalman filter, and K_1 and K_2 are, respectively, the horizontal and vertical Kalman gains.

Referring to Ramos et al. (2011), one can derive the basic 2-D CRSD subspace matrix equations through the iterative substitution procedures of (11) and (12) as follows:

$$Y_f = \Gamma_i^h X_f^h + H_i^h U_f + K_i^h E_f \tag{13}$$

$$Y_p = \Gamma_i^h X_p^h + H_i^h U_p + K_i^h E_p \tag{14}$$
$$X_f^h = A_1^i X_p^h + \Delta_i^h U_\mathrm{p} + \nabla_i^h E_\mathrm{p}, \tag{15}$$

where p and f denote the past and future. h and v denote the horizontal and the vertical. The future and past output block-Hankel matrices are arranged as follows:

$$Y_f(k) = \begin{bmatrix} y_{i,k} & y_{i+1,k} & \cdots & y_{i+j-1,k} \\ y_{i+1,k} & y_{i+2,k} & \cdots & y_{i+j,k} \\ \cdots & \cdots & \ddots & \cdots \\ y_{2i-1,k} & y_{2i,k} & \cdots & y_{2i+j-2,k} \end{bmatrix} \in \mathbb{R}^{li\times j},$$
$$Y_f = \begin{bmatrix} Y_f(0) & Y_f(1) & \cdots & Y_f(M) \end{bmatrix} \in \mathbb{R}^{li\times j(M+1)},$$
$$Y_p(k) = \begin{bmatrix} y_{0,k} & y_{1,k} & \cdots & y_{j-1,k} \\ y_{1,k} & y_{2,k} & \cdots & y_{j,k} \\ \cdots & \cdots & \ddots & \cdots \\ y_{i-1,k} & y_{i,k} & \cdots & y_{i+j-2,k} \end{bmatrix} \in \mathbb{R}^{li\times j},$$
$$Y_p = \begin{bmatrix} Y_p(0) & Y_p(1) & \cdots & Y_p(M) \end{bmatrix} \in \mathbb{R}^{li\times j(M+1)},$$

where i and j are the tunable, user-defined parameters. It is worth noting that the row dimension of Y_f can be different with that of Y_p. The future and past deterministic input block-Hankel matrices are defined as follows:

$$U_f(k) = \begin{bmatrix} u_{i,k} & u_{i+1,k} & \cdots & u_{i+j-1,k} \\ u_{i+1,k} & u_{i+2,k} & \cdots & u_{i+j,k} \\ \cdots & \cdots & \ddots & \cdots \\ u_{2i-1,k} & u_{2i,k} & \cdots & u_{2i+j-2,k} \end{bmatrix} \in \mathbb{R}^{mi\times j},$$
$$U_p(k) = \begin{bmatrix} u_{0,k} & u_{1,k} & \cdots & u_{j-1,k} \\ u_{1,k} & u_{2,k} & \cdots & u_{j,k} \\ \cdots & \cdots & \ddots & \cdots \\ u_{i-1,k} & u_{i,k} & \cdots & u_{i+j-2,k} \end{bmatrix} \in \mathbb{R}^{mi\times j},$$
$$U_f = \begin{bmatrix} U_f(0) & U_f(1) & \cdots & U_f(M) \\ 0 & U_f(0) & \cdots & U_f(M-1) \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & U_f(0) \end{bmatrix} \in \mathbb{R}^{m(M+1)i\times j(M+1)},$$
$$U_p = \begin{bmatrix} U_p(0) & U_p(1) & \cdots & U_p(M) \\ 0 & U_p(0) & \cdots & U_p(M-1) \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & U_p(0) \end{bmatrix} \in \mathbb{R}^{m(M+1)i\times j(M+1)}.$$

The future and past innovation block-Hankel matrices E_f and E_p are defined conformably with U_f and U_p. The horizontal state sequences X_f^h and X_p^h are defined as follows:

$$X_f^h(k) = \begin{bmatrix} x_{i,k}^h & x_{i+1,k}^h & \cdots & x_{i+j-1,k}^h \end{bmatrix} \in \mathbb{R}^{n_h\times j},$$
$$X_f^h = \begin{bmatrix} X_f^h(0) & X_f^h(1) & \cdots & X_f^h(M) \end{bmatrix} \in \mathbb{R}^{n_h\times j(M+1)},$$
$$X_p^h(k) = \begin{bmatrix} x_{0,k}^h & x_{1,k}^h & \cdots & x_{j-1,k}^h \end{bmatrix} \in \mathbb{R}^{n_h\times j},$$
$$X_p^h = \begin{bmatrix} X_p^h(0) & X_p^h(1) & \cdots & X_p^h(M) \end{bmatrix} \in \mathbb{R}^{n_h\times j(M+1)}.$$

In (13) and (14), the extended horizontal observability matrix Γ_i^h and the related information matrices H_i^h and K_i^h are given by

$$\Gamma_i^h = \begin{bmatrix} C_1 \\ C_1A_1 \\ \cdots \\ C_1A_1^{i-1} \end{bmatrix} \in \mathbb{R}^{li\times n_h},$$
$$H_i^h = \begin{bmatrix} H_T^h & G_T^{hv}C^v \end{bmatrix}, K_i^h = \begin{bmatrix} K_T^h & G_T^{hv}K^v \end{bmatrix},$$

where the lower block triangular Toeplitz matrices H_T^h, G_T^{hv}, K_T^h and extended vertical controllability-like matrices C^v and K^v are defined as follows:

$$H_T^h = \begin{bmatrix} D & 0 & \cdots & 0 \\ C_1B_1 & D & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ C_1A_1^{i-2}B_1 & C_1A_1^{i-3}B_1 & \cdots & D \end{bmatrix} \in \mathbb{R}^{li\times mi},$$
$$G_T^{hv} = \begin{bmatrix} C_2 & 0 & \cdots & 0 \\ C_1A_2 & C_2 & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ C_1A_1^{i-2}A_2 & C_1A_1^{i-3}A_2 & \cdots & C_2 \end{bmatrix} \in \mathbb{R}^{li\times n_v i},$$
$$K_T^h = \begin{bmatrix} I & 0 & \cdots & 0 \\ C_1K_1 & I & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ C_1A_1^{i-2}K_1 & C_1A_1^{i-3}B_1 & \cdots & I \end{bmatrix} \in \mathbb{R}^{li\times li},$$
$$C^v = \underbrace{\begin{bmatrix} (I_i\otimes B_2) & (I_i\otimes A_4B_2) & \cdots & \left(I_i\otimes A_4^{M-1}B_2\right) \end{bmatrix}}_{\in \mathbb{R}^{n_v i\times mMi}},$$
$$K^v = \underbrace{\begin{bmatrix} (I_i\otimes K_2) & (I_i\otimes A_4K_2) & \cdots & \left(I_i\otimes A_4^{M-1}K_2\right) \end{bmatrix}}_{\in \mathbb{R}^{n_v i\times lMi}},$$

where I_i denotes an $(i\times i)$ identity matrix and $\otimes$ denotes the Kronecker matrix product.

3.2 2-D closed-loop identification with SIMPCA

The main idea of the subspace identification model via PCA (SIMPCA) relies on the parity space (Wang and Qin (2006)). The instrument SIMPCA for a closed-loop 2-D CRSD Roesser system shows its advantages, as well as its consistent model estimations under the EIV situation.

Moving the term containing U_f from right to the left side of (13), the structure that both input and output variables are in the same side can be achieved. To calculate the related information matrices, the unknown term E_f should be eliminated. Because E_f is independent of the past input U_p and output Y_p, the past data combination $W_p = \begin{bmatrix} Y_p \\ U_p \end{bmatrix}$ can be used as the orthogonal projection instrumental variable. Orthogonally projecting (13) onto W_p yields:

$$\begin{bmatrix} I & -H_i^h \end{bmatrix} W_f/W_p = \Gamma_i^h X_f^h/W_p + K_i^h E_f/W_p, \tag{16}$$

where $W_f = \begin{bmatrix} Y_f \\ U_f \end{bmatrix} \in R^{(li+m(M+1)i)\times j(M+1)}$ is the future data combination. The last term of (16) is zero. Considering this, (16) can be simplified to

$$\begin{bmatrix} I & -H_i^h \end{bmatrix} W_f/W_p = \Gamma_i^h X_f^h/W_p. \tag{17}$$

By pre-multiplying $\left(\Gamma_i^{h\perp}\right)^T$, the transpose of the orthogonal complement of Γ_i^h, (17) can be transformed to

$$\left(\Gamma_i^{h\perp}\right)^T \begin{bmatrix} I & -H_i^h \end{bmatrix} W_f/W_p = 0. \tag{18}$$

Define $Z = W_f / W_p$ and perform singular value decomposition of Z as follows:

$$Z = [\, U_1 \ U_2 \,] \begin{bmatrix} S_1 & \\ & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}.$$

The column space of $\left(\Gamma_i^{h\perp}\right)^T [\, I \ -H_i^h \,]$ is equal to the orthogonal column space of Z.

$$\left(\left(\Gamma_i^{h\perp}\right)^T [\, I \ -H_i^h \,]\right)^T = U_2 M, \tag{19}$$

where $M \in R^{(li-n_h)\times(li-n_h)}$ is any constant nonsingular matrix and $U_2 M$ can be partitioned into $\begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$. Therefore, (19) can be formulated as follows:

$$\begin{bmatrix} \Gamma_i^{h\perp} \\ -(H_i^h)^T \Gamma_i^{h\perp} \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}. \tag{20}$$

The 2-D subspace identification method proposed by Ramos cannot deal with the noise. However, the SIMPCA can deal with the noise in a 1-D structure. Consequently, combining the two methods, one can estimate the associated system matrices A_1, A_2, A_4, B_1, B_2, C_1, C_2, D with a similarity transformation from Γ_i^h and H_i^h. Readers are referred to Ramos et al. (2011), Ramos and dos Santos (2011) for details on the realization procedures.

4. PERFORMANCE ASSESSMENT

After obtaining the identified model, the mentioned performance assessment methods for ILC can be conducted. For convenience, the following notations are introduced:

$$X(i+1) \triangleq \begin{bmatrix} x_{r+1,s}^h \\ x_{r,s+1}^v \end{bmatrix}, X(i) \triangleq \begin{bmatrix} x_{r,s}^h \\ x_{r,s}^v \end{bmatrix}. \tag{21}$$

The identified 2-D system can be described as follows:

$$\begin{aligned} X(i+1) &= AX(i) + B\varphi(i) \\ &\quad + W\delta_R w_{r,s} + H\delta_R v_{r,s+1}, \end{aligned} \tag{22}$$

and the 2-D LQG cost function can be presented as follows:

$$J_{LQG} = E\left\{\sum_{i=1}^{M} X^T(i)\, QX(i) + \sum_{i=1}^{M} r^T(i) r(i)\right\} \tag{23}$$

where $Q = \begin{bmatrix} \lambda_1 I_{n\times n} & 0 \\ 0 & \lambda_2 I_{1\times 1} \end{bmatrix}$. Solve the 2-D LQG problem to obtain the optimal state feedback control law as follows:

$$\varphi(i) = -KX(i), \tag{24}$$

where

$$K = (B^T PB + I)^{-1} B^T PA. \tag{25}$$

Which presents the state feedback gain matrix. Matrix P can be obtained by solving the following Riccati equation:

$$P = A^T PA - A^T PB(B^T PB + I)^{-1} B^T PA + Q. \tag{26}$$

Then the identified 2-D system can be expressed as follows:

$$\begin{aligned} \begin{bmatrix} x_{r+1,s}^h \\ x_{r,s+1}^v \end{bmatrix} &= (A - BK) \begin{bmatrix} x_{r,s}^h \\ x_{r,s}^v \end{bmatrix} + W\delta_R w_{r,s} \\ &\quad + H\delta_R v_{r,s+1}. \end{aligned} \tag{27}$$

The cost function describes a tradeoff between state variance and input variance when minimizing the cost function. A 2-D surface can be used to describe the tradeoff. Before obtaining the 2-D surface, let us define:

$$\begin{aligned} \varphi_{lqg} &= trace\left\{Var[\varphi_{r,s}]\right\}, \\ {x^h}_{lqg} &= trace\left\{Var[x_{r,s}^h]\right\}, \\ {x^v}_{lqg} &= trace\left\{Var[x_{r,s}^v]\right\}. \end{aligned} \tag{28}$$

By varying λ_1, λ_2 and obtaining the values of ${x^h}_{lqg}$, ${x^v}_{lqg}$, and φ_{lqg}, a plot of ${x^h}_{lqg}$, ${x^v}_{lqg}$ and φ_{lqg} describes the optimal 2-D LQG benchmark surface (Fig. 1), which can be used to assess the performance of the identified 2-D system.

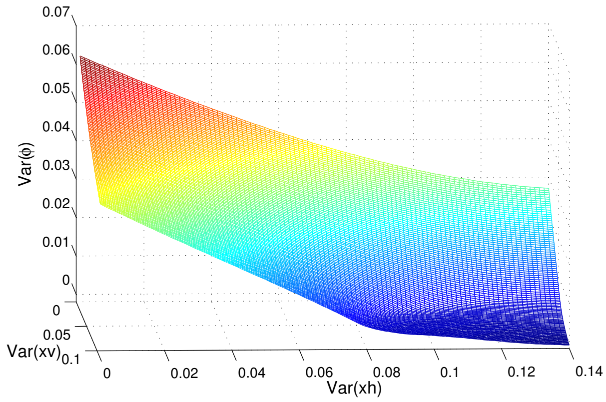

Fig. 1. 2-D performance assessment surface

The x-axis, $Var(x^h)$, presents the variance of the horizontal state. $Var(x^h)$ also refers to the variance of states in the original ILC system. Therefore, if the system converges faster, $Var(x^h)$ becomes smaller. The y-axis, $Var(x^v)$, presents the variance of the vertical state. It also refers to the variance of the tracking error in the original ILC system. As the tracking performance becomes better, $Var(x^v)$ becomes smaller. The z-axis, $Var(\varphi)$, presents the variance of the input signal in the 2-D system. $Var(\varphi)$ also means the variance of the updating law in the original ILC system. If the system costs less energy, $Var(\varphi)$ becomes smaller.

5. SIMULATIONS

5.1 Identification results

Here a linear batch process described as a time invariant state space model is considered

$$\begin{aligned} x_{r,s+1} &= A_0 x_{r,s} + B_0 u_{r,s} + w_{r,s}, \\ y_{r,s} &= C_0 x_{r,s} + v_{r,s}, \end{aligned} \tag{29}$$

where $A_0 = \begin{bmatrix} -0.8 & -0.22 \\ 1 & 0 \end{bmatrix}$, $B_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C_0 = [\,1 \ 0\,]$. Then, the corresponding 2-D CRSD Roesser system in its innovation representation can be formulated as follows:

$$\begin{aligned} \begin{bmatrix} x_{r+1,s}^h \\ x_{r,s+1}^v \end{bmatrix} &= \begin{bmatrix} I_{1\times 1} & -C_0 A_0 \\ 0_{2\times 1} & A_0 \end{bmatrix} \begin{bmatrix} x_{r,s}^h \\ x_{r,s}^v \end{bmatrix} + \begin{bmatrix} -C_0 B_0 \\ B_0 \end{bmatrix} \varphi_{r,s} \\ &\quad + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \theta_{r,s} \end{aligned}$$

$$y_{r,s} = \begin{bmatrix} I_{1\times1} & 0_{1\times2} \\ 0_{1\times1} & C_0 \end{bmatrix} \begin{bmatrix} x^h_{r,s} \\ x^v_{r,s} \end{bmatrix} + \theta_{r,s}$$

Thus, the 2-D CRSD model in this example is given by (11) and (12) with the following numerical values

$$A_1 = I_{1\times1} = [1]\,, A_2 = -C_0A_0 = [\,0.8\ 0.22\,]\,,$$

$$A_4 = A_0 = \begin{bmatrix} -0.8 & -0.22 \\ 1 & 0 \end{bmatrix},$$

$$A = \begin{bmatrix} A_1 & A_2 \\ 0 & A_4 \end{bmatrix} = \begin{bmatrix} 1 & 0.8 & 0.22 \\ 0 & -0.8 & -0.22 \\ 0 & 1 & 0 \end{bmatrix},$$

$$B_1 = -C_0B_0 = [-1]\,, B_2 = B_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0_{1\times1} \\ I_{1\times1} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_2 = \begin{bmatrix} C_0 \\ 0_{1\times2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C = [\,C_1\ C_2\,] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

To test the proposed data-driven method, the above-mentioned matrices are assumed to be unknown and are not used in the sequel. To verify the consistency and efficiency of the proposed methods for closed-loop 2-D CRSD system identification, a series of simulation tests were conducted. For illustration, 30 groups of Monte-Carlo tests were performed with a set of 2-D input-output data signals $\{\varphi_{r,s}, y_{r,s}\}$ for $r \in [\,0\ 30\,]$ and $s \in [\,0\ 100\,]$ taken in each test for implementation. User-defined parameters $i = 5$ and $j = 32$ were selected in these experiments. Finally, the identified 2-D CRSD system matrices were as follows:

$$\bar{A} = \begin{bmatrix} 1 & 0.0916 & 1.3618 \\ 0 & -0.0114 & 0.6529 \\ 0 & -0.3232 & -0.7886 \end{bmatrix}, \bar{B} = \begin{bmatrix} 1.7321 \\ 0.8163 \\ -1.0724 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} -0.5774 & 0.0000 & 0.0000 \\ 0 & 0.9656 & -0.1975 \end{bmatrix}$$

$$\bar{D} = 1.0\mathrm{e} - 014 * \begin{bmatrix} 0.1888 \\ 0.2284 \end{bmatrix}$$

Table 1. Eigenvalues of A and $\bar{A}$ along with performance index of mean square error

	True model	Identified model
α_1	1.0000	1.0000
α_2	-0.4000 + 0.2449i	-0.4000 + 0.2449i
α_3	-0.4000 - 0.2449i	-0.4000 - 0.2449i
MSE	–	0.1058

The mean value of the 2-D CRSD plant eigenvalues and mean square error (MSE) averaged from 30 runs are presented in Table 1. From previous results, it can be seen that consistent estimation and good accuracy are achieved by the proposed SIMPCA for the closed-loop 2-D CRSD models. Moreover, the tuning parameters i and j can be adjusted to improve the efficiency of the proposed closed-loop subspace identification method.

5.2 *Performance assessment cases*

After obtaining the identified 2-D model, the performance assessment surface can be obtained. Defining Q and R first, $Q = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}$, $R = 1$, and varying λ_1, λ_2, we solve the LQG problem to obtain different optimal state feedback gain matrix K. For the different K, there exists an optimal performance point. Based on these points, the performance assessment surface can be obtained. With the process input and output data, the current performance point of the original ILC system can be calculated. Then, the possibility of whether system performance can be improved is determined by using the performance assessment surface and the result is shown in Fig. 2.

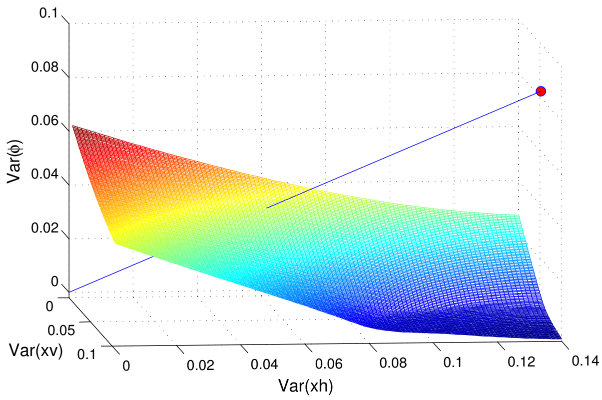

Fig. 2. Performance assessment for the original ILC system

Fig. 2. shows that the current performance point is located at the right top part above the surface, and there is considerable distance between the point and surface, which means the system will perform better by retuning parameter. Next for the sake of comparison, we use particle swarm optimization (PSO) algorithm to optimize the PID parameters of ILC controller. After retuning the parameters, the performance point can be obtained. The performance assessment result is shown in Fig. 3.

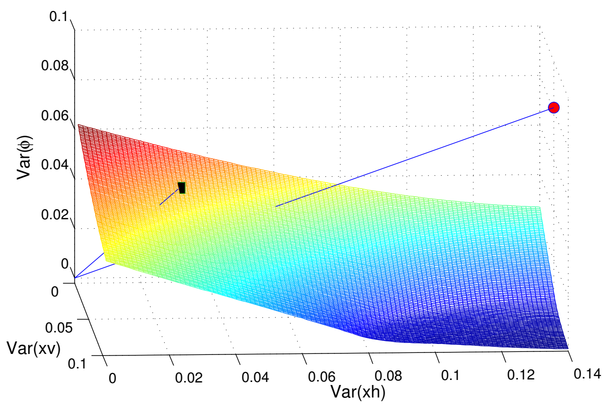

Fig. 3. Performance assessment of enhanced system. The black square point (0.0272, 0.0520, 0.0512) is the PSO enhanced ILC system performance point, and the red circle point is (0.1401, 0.0543, 0.0833).

Clearly, the PSO-enhanced ILC system performs better because its performance point is located closer to the surface. Along the $Var(xh)$ axis, the retuned performance point is smaller than the original performance point, which means that the retuned system converges faster. Along the $Var(\varphi)$ axis, the retuned system costs less energy than the original system. Along the $Var(xv)$ axis, the retuned performance point is bigger than the original performance point, which means the retuned system has worse tracking performance because of the tradeoff among tracking performance, system convergence performance and the input energy.

6. CONCLUSION

This study proposed a novel 2-D LQG benchmark based data-driven control performance assessment method for batch processes under ILC. As the process model parameters are unknown, this work converted the general ILC model into a two-dimensional Roesser model first. And then, this study proposed a novel closed-loop 2-D CRSD model subspace identification method for the transferred parameter unknown 2-D model. After identifying the 2-D model, an LQG benchmarkCbased ILC performance assessment method can be utilized. The proposed closed-loop 2-D CRSD model subspace identification method and the ILC performance assessment method are demonstrated via simulations.Actually, the proposed closed-loop 2-D subspace identification method only aims to the 2-D CRSD model, and a general closed-loop 2-D subspace identification will be our future research. The CPA for the ILC controlled batch processes with time delay will be considered in the future.

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