Preprints of the
9th International Symposium on Advanced Control of Chemical Processes
The International Federation of Automatic Control
June 7-10, 2015, Whistler, British Columbia, Canada

Multi-product Multi-stage Production Planning with Lead Time on a Rolling Horizon Basis

Shan Lu*, Hongye Su*, Yue Wang*, Lei Xie*, Quanling Zhang*

* *Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou, 310027, P.R.China.*
(email: lushan@zju.edu.cn; hysu@iipc.zju.edu.cn; yuewang@zju.edu.cn; leix@iipc.zju.edu.cn; qlzhang@iipc.zju.edu.cn)

Abstract: Responding to a scalable production system in a make-to-order environment requires increased effective decisions. This paper considers challenges brought by lead time existed in a multi-product multi-stage manufacturing system when making a short-term production planning. The order-based production routes enhance the production flexibility, and in the meanwhile complicate the decision-making for each production stage. We define a series of sets to aggregate the production route and then model the problem as a mixed integer linear program (MILP) problem. The model seeks to find optimal production lots with several practical extensions. To efficiently capture the production dynamics induced by the lead times and setup, the production planning is implemented on a rolling horizon basis. The model is divided into several sub-problems as the horizon is rolled forward, of which a fix-and-relax strategy is applied. A study is conducted using data from a real case to evaluate the performance.

Keywords: production planning, production route, lead time, rolling horizon

1. INTRODUCTION

Effective and cost-efficient operations of a manufacturing firm often rely on appropriate production planning and scheduling. A series of new efficient requirements in the modern manufacturing environment have been carried out in terms of production management. A production planning problem derived from a steel rolling plant is presented. This planning problem concerns multi-period lot sizing and scheduling of various products through a multi-stage production process. In particular, one raw material could be processed into different end item variants through different production routes. This can result that, for a certain production stage, it would handle the work-in-process (WIP) from one or more upstream production stages, and output to its successive stage based on the production routes. Decision-making under this scenario is not easy since the production operation environment is very flexible, and is subject to both varying customer requirements and manufacturing conditions(Lu et al., 2014a).

The production planning addressed in this paper concerns a time-varying order demand and each production stage also involves a time-varying capacity. Backlogging the order is allowed to deal with contradiction between limited capacity and customer requirements. The items are produced in batches of a fixed size to meeting the economic conditions of processing equipment. The setup activities associated with the production stages are carried out between batches, and are sequence independent. Since the production routes for the orders are designed in advance and distinguished with each other, we consider a number of sets to depict the material/information flows for a production stage between its successor and predecessor stages. These sets can be aggregated in terms of the order information to draw the designated production routes. For each period, the decision-makers have to not only determine lot sizing of the items for each production stage, but also schedule the manufacturing processes under the multi-stage scenarios.

From a practical point of view, another distinctive issue concerned is the lead time existed between initiation and execution for a production stage, or raw material supply and order delivery as well. The lead time considered is defined in three different contexts. Lead time for a production stage represents the time required to create a job from the time the requirement is received. Lead time for raw material supply is the time required to make the raw material available in the inventory from the time decision-makers learn the requirements. Delivery lead time is the time from the inventory ships the end items to customer order delivered.

The problem of finding optimized scheduling of production lots, as well as the sizing, have been approached through both large-bucket models like capacitated lot-sizing problem (CLSP) and small-bucket models like discrete lot-sizing and scheduling problem(Fleischmann, 1990, Karimi et al., 2003). From the review by Jans and Degraeve (2008) that outlined the lot sizing based modelling for production planning problems of various industrial extensions. The addressed multi-product multi-stage production system can be regarded as an instance of the multi-level capacitated lot-sizing problem (MLCLSP) with several extensions for practical

This work was supported in part by National Natural Science Foundation of P.R. China (NSFC: 61320106009) and National High Technology Research and Development Programming of China (863 Program 2012AA040307).

implementation. Stadtler (1996) introduced a novel mixed integer programming model formulation (MILP) for dynamic multi-item MLCLSP, based on modelling the changes of end-of-period inventory levels explicitly. Further, Fandel and Stammen-Hegene (2006) considered a multi-item MLCLSP with multiple machines and sequence dependent setup, and develop a big-bucket model. Altendorfer and Minner (2011) studied a two-stage make-to-order (MTO) production system, in which each stage is released with a planned lead time, and proposed an approach for simultaneous optimization of capacity and planned lead time. For solving the MLCLSP with general product structure, Ozdamar and Barbarosoglu (2000) combined the capability of Lagrangean relaxation to decompose the model and the capability of simulated annealing for intensive search to improve the heuristic performance. Helber and Sahling (2010) presented an iterative fix-and-optimize algorithm for the dynamic MLCLSP by optimizing the model over a small subset of binary variables. Almeder (2010) developed a hybrid optimization procedure by applying MAX-MIN ant system and MIP solver to solve the production lots of MLCLSP.

In this paper, we present a MILP model for the multi-period production planning in a multi-product multi-stage production system with three practical considerations of lead time. The orders are released and allocated through pre-specified production routes consisting of respective production stages. We apply the tool of sets to describe the adjacent production stages within a production route of an order, and then aggregate the sets to draw the production routes. Inventory of raw materials, WIPs and end items are considered separately to formulate the material and information flow explicitly. We assume that the unfulfilled orders could be backlogged to ensure a feasible solution. In a proper way for fast decisions, we implement the proposed production planning problem on a rolling horizon basis, and the impact of the lead time existed in different parts is considered. The problem is solved by a series of subproblems through a fix-and-relax based rolling horizon heuristic. As the production operation acts as a pull system, the proposed approach would reduce the computational efforts induced by the setup binaries with lead time.

2. PROBLEM STATEMENT AND MODEL FORMULATION

The production planning system addressed consists of three parts in terms of roles acted in the production system, including raw material supply, production stages and order delivery. For simplifying the model, the three parts are considered as nodes of the whole production system. The model is to minimize the total cost with a time-varying demand for a set of end items and a restricted capacity for each production stage over the planned periods. Lead time for each production stage and delivery act as pre-processing latency, such as paperwork or shipment; while lead time for raw material supply is defined in terms of the inventory, and is regarded as post-processing latency. In this way, there is latency from node to node when releasing the orders to the production system. For a certain order, the production system will respond with appropriate production decisions during respective period horizon of each node.

Before an order is release to the production system, its production route designed according to the process. Due to the flexible nature of the production system structure, the production routes of the orders may intersect with each other at one or more nodes. To formulate the characteristics, we firstly use two sets to depict successors and predecessors of a node within an order-based production route, and then aggregate the sets to construct the routes. Fig.1 gives an example to explain how to modelling production routes. We denote i and j respectively for items and production stages. Raw materials $i=1,2,3$ are to produce end items $i=7,8,9$, and set $\{j,i\}$ represents the production stage-item structure. The we define sets $H(j,i)$ and $\Omega(j,i)$ as immediate successor and predecessor production stages of j for producing i. We use $\{j,i\}=\{C,5\}$ as an example, item $i=5$ is the semi-product or WIP produced by production stage $j=C$ over the route 2-via-5-to-8 for producing end item $i=8$. In this case, the successor and predecessor $\{j,i\}=\{\mathrm{A},2\}$ and $\{j,i\}=\{\mathrm{E},8\}$ can be described as $H(j,i)=\mathrm{H}(5,8)$ and $\Omega(j,i)=\Omega(5,8)$.

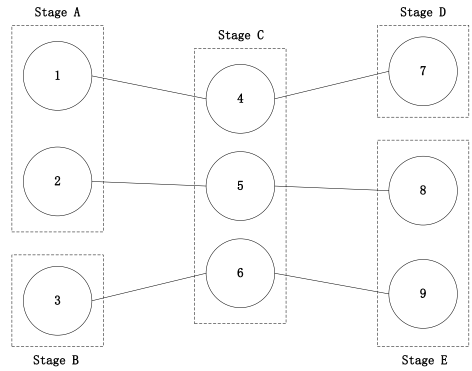

Fig. 1. Examples of production routes

In summary, the following notations are used in order to describe the mathematical model of the proposed problem.

Indices

i	index for raw materials, WIP and end items
i'	index for immediate downstream items of i
j	index for production stages
j'	index for immediate successor production stages j
t	index for time period

Sets

N	set for all the items
J	set for all the production stages
T	set for time horizon
R	subset of N denoting raw materials
W	subset of N denoting work-in-process
P	subset of N denoting end items
$C(i)$	set for the production stages that produce i

$O(i)$ set for the first production stage of i

$\mathrm{T}(i)$ set for the terminal production stage of i

$\mathrm{M}(j)$ set for the items produced by production stage j

$H(j,i)$ set for immediate successor production stages of j for producing i

$\Omega(j,i)$ set for the immediate predecessor production stages of j for producing i

$\Pi(j,i)$ set for the items produced from i by stage j

Parameters

η_{ij} production efficiency of stage j for producing i

AU_t hours in period t

cr_i cost of supplying one unit of raw material i

cba_i cost of backlogging one unit of end item i for one time period

cip_i cost of holding one unit of end item i in warehouse for one time period

cir_i cost of holding one unit of raw material i in warehouse for one time period

ciw_i cost of holding one unit of work-in-process i in warehouse for one time period

cp_{ij} cost of producing one unit of i at stage j

cs_{ij} setup cost of stage j for producing i

$IP_t^{\max}$ maximum inventory level of end item

$IR_t^{\max}$ maximum inventory level of raw material

$IW_t^{\max}$ maximum inventory level of WIP

l_i^P lead time required to deliver the order $i \in P$ from the end-item warehouse

l_i^R lead time required to supply raw materials $i \in R$

l_j lead time required to ship the item from the warehouse to stage j for processing

MB_i fixed supply batch size of raw material $i \in R$

MB_{ij} fixed production batch size of item $i \in W \cup P$

MD_{it} demand for end item $i \in P$ in period t

TM_{jt} planned maintenance time of stage j in period t

Continuous Variables

$MBA_{i,t,t_1(t_2)}$ backlogging quantity of end item $i \in P$ in period t that will be fulfilled in period $t_1(t_2)$

MR_{it} supply quantity of raw material $i \in R$ in period t

MX_{ijt} production quantity of item $i \in W \cup P$ at stage j

UP_{it} quantity of end item $i \in P$ shipped to the warehouse

Integer Variables

NI_{it} number of supply batch of raw material $i \in B$

NI_{ijt} number of production batch of item $i \in W \cup P$ at stage j in period t

X_{ijt} binary variable that indicates whether item $i \in W \cup P$ is produced at stage j in period t

The total cost is composed of production, setup, raw material supply, inventory holding and backorder penalty. For the inventory cost, holding the end items is more costly and holding the raw materials is cheaper, compared with the work-in-process. Penalty costs of backlogging the orders are rather high in comparison with other cost components. Since the production takes place in batches, setup for manufacturing a product variety is expensive in terms of costs. The objective function (1) is to minimize the total cost over the planning horizon

$$\begin{aligned} Min & \sum_{i\in M(j)}\sum_{j\in J}\sum_{t=1}^{T} MX_{ijt}\cdot cp_{ij} \\ & + \sum_{i\in W\cup P}\sum_{j\in C(i)}\sum_{t=1}^{T} X_{ijt}\cdot cs_{ij} + \sum_{i\in R}\sum_{t=1}^{T} MR_{it}\cdot cr_i \\ & + \sum_{i\in R}\sum_{t=1}^{T} IR_{it}\cdot cir_i + \sum_{i\in W}\sum_{t=1}^{T} IW_{it}\cdot ciw_i + \sum_{i\in P}\sum_{t=1}^{T} IP_{it}\cdot cip_i \\ & + \sum_{i\in P}\sum_{t=1}^{T}\sum_{t_1>t}^{T} (t_1-t)\cdot MBA_{i,t,t_1}\cdot cba_i \end{aligned} \tag{1}$$

s.t.

$$IR_{i,t+1} = IR_{i,t} + MR_{i,t-l_i^R} - \sum_{i'\in\Pi(j,i)}\sum_{j\in O(i)} MX_{i',j,t+l_j} \quad \forall i\in R, \forall t \tag{2}$$

$$\begin{aligned} IW_{i,t+1} = IW_{i,t} + \sum_{j\in C(i)} MX_{ijt} - \sum_{i'\in\Pi(j',i)}\sum_{j\in C(i)}\sum_{j'\in H(j,i)} MX_{i',j',t+l_j} \\ -UP_{it} \quad \forall i\in W \mid i', j' \notin \varnothing, \ \forall t \end{aligned} \tag{3}$$

$$\begin{aligned} IP_{i,t+1} - \sum_{t_2>t+l_i^P+1}^{T} MBA_{i,t+l_i^P+1,t_2} = IP_{it} - \sum_{t_1>t+l_i^P}^{T} MBA_{i,t+l_i^P,t_1} \\ +UP_{it} - MD_{i,t+l_i^P} \quad \forall i\in P, \forall t \end{aligned} \tag{4}$$

$$UP_{it} = \sum_{j\in C(i)} MX_{ijt} \quad \forall i\in P-W, \ \forall t \tag{5}$$

$$\sum_{i\in R} IR_{it}\cdot V_i \le IR_t^{\max} \quad \forall t \tag{6}$$

$$\sum_{i\in W} IW_{it}\cdot V_i \le IW_t^{\max} \quad \forall t \tag{7}$$

$$\sum_{i\in P} IP_{it}\cdot V_i \le IP_t^{\max} \quad \forall t \tag{8}$$

$$\sum_{i\in M(j)}\left(\frac{MX_{ijt}}{\eta_{ij}} + TS_{ij}\cdot X_{ijt}\right) \le AU_t - TM_{jt} \quad \forall j, \ \forall t \tag{9}$$

$$MX_{ijt} > 0 \iff X_{ijt} = 1 \quad \forall i\in W\cup P, \ \forall j, \ \forall t \tag{10}$$

$$MX_{ijt} = 0 \iff X_{ijt} = 0 \quad \forall i\in W\cup P, \ \forall j, \ \forall t \tag{11}$$

$$MR_{it} = NI_{it}\cdot MB_i \quad \forall i\in R, \ \forall t \tag{12}$$

$$MX_{ijt} = NI_{ijt}\cdot MB_i \quad \forall i\in M(j), \ \forall j, \ \forall t \tag{13}$$

$$X_{ijt} \in \{0,1\} \quad \forall i\in W\cup P, \ \forall j, \ \forall t \tag{14}$$

$$NI_{it} \in N \quad \forall i\in R, \ \forall t \tag{15}$$

$$NI_{ijt} \in N \ \ \forall i \in M(j), \ \forall j, \ \forall t \tag{16}$$

$$MBA_{i,t,t_1(t_2)}, MR_{it}, MX_{ijt}, UP_{it} \geq 0 \ \ \forall i, j, t, t_1, t_2 \tag{17}$$

Constraint (2) is to balance the material flow for raw material inventory. For period t, the raw material supply would delay the impacts on the inventory for l_i^R periods as it acts as postprocessing latency; however, the starting production stage should response l_j periods ahead of time to ship the raw materials from the inventory. Constraint (3) gives the material inputs and outputs of each production stage using sets $H(j,i)$ and $\Omega(j,i)$, and thus specifies the inner production route except for the starting and terminal production stage. For a production stage j, its successor production stage would also response l_j periods ahead as its nature of prepocessing latency. It should be noted that the output materials would be shipped to either its successor production stage or end item warehouse, depending on the respective production routes. Constraint (4) presents the material balance of end item warehouse, where backlogging quantity is regarded as a negative inventory level. Since both backorders and order demand are on the side of customers, their impacts on the end item inventory are l_i^P periods ahead. Constraint (5) indicates the relationship between the end items that are produced by its production stage and shipped to its warehouse. Constraints (6) ~ (8) specify the maximum inventory capacity for the three warehouses which cannot be exceeded. As per constraint (9), the total capacity for each production stage occupied by production and setup is limited by its designed capacity and maintenance. Constraints (10) and (11) define the relationship between setup binary and production quantity. As stated aforementioned, supply quantity of raw material $i \in R$ and production quantity of items for each production stage take place in batches with a fixed batch size in terms of the items. So $NI_{it} \mid \forall i \in R$ and $NI_{ijt} \mid \forall i \in M(j), \forall j$ are function of the batch size, number of batches supplies or produced, which are determined by constraints (12) and (13). Constraints (14) ~ (17) give the integral, binary and non-negative restrictions on the respective decision variables.

3. ROLLING HORIZON IMPLEMENTATION

Although the proposed MILP model can be solved to optimality by the state-of-the-art optimization techniques, the solution procedures still needs substantial computation efforts considering the massive binary and integer variables included. On the other hand, the nature of lead time existed in each production stage and raw material supply requires information on downstream stage with longer time horizon than that on the upstream. In another word, to obtain the planning decisions on an upstream stage, eg. raw material supply, the decision-maker should capture the information for its downstream stages, in advance enough to cover the effect of lead time. That means if the proposed model is solved for the time horizon of the terminal node, the resulting decisions on the upstream stage would be partially redundant.

As pointed by Clark and Clark (2000), the fluctuations may be added as the planning horizon is rolled forward, such as maintenance policies and uncertainty brought by forecast demand. Similar to model predictive control (MPC) algorithm, the rolling horizon strategy enables the system to make decision for the next period but taking into account the scenarios within a foreseen horizon(Perea-Lopez et al., 2003). On a rolling horizon basis, the decision maker implements only the decisions within a limited time horizon and advances the horizon forward. In this paper, we apply a rolling-horizon based strategy to reduce the integer constraints and redundant decision efforts, and then enhance the computational efficiency and model flexibility. This reduction is achieved by valid relaxation of the complicating variables or terms within a selected section of the horizon.

Looking back in the proposed MILP model, the integer variables consist mainly of batch number variables NI_{it} and NI_{ijt}, as well as setup variables X_{ijt}. Significantly, it is the integers that enhance the size of the branch-and-bound search tree and thus prevents obtaining solutions in quick responses. This fact prompts the reduction of the integer restrictions which can be either fixed or relaxed. To carry out this strategy on rolling horizon basis, the planning horizon It should be noted that, given the order demand of a certain period horizon, the corresponding decision horizon for each production stage can be different due to the lead time. Even so, the time horizons of each section for each production stage and raw material supply are the same and decomposed into three parts: a beginning section, a central section and an ending section.

Beginning section: this section is composed of the *Ta* first periods. Within this section, all the decision variables are fixed which include not only the integer variables mentioned above but also the continuous variables in the model.

Central section: this section consists of the *Tb* successor periods of the beginning section which are used for precise decision making. All the integrity or binary constraints on setup variables and other auxiliary variables are retained.

Ending section: this section includes the *Tc* successor periods of the central section until the last period. Within this section, all the binary variables are relaxed to be a continuous value between 0 and 1. The other integrity constraints are neglected as well. This relaxation means that these binary and integer variables will have less influence on the decisions on the central section.

To implement rolling horizon heuristic for the whole production system, three kinds of time horizon that should be defined before describing the algorithm. They are as follows:

(I) Optimization time period for one step TO: it refers to the fixed period length of which the proposed model is optimized for one step.

(II) Rolling step-length ΔRt: it refers to the periods of which the time horizon is rolled forward for each step.

(III) Planning horizon for each node T_j or $T_i \mid i \in R \cup P$: it refers to the periods of which decisions on each node are made based on the order information released.

For a certain production stage, the rolling horizon heuristic starts by solving the optimization model, where the variables in the beginning section are fixed and integer variables in the ending section are relaxed to continuous variables. The fixed value of the variables in the begging section is initially obtained by either actual manufacturing activities or solution of the predecessor rolling iteration. The approximated model can be solved using off-the-shelf optimization techniques,. The second step is to roll the horizon forward and repeat the approximation described. Hence, the central section shifts forward as the horizon rolls, and the feasible decisions for the whole planning horizon of a certain production stage can be achieved step by step. The procedures are applied to all the production stages as well as the raw material supply.

As the production network operates as pull driven system, considering the lead time, the procedures end when the feasible decisions of the raw material supply over its planning horizon is obtained. It can be noticed that the rolling step-length ΔRt would affect the relationship between two successive rolling steps k and $k+1$. Decisions of the central sections may overlap between the successive steps when ΔRt is too small. In this paper, we assume the rolling step-length ΔRt is equal to time horizon of the central section Tb, thus resulting consecutive decisions. Since the rolling horizon algorithm should also be implemented upon the production characteristics that pull the system driven by orders, the description of the rolling horizon algorithm is as follows.

Step1: Set rolling step $k=1$.

Step2: Given the information on order (correspond to end item $i \in P$) for periods $\left[L_i+1-Ta, L_i+1+Tb+Tc\right]$ and raw material supply status for Ta periods before, solve the sub-problem where beginning section is fixed and ending section is relaxed. Then the exact decisions on the raw material supply for periods $\left[1, Tb\right]$ can be obtained.

Step3: Set $k=k+1$. Roll the horizon forward with step-length ΔRt for all nodes, and solve the sub-problem. In this case, the order information is required for periods $\left[L_i+1-Ta+(k-1)\Delta Rt, L_i+1+Tb+Tc+(k-1)\Delta Rt\right]$, and exact decisions on the raw material supply for periods $\left[1, k\cdot Tb\right]$ are obtained.

Step4: Check whether the decisions for raw materials obtained cover the respective planning horizon. If $k\cdot Tb < T_i,\ \forall i \in R$, then return back to step3. Otherwise, the iteration ends.

4. COMPUTATIONAL RESULTS

We apply a numerical example extended by the case in Lu (2014b) to evaluate the applicability and performance of the proposed approach. The production system studied is inspired by a plant manufacturing steel plates, consisting of six production stages, each with respective capability and capacity. Ten kinds of products (end items) are considered and manufactured from three raw material species going through the designated production route. Table 1 shows the system structure and production routes of each product, as well as bill of material. In this table, we use a set *(sequence, output)* to depict the particular sequence of production stages in which the respective product is produced, and also give the output material of each production stage. For product V as an example, we can draw conclusions that product V is produced from raw material R2, and its production route is A-B-C-E, in which the four production stages output work in process of WA2, WB3, WC5 and WE5. Table 2 presents the fixed lead times existed in the raw material supply, production stages and order delivery.

To evaluate the performance of the proposed approaches in terms of the solution quality, we carry out two cases for comparison. In case I, the model is solved by the proposed approaches. In case II, the model is solved by the off-the-shelf optimization solver. In case III, the model is solved by relaxing the integers into continuous variables except for the first period. All the problems within the three cases are formulated and solved using LINGO 11 with branch-and-bound algorithm. The optimization time horizon for each step

Table 1. Production system for the example

Bill of material		Production stage					
Product	RM	A	B	C	D	E	F
I	R1	(1,WA1)	(2,WB1)	(3,WC1)	-	(4,WE1)	-
II	R1	(1,WA1)	(2,WB1)	(3,WC2)	-	(4,WE2)	-
III	R1	(1,WA2)	(2,WB2)	(3,WC3)	-	(4,WE3)	-
IV	R1	(1,WA2)	(2,WB2)	(3,WC4)	-	(4,WE4)	-
V	R2	(1,WA3)	(2,WB3)	(3,WC5)	-	(4,WE5)	-
VI	R2	(1,WA4)	(2,WB4)	-	(3,WD1)	(4,WE6)	-
VII	R3	(1,WA5)	(2,WB5)	-	(3,WD2)	(4,WE7)	-
VIII	R3	(1,WA5)	(2,WB5)	-	(3,WD3)	(4,WE8)	-
IX	R1	(1,WA1)	(2,WB1)	(3,WC1)	-	(4,WE1)	(5,WF1)
X	R2	(1,WA4)	(2,WB4)	-	(3,WD1)	(4,WE6	(5,WF2)

Note: RM means raw material.

Table 2. Lead time in each node of the production system

Node	R1	R2	R3	A	B	C	D	E	F	Delivery
Lead time/day	2	2	2	1	1	1	1	1	1	1
Planning horizon/day	4	4	4	5	6	7	8	9	10	-

Table 3. Computational results for the cases

	Case I	Case II	Case III
LB/10^3 *RMB*	9602.49	9584.11	9559.53
UB/10^3 *RMB*	9602.49	9661.61	9559.53
GAP	0%	0.8%	0%
CPU Time/*s*	14	1000	7
Objectives/10^3 RMB			
Production	2507.55	2508.35	2497.75
Setup	49.5	47.3	48.6
Raw Material	6987.8	7006.8	6960.6
Inventory	1.44	22.16	0.26
Backlogs	56.2	77	52.32

TO is set to be 5 periods. For case I, we set $Ta=1$, $Tb=2$, $Tc=2$ and $\Delta Rt=2$ for all iterations. To analyse the solution quality, we induce an indicator $GAP=(UB-LB)/UB$, where UB and LB is respectively the lower bound and upper bound values at the termination of the solution procedures. It should be noted that UB is the current optimal results. We set a CPU time limit of 1000s. The obtained results for the three cases are presented in table 3.

From the solution time perspective, models in case I and III can be solved optimally with significantly lower CPU time. The model in case II is not able to obtain an optimal solution within the CPU time limit, and thus with a GAP of 0.8%. This is due to the integer restrictions on batch sizes of production and raw material supply that complicate the solution process. It is obvious that relaxation on integer or binary variables would lead to great savings in computational time. Since the model in case I is implemented on a rolling horizon basis, the nature of its iterative solution procedures should theoretically take more efforts. But the results show that the CPU time for case I is rather low and acceptable.

If considering the objective values, case III relaxing the integer approximates the problem for the unimplemented periods, and results in lowest total cost. Therefore, the production planning decisions obtained are actually not feasible throughout the planning horizon. In contrast, solutions for case I and II are both feasible. But case I outperforms in total cost, and can achieve lower cost of production, raw material supply, inventory holding and backlogs. Out of the three cases, case I compromises both the solution time and quality.

5. CONCLUSIONS

In this paper, we investigate on planning production lots of a multi-product multi-stage production system considering lead time in various forms, as well as practical extensions of the system characteristics. A series of sets are defined to depict the flexible nature of the order-based production routes in terms of aggregating the respective production stages and raw material supply. A MILP model is proposed to formulate the addressed problem in a multi-period horizon that takes into account several industrial constraints. To solve the problem efficiently, we propose a rolling horizon based approach to explore the model structure and we decompose the model by periods into several sub-horizons over which a fix-and-relax strategy is used. Computational experiments show that the proposed approach provides an efficient tool for planning decision-making in a dynamic and flexible environment.

REFERENCES

Almeder, C. (2010). A hybrid optimization approach for multi-level capacitated lot-sizing problems. *European Journal of Operational Research,* **200,** 599-606.

Altendorfer, K. and Minner, S. (2011). Simultaneous optimization of capacity and planned lead time in a two-stage production system with different customer due dates. *European Journal of Operational Research,* **213,** 134-146.

Clark, A. R. and Clark, S. J. (2000). Rolling-horizon lot-sizing when set-up times are sequence-dependent. *International Journal of Production Research,* **38,** 2287-2307.

Fandel, G. and Stammen-Hegene, C. (2006). Simultaneous lot sizing and scheduling for multi-product multi-level production. *International Journal of Production Economics,* **104,** 308-316.

Fleischmann, B. (1990). The discrete lot-sizing and scheduling problem. *European Journal of Operational Research,* **44,** 337-348.

Helber, S. and Sahling, F. (2010). A fix-and-optimize approach for the multi-level capacitated lot sizing problem. *International Journal of Production Economics,* **123,** 247-256.

Jans, R. and Degraeve, Z. (2008). Modeling industrial lot sizing problems: a review. *International Journal of Production Research,* **46,** 1619-1643.

Karimi, B., Fatemi Ghomi, S. and Wilson, J. (2003). The capacitated lot sizing problem: a review of models and algorithms. *Omega,* **31,** 365-378.

Lu, S., Su, H., Johnsson, C. and Xie, L. 2014. A Production Planning Model for a Steel Plate Fabrication Plant with Flexible Customization and Manufacturing. The 19th World Congress of the International Federation of Automatic Control, 2014a Cape Town, South Africa. 3315-3320.

Lu, S., Su, H., Wang, Y., Xie, L. and Zhu, L. 2014. Multi-product Multi-route Production Planning Simultaneously Considering Partial Backlogs and Lost Sales. The 5th International Symposium on Advanced Control of Industrial Processes, 2014b Hiroshima, Japan. 278-283.

Ozdamar, L. and Barbarosoglu, G. (2000). An integrated Lagrangean relaxation-simulated annealing approach to the multi-level multi-item capacitated lot sizing problem. *International Journal of Production Economics,* **68,** 319-331.

Perea-Lopez, E., Ydstie, B. E. and Grossmann, I. E. (2003). A model predictive control strategy for supply chain optimization. *Computers & Chemical Engineering,* **27,** 1201-1218.

Stadtler, H. (1996). Mixed integer programming model formulations for dynamic multi-item multi-level capacitated lotsizing. *European Journal of Operational Research,* **94,** 561-581.

