Production Optimization under Uncertainty - Applied to Petroleum Production

Kristian G. Hanssen^{*,**} Bjarne Foss^{*}

* Norwegian University of Science and Technology, Trondheim, Norway ** e-mail:kristian.gaustad.hanssen@itk.ntnu.no

Abstract: A key challenge in production optimization is handling of model uncertainty. Traditionally, production optimization is done in a deterministic setting, ignoring the uncertainty. In this work, we formulate the problem as a two-stage stochastic programming problem. The solution to the problem is a strategy for operating the wells, instead of a single setpoint obtained from the deterministic problem. This strategy is easy to follow for the operator. A synthetic case study shows how the proposed approach increases the expected oil production by 1.5 percent.

Keywords: Production optimization, uncertainty, stochastic programming, optimal control

1. INTRODUCTION

In the exploitation of oil and gas, Real Time Optimization (RTO) can be used to optimize the production. RTO is a widely studied topic, see Tosukhowong et al. (2004), and although no widely accepted formal definition of RTO exists, it is used to denote a workflow where some of the decision variables are optimized by the use of mathematical optimization. A control hierarchy is often structured in layers according to time scales. In the context of upstream production, this hierarchy is divided into the four layers, asset management, long-term reservoir management, production optimization in daily operation, and control and automation (Foss and Jensen, 2011). We will in this work focus on production optimization, where typical control inputs include production choke opening and gas-lift rates. However, this layer is closely linked to the other layers, especially reservoir management. An early reference in the context of petroleum production is Saputelli et al. (2003), and a later overview of RTO can be found in Bieker et al. (2007a). The remainder of this paper will focus on this application domain.

In RTO, a mathematical model is employed when optimizing the performance of the system. This model is used to predict the outcome when changing decision variables, e.g. a model may describe an oil well by predicting flow rate for various choke openings. However, the model may fail to accurately predict the outcome due to model uncertainty. For example when the model is based on recent production data, it will often be accurate in the region around the current operating point, but poor when evaluated further away from this operating point. Models used in production optimization and reservoir management are inherently uncertain. This is due to the complexity of the system, difficulty in modeling multiphase flow and sparsity of well tests. If special precautions are not taken, the solution to the optimization problem might be in a region where we do not trust the model, and the output might thus have to be disregarded. The model uncertainty challenge was articulated in Bieker et al. (2007a); "The handling of model uncertainty is a key challenge for the success of RTO".

Although models are uncertain, this is often neglected when solving RTO problems. The most common approach is to solve what is known as the expected value problem. That is using the expected value of the uncertain parameters, e.g. using the gas-oil-ratio (GOR) and water cut (WC) from the most recent well test of each well. Thus, the fact that these values are uncertain does not enter into the formulation of the optimization problem. For an unconstrained problem, this might still be a viable approach, although no guarantee of optimality can be given. For a constrained problem, however, this approach has some serious flaws. Consider the production optimization problem where the objective is to maximize oil production, subject to a constraint on the gas processing capacity. When the gas processing capacity is limiting the production, the solution to the optimization problem will be at the constraint, that is, the modeled gas flow at the solution will be equal to the capacity constraint. If the solution were to be implemented directly, there is a chance that the constraint will be violated, but also a chance for the constraint to be inactive, such that there is spare capacity left. This happens because of model uncertainty, where the actual response of the system deviate from the model output.

The operator will, in a petroleum production setting, adjust the controls iteratively in order to reach the suggested setpoint. Thus, when there are multiple wells, there are multiple paths in order to reach the setpoint. If the operator discovers that he can not reach it, meaning it

Acknowledgements: This paper was supported by the IO Center at the Norwegian University of Science and Technology - NTNU.

is infeasible, he might simply disregard it, and end up somewhere in between the prior operating condition and the suggested one. Thus, it is clear that the selected path will affect the outcome. However, this fact is actually not included in the optimization problem.

When solving the expected value problem, often denoted as the deterministic problem, we make the assumption that everything about the problem is perfectly known. We can then find a setpoint for all the control inputs, which will be optimal for the formulated problem. In this context, the solution will be feasible provided a feasible point exists. However, because of model uncertainty, there is a great chance the solution is unreachable. To overcome this, we can formulate the problem such that the solution will be feasible with a high probability. This can be done by using chance constraints, or in an even more conservative way, by applying a robust formulation. With chance constraints or a robust formulation, we can be quite certain that the solution will be feasible in practice, the drawback, however, is that the solution may be quite conservative.

In the deterministic world, where everything is known, it makes sense that the solution to the optimization problem is a setpoint for all the wells. In the real world, however, it is sensible to challenge this approach. Thus, in this paper, we propose a two-stage optimization formulation that defines an operational strategy rather than a single operating point. We also argue that such a strategy fits nicely into the mindset of operators.

We give a short overview of previous work in Section 2, before focusing on stochastic programming in Section 3. The mathematical formulation of our approach is given in Section 3.1. We then evaluate the approach on 3 different synthetic cases with increasing complexity in Section 4, ahead of a discussion of the results and conclusion in Section 5.

2. PREVIOUS WORK

There exists numerous publications on reservoir management under uncertainty, amongst others (van Essen et al., 2009; Chen et al., 2011). Uncertainty usually enters the reservoir optimization problems by the use of multiple realizations to span subsurface uncertainty. Published work does not, to the authors' knowledge, include capacity constraints, except for Chen et al. (2011), where they use a robust formulation to handle such constraints.

There are only a few published papers on short term production optimization under uncertainty. In Elgsæter et al. (2010), a structured approach for changing the setpoint when there is uncertainty is proposed. The uncertainty is, however, not considered in the optimization itself, only to assess the solution. To our knowledge, the only publication where the uncertainty is explicitly handled in the optimization problem is by Bieker et al. (2007b). They propose to formulate the optimization solution as a priority list between the wells. This list represents an operational strategy, thus whenever there is spare capacity or the opposite, the priority list is applied.

Although many deterministic formulations result in a single operating point, there are some methods which naturally extends to a strategy. The ideas of using incremental GOR for rate dependent wells in Urbanczyk and Wattenbarger (1994) and Barnes et al. (1990), can be thought of as strategies rather than providing specific operating points. However, these methods works for only one constraints, and are not easily extended for multiple constraints.

3. PRODUCTION OPTIMIZATION BY STOCHASTIC PROGRAMMING

A general deterministic optimization problem can be formulated as

$$\min J(x) \text{ s.t. } c(x) \le 0 \tag{1}$$

When the problem contains uncertainty, both the objective and the constraints can be dependent on a stochastic parameters, denoted ω . Thus, the objective and constraints are no longer deterministic, but rather stochastic variables. To compare two different stochastic objective functions, we must compare distributions instead of scalars. A natural approach is therefore to compare expected values. Methods emphasizing some quantile of the distribution is also typical. Since the short term production optimization problem is solved on a daily basis over many years, it is reasonable to use the expected value. For the reservoir optimization problem, however, a more conservative approach could be more reasonable.

While handling uncertainty in the objective boils down to assessing distributions instead of scalars, constraints are fundamentally different. The satisfaction of a constraint on average is often inadequate, while a robust formulation ensuring that the constraints hold with probability 1, can lead to an overly conservative solution. A middle of the road approach is to use chance constraints, formulating the problem as

$$\min_{x} \mathbb{E}[J(x,\omega)] \text{ s.t. } \Pr\{c(x,\omega) \le 0\} \ge \alpha$$
 (2)

so that the solution must hold with a predefined probability α . The solution of (2) will, however, with probability α have a margin to the constraint. In the production optimization problem with capacity constraints, this means there will probably be spare capacity when the solution is implemented. The operator might try to utilize this, but it is not included in the optimization problem. Thus, the final implemented operating point is dependent on the optimized solution and the operators implementation strategy.

Since the constraints are uncertain, it is not possible to provide the operator with a single setpoint for all the wells, such that it is feasible with probability 1, while at least one capacity constraint is active. However, we could specify a setpoint for each well, and in addition specify which well to turn up to utilize additional spare capacity. The initial setpoint should be feasible with a high probability. This is similar to how many fields are operated today, using a swing producer to utilize any spare capacity. The difference is that we include this information into the optimization problem itself, and the setpoint is calculated with awareness of this second phase. Which well to use as a swing producer will also be part of the solution to the optimization problem. Because of different well properties and dynamics, certain wells might be more suitable to use as swing producer. This can easily be included in the problem formulation by constraints.

If there is substantial uncertainty in the well models, it might be difficult to provide a setpoint with a high probability of being feasible, while guaranteeing that all spare capacity is utilized by the swing producer. It is then necessary to use multiple swing producers. The solution can then be a list of setpoints to the operator. At each step, the operator should turn up one well to a specified opening, until he reaches a capacity constraint, or, optionally, that all wells are fully open. The initial setpoint to each well must, however, be a feasible configuration.

The above approach can be seen as a generalization of the priority list by Bieker et al. (2007b). By restricting the initial setpoint such that all wells are closed, the last setpoint leaving all wells fully open, the number of steps equal the number of wells, and only one capacity constraint, the approaches are identical.

In the case of multiple capacity constraints, we assume that the operator will follow the list until one of the constraints becomes active, and stop there. We assume the operator can determine if a constraint is active or not and do not rely on measurements of the distance to a constraint, for instance measurements of total produced gas rate. If such measurements are reliable and available, it opens up for a wide range of on-line optimization and extremum seeking approaches, as it permits on-line identification. This would result in a complex operating strategy for the operator, while our approach yields a simple strategy.

3.1 Mathematical formulation

We can formulate our approach as a two stage-problem, by the use of Sample Average Approximation (SAA). The first stage decision is the strategy on how to operate the controls, while the second stage variables are the actual operating points for different realizations of the unknown parameters. Further, the second stage variables have to obey the strategy defined by the first stage variables. It is the first stage variables we are really interested in, but to evaluate the effect of them, we must consider a range of different scenarios.

We assume linear well models, and the decision variable of each well is a setpoint specifying an opening between 0 and 1, where 1 means it is fully open. The deterministic problem can then be modeled as

$$\max_{x^d} \sum_{i \in \mathcal{I}} c_i x_i^d \text{ s.t. } \sum_{i \in \mathcal{I}} a_{i,k} x_i^d \le b_k \ \forall \ k \in \mathcal{B}$$
(3)

where x_i^d is the setpoint for well *i*. Superscript *d* denotes deterministic. In general, subscript is used for denoting index sets, and superscript for description of variables or parameters. The sets are defined in Table 1. c_i and $a_{i,k}$ include the model parameters, calculated by using the mean of any uncertain well parameter. b_k defines the k'th capacity constraint.

For the stochastic formulation, the strategy is given by n^s steps. These steps are defined by $n^s + 1$ points. The first stage decision variables are then

$$0 \le x_{i,j} \le 1 \ \forall \ i \in \mathcal{I}, j \in \mathcal{J} \tag{4}$$

Table 1. Sets used in model

$\mathcal{I} = \{1, \dots, n^d\}$	set of wells
$\mathcal{J} = \{1, \dots, n^s + 1\}$	set of points
$\mathcal{S} = \{1, \dots, N\}$	set of realizations
$\mathcal{B} = \{1, \dots, n^c\}$	set of capacity constraints

where $x_{i,j}$ is the well opening of well *i* at point *j*. At each step, only one decision variable can be changed. This is handled by introducing the binary variable $y_{i,j}$, which is 1 if decision variable *i* can change in step *j*, from point *j* to j + 1. This is modeled by

 $x_{i,j}-y_{i,j} \leq x_{i,j+1} \leq x_{i,j}+y_{i,j} \forall i \in \mathcal{I}, j \in \mathcal{J} \setminus \{n^s+1\}$ (5) When a well can not be turned back at any step, the left inequality reduces to

$$x_{i,j} \le x_{i,j+1} \ \forall \ i \in \mathcal{I}, j \in \mathcal{J} \setminus \{n^s + 1\}$$
(6)

To ensure that only one well is adjusted at each step, we require

$$\sum_{i \in \mathcal{I}} y_{i,j} = 1 \ \forall \ j \in \mathcal{J} \setminus \{n^s + 1\}$$
(7)

The second stage variables must obey the strategy defined by the first stage variables, and are the operating point as if the parameters was known precisely. $z_{i,s}$ is the opening of well *i* in scenario *s*. To ensure that $z_{i,s}$ obeys the strategy defined by $x_{i,j}$, we introduce the binary variables $v_{j,s}$. $v_{j,s}$ is 1 when $z_{i,s}$ is on the line between $x_{i,j}$ and $x_{i,j+1}$. This can be imposed by

$$x_{i,j} - (1 - v_{j,s}) \le z_{i,s} \le x_{i,j+1} + (1 - v_{j,s})$$

$$\forall i \in \mathcal{I}, j \in \mathcal{J} \setminus \{n^s + 1\}, s \in \mathcal{S} \quad (8)$$

and only one $v_{i,s}$ can be nonzero for a scenario, thus

$$\sum_{j \in \mathcal{J} \setminus \{n^s + 1\}} v_{j,s} = 1 \ \forall \ s \in \mathcal{S}$$
(9)

Note that this only works when there is a single well changing in step j.

The capacity constraint must be satisfied for all second stage variables, so that

$$\sum_{i \in \mathcal{I}} a_{i,s,k} z_{i,s} \le b_{k,s} \ \forall \ s \in \mathcal{S}, k \in \mathcal{B}$$
(10)

where $b_{k,s}$ is the capacity constraint k of scenario s. It will typically be one constraint for each phase. $a_{i,s,k}$ describes how well i in scenario s influences constraint k.

We must also ensure that all previously visited points defined by the strategy are feasible. This can be done by

$$\sum_{i \in \mathcal{I}} a_{i,s,k} x_{i,j} \leq b_{k,s} + \left(\sum_{i \in \mathcal{I}} a_{i,s,k} - b_{k,s}\right) \left(1 - \sum_{t=j}^{n^s} v_{t,s}\right)$$
$$\forall \ j \in \mathcal{J}, s \in \mathcal{S}, k \in \mathcal{B} \quad (11)$$

However, when the setpoints are non-decreasing for all wells, and all elements of $a_{i,s,k}$ are non-negative, these constraints can be omitted.

The objective, which we want to maximize, is defined as

$$J = \frac{1}{N} \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}} c_{i,s} z_{i,s}$$
(12)

where $c_{i,s}$ is the oil potential of well *i* in scenario *s*.

4. CASE STUDY

The model formulated in section 3.1 is implemented in MATLAB using YALMIP (Löfberg, 2004), while CPLEX

Copyright © 2015 IFAC

Table 2.	Well	properties,	case	1
----------	------	-------------	------	---

	$(\mu^{GOR}, \sigma^{GOR})$	Oil potential [Sm3/d]
Well 1	(64, 20)	200
Well 2	(64.1, 0)	300

Table 3. Well properties, case 2 and 3

	$(\mu^{GOR}, \sigma^{GOR})$	(μ^{WC}, σ^{WC})	Oil potential [Sm3/d]
Well 1	(337, 22)	(0.47, 0.10)	518
Well 2	(400, 16)	(0.32, 0.06)	885
Well 3	(360, 24)	(0.45, 0.01)	662
Well 4	(397, 69)	(0.32, 0.08)	630
Well 5	(393, 17)	(0.58, 0.02)	545
Well 6	(336, 31)	(0.30, 0.02)	950
Well 7	(380, 22)	(0.32, 0.10)	253
Well 8	(353, 41)	(0.54, 0.05)	971

is used for solving the optimization problem. CPLEX is a state of the art MILP solver. The problem is solved in its *extensive form*, and no attempt to exploit the structure of the problem is made. Exploiting this structure is complicated because of the binary second stage variables and uncertainty in the recourse matrix. All calculations are performed on a quad-core laptop running at 2.5 GHz with 16 GB memory.

The case study consists of 3 synthetic cases. We start with 2 wells and 1 constraint, then extend this to 8 wells and 1 constraint, before we introduce a second constraint, ending up with 8 wells and 2 constraints. In all cases, we assume that there is uncertainty in the GOR and WC estimates. We assume they are given by normal distributions with known mean and variance, and no correlation between the GOR and WC estimates, or correlations at the well level. The oil potential and capacity constraints are assumed to be known precisely. In all cases, we are interested in comparing the stochastic approach to the deterministic solution. This is however not straight forward, since the deterministic solution is a single point, which might be infeasible. When there is only one constraint, there is a natural prioritizing of the wells. The deterministic solution is then evaluated by following this prioritizing, until a constraint is hit or all wells are fully open. However, there is no natural extension to multiple constraints.

For a real life application, great effort must be put into describing the uncertainty. In Elgsæter et al. (2008), bootstrapping is used for obtaining parameter and uncertainty estimates. The focus of this work is, however, solving the resulting optimization problem.

Case 1; 2 wells, 1 constraint: Two synthetic linear well models are defined by the properties in Table 2. The total gas processing capacity is set to 60% of the expected total gas when both wells are fully open. The stochastic problem is defined to be a single step, resulting in two points.

The deterministic solution is operated by turning up well 1 first, since it has a marginally lower expected GOR, before opening well 2.

Case 2; 8 wells, 1 constraint: 8 linear well models are defined by the properties in Table 2. The WC is not used in this case. Again, the deterministic solution is operated by prioritizing according to expected GOR. The total gas processing capacity is set to 60% of the expected total gas when all wells are fully open.

Case 3; 8 wells, 2 constraints: Case 2 is extended by also including a water handling capacity, set to 60% of the expected total water production when all wells are fully open. The uncertain WC properties are given by normal distribution and parameters are listed in Table 3.

4.1 Results

Because of the computational complexity, the stochastic problems are solved with a relatively small number of scenarios. However, all solutions are evaluated over 10 000 realizations generated from identical distributions, which we denote as the statistical ensemble.

The first point of the stochastic solutions will be feasible for all scenarios considered in the optimization problem. However, there is no guarantee it will be feasible for all scenarios in the statistical ensemble. To make a fair comparison, we extend the strategy from the stochastic solution, so that is turned up sequentially from the origin to the first point. This is done by the well numbering. Similarly, we extend the solution to sequentially set all wells to fully open after the last point.

Case 1: Figure 1 illustrates how the two wells are operated for the different strategies. The black lines shows the probability for the capacity constraint being satisfied. The deterministic solution is the single point plotted as a circle. The operational strategy, however, implies opening well 1 first and subsequently well 2 until production hits the capacity limit. The stochastic problem was defined by a single step, which is the horizontal part of the path, but as already explained, it is extended to make the comparison fair. The stochastic based strategy entails opening well 2 up to about 70% before opening well 1 until the capacity is fully exploited. Both strategies can therefor be seen as a path from fully closed to fully open wells.

In Figure 2, we observe a box plot of the actual well opening, evaluated on the statistical ensemble. The red line is the median, the edges of the box are 25th and 75th percentile, while the whiskers show the most extreme values. For the deterministic case, the red cross is the setpoint from the deterministic problem.

In Figure 3, we show the histogram of the total oil production for the different approaches. The mean for the deterministic approach is at 300 [Sm3/d], while it is 310 [Sm3/d] for the stochastic approach, which yields an improvement in expected oil production of about 3.3%.

The stochastic problem was solved with 100 realizations. The solver returned a proven global optimum in less than a second.

Case 2: In Figure 4 shows the histograms for the total oil production. The mean for the deterministic approach is 3411 [Sm3/d], while it is 3414 [Sm3/d] for the stochastic approach.

The stochastic problem was solved with 100 scenarios, using two steps. After one hour run time, the optimization was terminated with an optimality gap of 0.4%. The best integer feasible solution had not been improved during the latter 50 minutes.



Fig. 1. Illustrating the different strategies/paths for case 1



Fig. 2. Statistics of actual well opening for case 1



Fig. 3. Histogram of total oil production for case 1



Fig. 4. Histogram of total oil production for case 2

Case 3: When there are multiple constraints, there is no natural prioritization of the wells, when evaluating the deterministic solution. One option is to order by either GOR or WC, or some weighted average of these. A wide range of orderings were tested, and it turned out that ordering by WC seemed to be a reasonable choice. We include results based on GOR and WC priority, respectively, for the deterministic problem. Again, the stochastic problem was solved with 100 scenarios, using two steps. After one hour run time, the optimization was terminated with an optimality gap of 0.8%.

In Figure 5, we see a boxplot of the well openings for the different scenarios. This resulted in a mean total oil production of 3116 [Sm3/day] for the deterministic GOR priority approach, 3252 [Sm3/day] for the deterministic WC priority approach, and 3301 [Sm3/day] for the stochastic approach. This is an improvement of 1.51% or 5.94%, depending on the chosen baseline approach. In Figure 6, the histogram for total oil production is shown. To keep the figure readable, only the deterministic WC priority and stochastic approach are shown.

5. DISCUSSION AND CONCLUSION

In the small constructed case 1, we saw that we could achieve an increase in expected oil production of about 3.3%. This was done by solving the stochastic two stage problem, which takes the uncertainty into account. The two wells are very similar in expected value, but well 1 is slightly "better" then well 2, although the estimate of well 1 is uncertain. The deterministic approach is unaware of this uncertainty, while the stochastic approach is able to exploit this information. By first setting well 2 to about 70%, and then opening well 1, we are able to limit the effect when well 1 is worse than expected, while most of its potential is utilized if it is better than expected. This occurs because of how we reach the constraint. However, when we look at the 8 wells in case 2, we no longer have any significant benefit from considering the stochastic solution. When there are multiple uncertain wells, we are not able to control the final approach towards the constraint accurately, and we can not benefit from the effect as in case 1. Furthermore, if it were only one uncertain well, and we could control the final approach accurately, the benefit does not scale with problem size.

When there are multiple constraints, there is an advantage of using the stochastic approach. The performance of the



Fig. 5. Statistics of actual well opening for case 3



Fig. 6. Histogram of total oil production for case 3

deterministic solution depends on the chosen operational strategy. We tried various priorities of the deterministic solution, and the WC prioritizing worked best in this case study. However, even in this case, there was an expected increase in total oil production by about 1.5% when considering the stochastic solution. From the histograms in Figure 6, we note that we are able to limit the worst case behavior.

We have assumed GOR and WC were given by normal distributions. GOR enters directly in the capacity constraint, while WC is used to calculate the water-oil ratio (WOR) by $WOR = \frac{WC}{1-WC}$. This means WOR is not given by a normal distribution, but a distribution with a longer tail towards high WOR values. This explains why the WC prioritization works well in this case. In a real application, estimating these distributions is very important.

We have shown that the stochastic approach has a potential for increasing expected total oil production, when considering linear wells and multiple constraints, compared to the deterministic solution. The method is computational demanding, thus further research will study how to exploit problem structure. The methodology can be extended to nonlinearly behaving wells, although this will increase the complexity significantly.

REFERENCES

- Barnes, D., Humphrey, K., and Muellenberg, L. (1990). A production optimization system for western prudhoe bay field, alaska. In SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers, New Orleans, Louisiana.
- Bieker, H., Slupphaug, O., and Johansen, T. (2007a). Realtime production optimization of oil and gas production systems: A technology survey. *SPE Production & Operations*, 22(4).
- Bieker, H.P., Slupphaug, O., and Johansen, T.A. (2007b). Well management under uncertain gas/ or water/oil ratios. In *Proceedings of Digital Energy Conference and Exhibition*. Society of Petroleum Engineers, Houston, Texas, U.S.A.
- Chen, C., Li, G., and Reynolds, A.C. (2011). Robust constrained optimization of short and long-term npv for closed-loop reservoir management. In *SPE Reservoir Simulation Symposium*. Society of Petroleum Engineers, The Woodlands, Texas, USA.
- Elgsæter, S., Slupphaug, O., and Johansen, T.A. (2008). Production optimization; system identification and uncertainty estimation. In *Intelligent Energy Conference* and *Exhibition*, February. Society of Petroleum Engineers, Amsterdam, The Netherlands.
- Elgsæter, S.M., Slupphaug, O., and Johansen, T.A. (2010). A structured approach to optimizing offshore oil and gas production with uncertain models. *Computers & Chemical Engineering*, 34(2), 163–176.
- Foss, B.A. and Jensen, J.P. (2011). Performance analysis for closed-loop reservoir management. SPE Journal, 16(01), 183–190.
- Löfberg, J. (2004). Yalmip. In Proceedings of the CACSD Conference. Taipei, Taiwan.
- Saputelli, L., Mochizuki, S., Hutchins, L., Cramer, R., Anderson, M., Mueller, J., Escorcia, A., Harms, A., Sisk, C., Pennebaker, S., Han, J., Brown, A., Kabir, C., Reese, R., Nunez, G., Landgren, K., McKie, C., and Airlie, C. (2003). Promoting real-time optimization of hydrocarbon producing systems. In *Offshore Europe*. Society of Petroleum Engineers, Aberdeen, U.K.
- Tosukhowong, T., Lee, J.M., Lee, J.H., and Lu, J. (2004). An introduction to a dynamic plant-wide optimization strategy for an integrated plant. *Computers & Chemical Engineering*, 29(1), 199–208.
- Urbanczyk, C. and Wattenbarger, R. (1994). Optimization of well rates under gas coning conditions. SPE Advanced Technology Series, 2(02), 61–68.
- van Essen, G., Zandvliet, M., Van den Hof, P., Bosgra, O., and Jansen, J.D. (2009). Robust waterflooding optimization of multiple geological scenarios. SPE Journal, 14(01), 202–210.