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Dissipativity-based Analysis of Controller Networks with Reduced Rate Communication

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Abstract: Distributed control is a computationally efficient and high performance approach for plantwide process systems, where the interactions between process units can be dealt with using the information exchanged among controllers. In this work we develop an approach to analyze distributed control systems where the controllers communicate at a rate lower than the sampling rate for the local process units. This is motivated by the observation that many chemical processes have plant-wide dynamics on a time-scale slower than that of the individual processes' dynamics. Based on the dissipative systems theory, this approach can be used to determine the impact of the reduced communication rate on the stability and control performance of the plant-wide system.

Keywords: Dissipative Systems Theory, Multi-Rate Control, Distributed Control

1. INTRODUCTION

Dissipativity (or passivity, as a special case of dissipativity) is an input-output property of dynamical systems, which may be related to ℓ_2 gain and phase properties of systems (Bao and Lee , 2007; Willems , 1972). Dissipativity theory is an effective tool for the quantitative stability and performance analysis of large-scale interconnected systems (Moylan and Hill , 1978), wherein the problem is decomposed into the analysis of the dissipativity of the subsystems and interconnection topology, (e.g. (Tippett and Bao , 2014c; Hioe et al. , 2013)). A dissipativity-based distributed control approach for plant-wide process systems was first developed in (Xu and Bao , 2010, 2011), where the closed-loop dissipativity constraint guarantees plant-wide stability and minimum plant-wide performance. Furthermore, dissipative distributed MPC was developed in (Tippett and Bao , 2013) by adopting dynamic supply rates in quadratic difference form (QdF).

When implementing a control system on a large-scale system, communication issues may be inevitable such as limited communication capacity, data losses and irregular time delays. For example, (Matveev and Savkin , 2009) provided a result on how channel capacity limits observability and stabilizability by employing the concept of topological entropy. In order to reduce communication requirement in spacecraft systems (Lavaei et al. , 2008) reformulated the distributed controller into a decentralized fashion. (Maestre et al. , 2009) proposed an algorithm to resolve the problem caused by communication errors, where the controllers can operate in a decentralized way over a low-reliability communication network. (Sun and El-Farra , 2008) formulated a plant-wide control system with a communication network as a hybrid system, where the maximum allowable update period can be determined without losing the exponential stability of the closed-loop plant-wide system. By using a dissipativity-based approach, the analysis of the communication issues can be formulated into the study of interconnected input-output properties of a plant-wide system with communication network, such as (Hirche et al. , 2009) and (Tippett and Bao , 2014a).

In this paper, we develop a new approach for the analysis of the effects of reduced communication rates on plant-wide stability and performance, wherein distributed controllers communicate with one another at a rate slower than the sampling rate of the individual subsystems. The proposed analysis is explicitly formulated in terms of the dissipativity properties of the plant-wide system, by lifting it into a slower sampling rate. That is, by describing the system (and its dissipativity properties) in a slower sampling rate which is an integer multiple of the original sampling rate. The dissipativity of this lifted system can then be handled using QdFs as supply rates. This analysis facilitates the design of dissipativity-based distributed MPC (as an extension of the approach developed in (Tippett and Bao , 2013)) with reduced communication rates. Or the analysis of the effect of reduced communication rates on plant-wide stability and performance.

The following notation is used throughout this paper. $\mathbb{R}^{\cdot\times\cdot}[\zeta,\eta]$ denotes the ring of two variable polynomial matrices with real coefficients and arbitrary dimensions. $\|\cdot\|_\ell$ denotes the ℓ-norm. The symbols $\overline{\sigma}(A)$ and $\underline{\sigma}(A)$

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denote the maximum and minimum singular values of the matrix A, respectively. The symbol $\mathbb{Z}^+$ denotes the set of nonnegative integers, and $\mathtt{diag}_{\tilde{\tau}}(Q)$ represents a block diagonal matrix with $\tilde{\tau}$ diagonal blocks of Q.

2. BACKGROUND MATERIAL

2.1 Distributed Control of Process Networks

The structure of the plant-wide system with a distributed controller is depicted in Fig. 1. The plant-wide system consists of the collections of individual processes and local controllers, which are introduced later. All physical flows are exchanged via the process topology H_p. The controllers communicate with network topology defined by H_c.

The plant-wide process $\mathcal{P}$ refers to the diagonal stacking of individual processes (i.e. $\mathtt{diag}(\mathcal{P}_1, \ldots, \mathcal{P}_{n_p})$) in a chemical plant with n_p processes, where the i-th process $\mathcal{P}_i$ is governed by state space representation

$$\mathcal{P}_i : \begin{pmatrix} x_i(k+1) \\ y_i(k) \end{pmatrix} = \begin{pmatrix} A_i & B_{1i} & B_{2i} & B_{3i} \\ C_i & D_{1i} & D_{2i} & D_{3i} \end{pmatrix} \begin{pmatrix} x_i(k) \\ u_{\mathrm{p}_i}(k) \\ u_{\mathrm{c}_i}(k) \\ d_i(k) \end{pmatrix}. \quad (1)$$

It is assumed throughout that the systems are controllable and zero state detectable. Denote y_i, u_{p_i}, u_{c_i} and d_i as its process output, input, controlled input and disturbance, respectively. The process input u_{p_i} is a physical flow, (such as material and energy). The controlled input u_{c_i} is local optimal control actions and determined by the i-th local controller. From a plant-wide system point of view, those signals are stacked, such as $y = (y_1^T, \ldots, y_{n_p}^T)^T$.

Similarly, the stacked controller $\mathcal{C}$ refers to the collection of individual controllers without communication (i.e. $\mathcal{C} = \mathtt{diag}(\mathcal{C}_1, \ldots, \mathcal{C}_{n_p})$). With local/remote controller input $u_\mathrm{l}/u_\mathrm{r}$ and local/remote controller output $y_\mathrm{l}/y_\mathrm{r}$, respectively. The pair of remote signals, $y_\mathrm{r}/u_\mathrm{r}$, is sent/received predicted trajectories that exchanges among controllers. The process network topology H_p and controller network topology H_c are constant matrices, which describe the interconnection structure in the plant-wide process system. The filters, F_p and F_I, select interconnecting and measured outputs, respectively. In this work, they are considered as constant matrices with elements of 1 or 0. As such, u_p

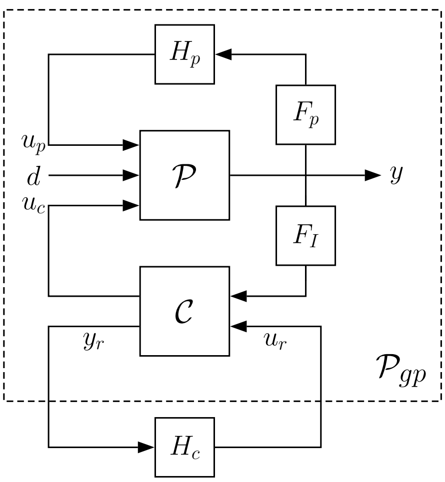

Fig. 1. Structure of the plant-wide system

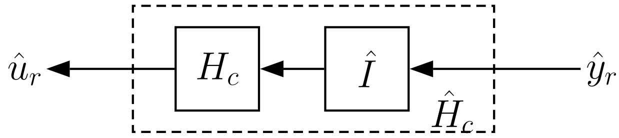

Fig. 2. Communication Network

is selected and interconnected via topology $u_\mathrm{p} = H_\mathrm{p} F_\mathrm{p} y$. The predicted trajectories are exchanged via controller topology H_c. When communication occurs, the remote input of i-th controller, u_{r_i}, is the collection of the remote outputs from other controllers. Such remote input can be represented as the composite vector of remote outputs y_{r_j} for some j, as such, u_r becomes $H_\mathrm{c} y_\mathrm{r}$ for the case of the plant-wide system. These pairs of remote signals exchange trajectories via the communication network $\hat{H}_\mathrm{c}$, which is modelled as a switched communication network, as shown in Fig. 2. $\hat{H}_\mathrm{c}$ consists of the controller topology H_c and a switch $\hat{I}$. Normally, the unit communication rate may be designed as the same as process sampling rate. In this case, under the switching law $\hat{I}$, information only exchanges after $\tilde{\tau}$ sampling periods.

2.2 Dissipative Systems Theory

Dissipativity theory can be used to capture the dynamic features of systems (Willems , 1972). It allows for large-scale systems to be analysed in terms of their subsystems and interconnection topology by studying the dissipativity properties of these subsystems. Loosely speaking, a system is said to be dissipative if the change of 'energy' of the system is bounded by the net supply from the environment through the inputs and outputs. A discrete-time dynamic system is said to be dissipative with a supply rate, $s(y(k), u(k))$, if there exists a positive semi-definite storage function, $V(x(k))$, satisfying the dissipation inequality:

$$V(x(k+1)) - V(x(k)) \leq s(y(k), u(k)) \qquad \forall k \geq 0 \quad (2)$$

where $x(k) \in \mathbb{R}^m$, $y(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^q$ are state, output and input at time instant k, respectively. Supply rates can be written in a quadratic form as

$$s(y(k), u(k)) = \begin{pmatrix} y(k) \\ u(k) \end{pmatrix}^T \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} y(k) \\ u(k) \end{pmatrix}, \quad (3)$$

where Q, S and R are matrices with appropriate dimensions. However, analysis based on such results can lead to conservative bounds on stability and performance. In (Kojima and Takaba , 2005), quadratic difference forms (QdFs) can be used to overcome this issue by including additional system information. Denote an $\hat{n}_\mathrm{d}$-degree extended signal space by $\hat{w}(k) = (\hat{y}^T(k), \hat{u}^T(k))^T$, where

$$\hat{y}(k) = (y^T(k), y^T(k+1), \ldots, y^T(k+\hat{n}_\mathrm{d}))^T \quad (4a)$$

$$\hat{u}(k) = (u^T(k), u^T(k+1), \ldots, u^T(k+\hat{n}_\mathrm{d}))^T. \quad (4b)$$

A dynamic supply rate, $Q_\Phi(\hat{w}(k)) = \hat{w}^T(k)\Phi\hat{w}(k)$, is defined in QdF induced by a polynomial matrix $\Phi(\zeta, \eta)$. Here, η denotes the forward shift operator of unit time, that is, $\eta w(k) = w(k+1)$. Similarly, ζ is defined as a forward shift operator of unit time on $w^T(k)$. They enjoy the property that $\zeta^T = \eta$. QdFs are able to be systematically represented as $Q_\Phi = w^T(k)\Phi(\zeta, \eta)w(k)$ using two-variable polynomial matrices, which are said to be induced by the two-variable polynomial matrix

$$\Phi(\zeta,\eta)=\sum_{i=0}^{\hat{n}_\mathrm{d}}\sum_{j=0}^{\hat{n}_\mathrm{d}}\zeta^i\eta^j\phi_{ij}\in\mathbb{R}^{\cdot\times\cdot}[\zeta,\eta]\,. \tag{5}$$

Theorem 1. [(Kojima and Takaba , 2005)] A discrete linear time-invariant (LTI) system is *asymptotically stable*, if and only if there exists a symmetric nonnegative two-variable polynomial matrix $\psi(\zeta,\eta)$ satisfying the following inequality for all allowable output trajectories $\Delta\psi(\zeta,\eta)<0$, where $\Delta\psi(\zeta,\eta)$ is the forward difference operator, defined as $(\zeta\eta-1)\psi$.

Theorem 1 is also sufficient for the nonlinear case. The following result provides a condition for determining the dissipativity properties of an LTI system.

Proposition 1. ((Tippett and Bao , 2013)). A discrete time LTI system with state space representation $(A,\,B,\,C,\,D)$ is dissipative with supply rate and (positive semidefinite) storage function pair induced by $\phi(\zeta,\eta)$ and $\psi(\zeta,\eta)$, respectively, with the corresponding coefficient matrices $\tilde{\phi}$ and $\tilde{\psi}$ partitioned as $\tilde{\phi}=\begin{pmatrix}\tilde{\phi}_\mathrm{Q} & \tilde{\phi}_\mathrm{S}\\ \tilde{\phi}_\mathrm{S}{}^T & \tilde{\phi}_\mathrm{R}\end{pmatrix}$ and $\tilde{\psi}=\begin{pmatrix}\tilde{\psi}_\mathrm{X} & \tilde{\psi}_\mathrm{Y}\\ \tilde{\psi}_\mathrm{Y}{}^T & \tilde{\psi}_\mathrm{Z}\end{pmatrix}$, if and only if the following LMIs are satisfied:

$$\begin{pmatrix}\mathbb{T}_{11} & \mathbb{T}_{12}\\ \mathbb{T}_{12}^T & \mathbb{T}_{22}\end{pmatrix}\geq 0; \tag{6}$$

$$\tilde{\psi}\geq 0, \tag{7}$$

with

$$\hat{C}=\begin{pmatrix}C\\ CA\\ \vdots\\ CA^N\end{pmatrix}, \tag{8}$$

$$\hat{D}=\begin{pmatrix}D & 0 & \dots & 0 & 0\\ CB & D & \dots & 0 & 0\\ \vdots & \ddots & \ddots & \ddots & \vdots\\ CA^{N-1}B & CA^{N-2}B & \dots & CB & D\end{pmatrix}, \tag{9}$$

$$\mathbb{T}_{11}=\hat{C}^T[\tilde{\phi}_\mathrm{Q}-\tilde{\nu}_\mathrm{X}]\hat{C}, \tag{10}$$

$$\mathbb{T}_{12}=\hat{C}^T[\tilde{\phi}_\mathrm{Q}-\tilde{\nu}_\mathrm{X}]\hat{D}+\hat{C}^T[\tilde{\phi}_\mathrm{S}-\tilde{\nu}_\mathrm{Y}], \tag{11}$$

$$\mathbb{T}_{22}=\hat{D}^T[\tilde{\phi}_\mathrm{Q}-\tilde{\nu}_\mathrm{X}]\hat{D}+\hat{D}^T[\tilde{\phi}_\mathrm{S}-\tilde{\nu}_\mathrm{Y}], \tag{12}$$

$$+[\tilde{\phi}_\mathrm{S}-\tilde{\nu}_\mathrm{Y}]^T\hat{D}+[\tilde{\phi}_\mathrm{R}-\tilde{\nu}_\mathrm{Z}], \tag{13}$$

where N is the degree of the supply rate and $\nu(\zeta,\eta)=\Delta\psi(\zeta,\eta)$.

A reduced communication rate can be formulated by imposing a lengthened communication period, $\tilde{\tau}$, so that the communication only occurs every $\tilde{\tau}+1$ time units. The dynamic behaviour of this system can be captured by the signals after lifting, wherein the signal $\hat{y}(k)$ becomes $(\hat{y}^T(k),\ldots,\hat{y}^T(k+\tilde{\tau}))^T$. The following result allows for the dissipativity properties of a lifted system to be systemically determined.

Lemma 1. [Lifted Dissipativity (Tippett and Bao , 2014a)] Assume the supply rate of a dynamic system Σ, sampled at unit time, is induced by polynomial matrix $\Phi(\zeta,\eta)$ in Eq (5). Let $\Sigma_{\tilde{\tau}}$ denote the lifted dynamic system in the sampling period $\tilde{\tau}$. Then, the QdF-supply rate of this system is induced by the polynomial $\tilde{\Phi}(\zeta,\eta)$

$$\tilde{\Phi}(\zeta,\eta)=\mathbb{L}_{\tilde{\tau}}^T(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Phi(\zeta,\eta))\mathbb{L}_{\tilde{\tau}}(\eta). \tag{14}$$

with the lifting operator $\mathbb{L}_{\tilde{\tau}}(\zeta)=(I\ I\zeta\ \cdots\ I\zeta^{\tilde{\tau}})$.

To facilitate the dissipativity-based analysis of the effect of communication rate on distributed MPC performance, we adopt the following concept of a dissipative trajectory, which follows from our previous work (Tippett and Bao , 2013).

Definition 1. [Dissipative Trajectory (Tippett and Bao , 2013)] The i-th controller $\mathcal{C}_i$ is said to trace a dissipative trajectory with respect to a supply rate, $Q_{\Phi_{\mathrm{c}_i}}$, at all instants k, if the following inequality is satisfied within $t\in\mathbb{Z}^+$.

$$\mathcal{W}_{\mathrm{c}_i}=\sum_{k=0}^{t}Q_{\Phi_{\mathrm{c}_i}}(y(k),u(k))\geq 0. \tag{15}$$

3. MAIN RESULTS

In our previous studies (Tippett and Bao , 2013, 2014b), the predicted process trajectories are exchanged among controllers at every instant k. Chemical plants, however, often have slower plant-wide dynamics than the dynamics of all local processes (Kumar and Daoutidis , 2002). Therefore, it is possible to use a slower communication rate. In this section, we study the effects of reduced communication rate on a plant-wide system by studying its dissipativity properties. The reduced rate communication can be modelled using a communication switch, which is introduced in Section 3.2. This allows for the effect of slower communication rates on plant-wide stability and performance to be studied, and allows for the redesign of existing control systems with slower communication rates.

3.1 Dissipative Formulation of Individual Subsystems

Let $Q_{\Phi_i}(\hat{y},\hat{u})$ represents the QdF-supply rate of the i-th process with respect to the extended signal space $(\hat{y}_i^T,\hat{u}_{\mathrm{p}_i}^T,\hat{u}_{\mathrm{c}_i}^T,\hat{d}_i^T)$, this supply rate is induced by the polynomial matrix

$$\Phi_i(\zeta,\eta)=\begin{pmatrix}\mathcal{Q}_i(\zeta,\eta) & \mathcal{S}_i(\zeta,\eta)\\ \mathcal{S}_i^T(\zeta,\eta) & \mathcal{R}_i(\zeta,\eta)\end{pmatrix}, \tag{16}$$

with

$$\mathcal{Q}_i=Q_i(\zeta,\eta), \tag{17}$$

$$\mathcal{S}_i=(S_{p_i}(\zeta,\eta),S_{\mathrm{c}_i}(\zeta,\eta),S_{\mathrm{d}_i}(\zeta,\eta))\,, \tag{18}$$

$$\mathcal{R}_i=\begin{pmatrix}R_{\mathrm{pp}_i}(\zeta,\eta) & R_{\mathrm{pc}_i}(\zeta,\eta) & R_{\mathrm{pd}_i}(\zeta,\eta)\\ R_{\mathrm{pc}_i}^T(\zeta,\eta) & R_{\mathrm{cc}_i}(\zeta,\eta) & R_{\mathrm{cd}_i}(\zeta,\eta)\\ R_{\mathrm{pd}_i}^T(\zeta,\eta) & R_{\mathrm{cd}_i}^T(\zeta,\eta) & R_{\mathrm{dd}_i}(\zeta,\eta)\end{pmatrix}. \tag{19}$$

Analogously, the QdF-supply rate of the i-th controller $\mathcal{C}_i$ is induced by Φ_{c_i} with respect to the extended signal space $(\hat{y}_{\mathrm{l}_i}^T,\hat{y}_{\mathrm{r}_i}^T,\hat{u}_{\mathrm{l}_i}^T,\hat{u}_{\mathrm{r}_i}^T)^T$, where

$$\Phi_{\mathrm{c}_i}=\begin{pmatrix}\mathcal{Q}_{\mathrm{c}_i} & \mathcal{S}_{\mathrm{c}_i}\\ \mathcal{S}_{\mathrm{c}_i}^T & \mathcal{R}_{\mathrm{c}_i}\end{pmatrix}, \tag{20}$$

with

$$\mathcal{Q}_{\mathrm{c}_i}=\begin{pmatrix}Q_{\mathrm{ll}_i}(\zeta,\eta) & Q_{\mathrm{lr}_i}(\zeta,\eta)\\ Q_{\mathrm{lr}_i}^T(\zeta,\eta) & Q_{\mathrm{rr}_i}(\zeta,\eta)\end{pmatrix}, \tag{21}$$

$$\mathcal{S}_{\mathrm{c}_i}=\begin{pmatrix}S_{\mathrm{ll}_i}(\zeta,\eta) & S_{\mathrm{lr}_i}(\zeta,\eta)\\ S_{rl_i}(\zeta,\eta) & S_{\mathrm{rr}_i}(\zeta,\eta)\end{pmatrix}, \tag{22}$$

$$\mathcal{R}_{\mathrm{c}_i}=\begin{pmatrix}R_{\mathrm{ll}_i}(\zeta,\eta) & R_{\mathrm{lr}_i}(\zeta,\eta)\\ R_{\mathrm{lr}_i}^T(\zeta,\eta) & R_{\mathrm{rr}_i}(\zeta,\eta)\end{pmatrix}. \tag{23}$$

3.2 Analysis of Reduced-Rate Communication Network

To aid the analysis of the effects of the controller communication, the dissipativity of the process network with controllers and *without* communication is studied below.

Lemma 2. Consider system $\mathcal{P}_{\mathrm{gp}}$, as shown in Fig. 1, which consists of the plant-wide process $\mathcal{P}$ and the stacked controller $\mathcal{C}$ with supply rates Q_Φ and Q_{Φ_c}, respectively. Denote Q_Ψ the nonnegative storage function of $\mathcal{P}$. The dissipativity of all processes are given in Proposition 1 and all controllers trace dissipative trajectories (as in Definition 1). If $\mathcal{P}_{\mathrm{gp}}$ traces dissipative inequality at $t \in \mathbb{Z}^+$, such that

$$\sum_{k=0}^{t} Q_M = \sum_{k=0}^{t} Q_\Phi + Q_{\Phi_c} \geq Q_\Psi \geq 0. \tag{24}$$

Then, the supply rate of $\mathcal{P}_{\mathrm{gp}}$, Q_M, is induced by

$$M = \begin{pmatrix} \mathbb{X}_{11} & \mathbb{X}_{12} & \mathbb{X}_{13} \\ \mathbb{X}_{12}^T & \mathbb{X}_{22} & \mathbb{X}_{23} \\ \mathbb{X}_{13}^T & \mathbb{X}_{23}^T & \mathbb{X}_{33} \end{pmatrix}, \tag{25}$$

where

$$\begin{aligned}
\mathbb{X}_{11} &= \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12}^T & \Gamma_{22} & \Gamma_{23} \\ \Gamma_{13}^T & \Gamma_{23}^T & \Gamma_{33} \end{pmatrix}, \\
\mathbb{X}_{12} &= (R_{\mathrm{lr}}^T F_{\mathrm{I}} \; S_{\mathrm{lr}}^T \; S_{\mathrm{rr}}^T)^T, \\
\mathbb{X}_{13} &= (S_{\mathrm{d}}^T + R_{\mathrm{pd}}^T H_{\mathrm{p}} F_{\mathrm{p}} \; R_{\mathrm{cd}}^T \; \mathbf{0})^T, \\
\mathbb{X}_{22} &= R_{\mathrm{rr}}, \\
\mathbb{X}_{23} &= \mathbf{0}, \\
\mathbb{X}_{33} &= R_{\mathrm{dd}}, \\
\Gamma_{11} &= Q + S_{\mathrm{p}} H_{\mathrm{p}} F_{\mathrm{p}} + F_{\mathrm{p}}^T H_{\mathrm{p}}^T S_{\mathrm{p}}^T + F_{\mathrm{p}}^T H_{\mathrm{p}}^T R_{\mathrm{pp}} H_{\mathrm{p}} F_{\mathrm{p}}, \\
&\quad + F_I^T R_{ll} F_I \\
\Gamma_{12} &= S_{\mathrm{c}} + F_{\mathrm{p}}^T H_{\mathrm{p}}^T R_{\mathrm{pc}} + F_{\mathrm{I}}^T S_{\mathrm{ll}}^T, \\
\Gamma_{13} &= F_{\mathrm{I}}^T S_{\mathrm{lr}}, \\
\Gamma_{22} &= R_{\mathrm{cc}} + Q_{\mathrm{ll}}, \\
\Gamma_{23} &= Q_{\mathrm{lr}}, \\
\Gamma_{33} &= Q_{\mathrm{rr}}.
\end{aligned}$$

Proof. The dissipativity of $\mathcal{P}$ and $\mathcal{C}$ can be defined by the diagonally stacked of the supply rates of their substituent subsystems dissipativity, i.e. $Q = \mathtt{diag}(Q_1, \ldots, Q_i, \ldots, Q_{n_{\mathrm{p}}})$. The controllers trace dissipative trajectories with the supply rate Q_{Φ_c}, and the plant-wide process $\mathcal{P}$ is dissipative with respect to the supply rate Q_Φ. Therefore, $\sum Q_M = \sum Q_\Phi + Q_{\Phi_c} \geq Q_\Psi \geq 0$ for all $k > 0$. Denote $w_{dp} = \left(\hat{y}^T,\, \hat{y}_{\mathrm{l}}^T,\, \hat{y}_{\mathrm{r}}^T,\, \hat{u}_{\mathrm{r}}^T,\, \hat{d}^T\right)^T$. The supply rate of the plant-wide process with the stacked controller in Fig. 1 can rewritten in terms of w_{dp}, which is induced by the polynomial matrix M in (25). □

Remark 1. The above result includes the dissipativity of the remote signals. If controllers acts in a decentralized manner, i.e., $\hat{u}_{\mathrm{r}}(k) = \hat{y}_{\mathrm{r}}(k) = \mathbf{0}$, (25) reduces to $\begin{pmatrix} \mathbb{X}_{11} & \mathbb{X}_{13} \\ \mathbb{X}_{13}^T & \mathbb{X}_{33} \end{pmatrix}$.

The communication network $\hat{H}_c$ described in Section 2.1 can be represented as a time invariant system after lifting into the period $\tilde{\tau} + 1$ as

$$\hat{H}_{\mathrm{c}}(\eta) = H_{\mathrm{c}} \begin{pmatrix} P_0(\eta) I & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ P_{\tilde{\tau}}(\eta) I & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}. \tag{26}$$

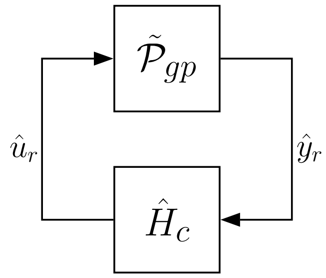

Fig. 3. Plant-wide system with communication network

In other words, the lifted input-output relationship of this communication network is governed by the following

$$\mathbb{L}_{\tilde{\tau}}(\eta) \hat{u}_{\mathrm{r}}(k) = \hat{H}_{\mathrm{c}}(\eta) \mathbb{L}_{\tilde{\tau}}(\eta) \hat{y}_{\mathrm{r}}(k), \tag{27}$$

where $\mathbb{L}_{\tilde{\tau}}$ is lifting operator defined in Lemma 1.

Remark 2. $P_\ell(\eta)$ in (26) describes the dynamics of the switch. The switch can act as zeroth or first-order hold, i.e. $P_\ell(\eta) = I$, $\forall \ell \in \mathbb{Z}^+$ or $P_\ell(\eta) = \eta^\ell I$, $\forall \ell \in \mathbb{Z}^+$, respectively.

The lifted dissipativity of the closed-loop is as follows.

Theorem 2. Consider a lifted plant-wide system represented as the closed loop of $\tilde{\mathcal{P}}_{\mathrm{gp}}$ and a reduced-rate communication network $\hat{H}_{\mathrm{c}}$ (based on H_{c} in (26)), as shown in Fig. 3, where $\tilde{\mathcal{P}}_{\mathrm{gp}}$ is the lifted system $\mathcal{P}_{\mathrm{gp}}$ with the communication period $\tilde{\tau} + 1$. Consider conditions such that (1) the process model $\mathcal{P}$ (in Fig. 1) is dissipative with a supply rate Q_ϕ with storage function $Q_\Psi \geq 0$; (2) the stacked controller $\mathcal{C}$ traces a nonnegative dissipative trajectory; (3) the communication network $\hat{H}_{\mathrm{c}}$ is dissipative with respect to the supply rate $Q_{\mathrm{M_{com}}}$ induced by $M_{\mathrm{com}} = \begin{pmatrix} \mathcal{Q}_{\mathrm{com}} & \mathcal{S}_{\mathrm{com}} \\ \mathcal{S}_{\mathrm{com}}^T & \mathcal{R}_{\mathrm{com}} \end{pmatrix}$. For all external disturbances d and all instants $t \in \mathbb{Z}^+$, the plant-wide system is dissipative with respect to the supply rate $Q_{\tilde{M}}$ induced by

$$\tilde{M} = \begin{pmatrix} \tilde{\mathbb{X}}_{11} & \tilde{\mathbb{X}}_{13} \\ \tilde{\mathbb{X}}_{13}^T & \tilde{\mathbb{X}}_{33} \end{pmatrix}, \tag{28a}$$

where

$$\begin{aligned}
\tilde{\mathbb{X}}_{11} &= \begin{pmatrix} \tilde{\Gamma}_{11} & \tilde{\Gamma}_{12} & \tilde{\Gamma}_{13} \\ \tilde{\Gamma}_{12}^T & \tilde{\Gamma}_{22} & \tilde{\Gamma}_{23} \\ \tilde{\Gamma}_{13}^T & \tilde{\Gamma}_{23}^T & \tilde{\Gamma}_{33} \end{pmatrix}, \\
\tilde{\mathbb{X}}_{13} &= \mathbb{L}_{\tilde{\tau}}(\zeta) \mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{13}) \mathbb{L}_{\tilde{\tau}}(\eta), \\
\tilde{\mathbb{X}}_{33} &= \mathbb{L}_{\tilde{\tau}}(\zeta) \mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{33}) \mathbb{L}_{\tilde{\tau}}(\eta), \\
\tilde{\Gamma}_{11} &= \mathbb{L}_{\tilde{\tau}}(\zeta) \mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{11}) \mathbb{L}_{\tilde{\tau}}(\eta), \\
\tilde{\Gamma}_{12} &= \mathbb{L}_{\tilde{\tau}}(\zeta) \mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{12}) \mathbb{L}_{\tilde{\tau}}(\eta), \\
\tilde{\Gamma}_{13} &= \mathbb{L}_{\tilde{\tau}}(\zeta) (\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{13}) + \mathtt{diag}_{\tilde{\tau}+1}(F_I^T R_{\mathrm{lr}}) \hat{H}_{\mathrm{c}}) \mathbb{L}_{\tilde{\tau}}(\eta), \\
\tilde{\Gamma}_{22} &= \mathbb{L}_{\tilde{\tau}}(\zeta) \mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{22}) \mathbb{L}_{\tilde{\tau}}(\eta), \\
\tilde{\Gamma}_{23} &= \mathbb{L}_{\tilde{\tau}}((\zeta) \mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{23}) + \mathtt{diag}_{\tilde{\tau}+1}(S_{\mathrm{lr}}) \hat{H}_{\mathrm{c}}) \mathbb{L}_{\tilde{\tau}}(\eta), \\
\tilde{\Gamma}_{33} &= \mathbb{L}_{\tilde{\tau}}(\zeta) (\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{33} + \mathcal{R}_{\mathrm{com}}) \\
&\quad + \hat{H}_{\mathrm{c}}(\zeta) \mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{22} + \mathcal{Q}_{\mathrm{com}}) \hat{H}_{\mathrm{c}}(\eta) \\
&\quad + \mathtt{diag}_{\tilde{\tau}+1}(\mathcal{S}_{\mathrm{rr}}^T + \mathcal{S}_{\mathrm{com}}^T) \hat{H}_c(\eta) \\
&\quad + \hat{H}_{\mathrm{c}}(\zeta) \mathtt{diag}_{\tilde{\tau}+1}(\mathcal{S}_{\mathrm{rr}} + \mathcal{S}_{\mathrm{com}})) \mathbb{L}_{\tilde{\tau}}(\eta).
\end{aligned}$$

Proof. The overall supply rate of the plant-wide system is the linear combination of their lifted supply rates, which is induced by $\tilde{M} = \mathbb{L}_{\tilde{\tau}}(\zeta) \mathtt{diag}_{\tilde{\tau}+1}(M + M_{\mathrm{com}}) \mathbb{L}_{\tilde{\tau}}(\eta)$.

According to Lemma 2, the closed-loop system satisfies the dissipative inequalities $\sum_{k=0}^{t} Q_{\tilde{M}} \geq Q_{\tilde{\Psi}} \geq 0$. The above result can be obtained by using those definitions and formulation of network in (26) and (27). □

With Theorem 1, the following result provides the plant-wide stability condition for the lifted plant-wide system.

Theorem 3. Consider a lifted plant-wide system with $\mathcal{P}_{\rm gp}$ and the controller network with conditions, as described in Theorem 2. This plant-wide system from external disturbances $\hat{d}$ to plant-wide output $\hat{y}_{pw} = (\hat{y}^T, \hat{y}_1^T, \hat{y}_{\rm r}^T)^T$ is asymptotically stable if

$$\tilde{\mathbb{X}}_{11} < 0. \tag{29}$$

Proof. To aid readability only a sketch of the proof is given. For vanishing disturbances, the lifted supply rate $Q_{\tilde{M}}$ becomes negative definite if $\tilde{\mathbb{X}}_{11} < 0$. Then, the dissipation inequality $Q_{\tilde{M}} \geq Q_{\nabla\tilde{\Psi}}$ implies $Q_{\nabla\tilde{\Psi}} < 0$. Together with Theorem 1 and $\Psi > 0$, this lifted plant-wide system from external disturbances to plant-wide output is asymptotically stable. □

The following result is derived in the special case that the communication switch $\hat{I}$ acts as zero-order hold.

Corollary 1. Consider a lifted plant-wide system is represented as a closed loop of $\tilde{\mathcal{P}}_{\rm gp}$ and the communication network $\hat{H}_c = \begin{pmatrix} H_{\rm c} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ H_{\rm c} & \mathbf{0} & \dots & \mathbf{0} \end{pmatrix}$, as shown in Fig. 3. All processes, controllers and the communication network follow the conditions as given in Theorem 2. Then, the lifted closed-loop system is dissipative with respect to the supply rate $Q_{\tilde{M}'}$ induced by

$$\tilde{M}' = \begin{pmatrix} \tilde{\mathbb{X}}_{11} & \tilde{\mathbb{X}}_{13} \\ \tilde{\mathbb{X}}_{13}^T & \tilde{\mathbb{X}}_{33} \end{pmatrix}, \tag{30}$$

where

$$\tilde{\mathbb{X}}_{11} = \begin{pmatrix} \tilde{\Gamma}_{11} & \tilde{\Gamma}_{12} & \tilde{\Gamma}_{13} \\ \tilde{\Gamma}_{12}^T & \tilde{\Gamma}_{22} & \tilde{\Gamma}_{23} \\ \tilde{\Gamma}_{13}^T & \tilde{\Gamma}_{23}^T & \tilde{\Gamma}_{33} \end{pmatrix},$$
$$\tilde{\mathbb{X}}_{13} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{13})\mathbb{L}_{\tilde{\tau}}(\eta),$$
$$\tilde{\mathbb{X}}_{33} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{33})\mathbb{L}_{\tilde{\tau}}(\eta),$$
$$\tilde{\Gamma}_{11} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{11})\mathbb{L}_{\tilde{\tau}}(\eta),$$
$$\tilde{\Gamma}_{12} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{12})\mathbb{L}_{\tilde{\tau}}(\eta),$$
$$\tilde{\Gamma}_{13} = \mathbb{L}_{\tilde{\tau}}(\zeta)\begin{pmatrix} \Gamma_{13}+(\tilde{\tau}+1)F_1^T R_{\rm lr} H_{\rm c} & \mathbf{0} \\ \mathbf{0} & \mathtt{diag}_{\tilde{\tau}}(\Gamma_{13})) \end{pmatrix}\mathbb{L}_{\tilde{\tau}}(\eta),$$
$$\tilde{\Gamma}_{22} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{22})\mathbb{L}_{\tilde{\tau}}(\eta),$$
$$\tilde{\Gamma}_{23} = \mathbb{L}_{\tilde{\tau}}(\zeta)\begin{pmatrix} \Gamma_{23}+(\tilde{\tau}+1)S_{\rm lr}H_{\rm c} & \mathbf{0} \\ \mathbf{0} & \mathtt{diag}_{\tilde{\tau}}(\Gamma_{23})) \end{pmatrix}\mathbb{L}_{\tilde{\tau}}(\eta)$$
$$\tilde{\Gamma}_{33} = \mathbb{L}_{\tilde{\tau}}(\zeta)\begin{pmatrix} \mathcal{Z} & \mathbf{0} \\ \mathbf{0} & \mathtt{diag}_{\tilde{\tau}}(\Gamma_{33}+\mathcal{R}_{\rm com})) \end{pmatrix}\mathbb{L}_{\tilde{\tau}}(\eta),$$

and

$$\mathcal{Z} = \Gamma_{33} + \mathcal{R}_{\rm com} + (\tilde{\tau}+1)(\mathcal{Q}_{\rm com} + H_{\rm c}^T(\mathbb{X}_{22} + \mathcal{Q}_{\rm com})H_{\rm c} + H_{\rm c}^T(\mathcal{S}_{\rm rr} + \mathcal{S}_{\rm com}) + (\mathcal{S}_{\rm rr}^T + \mathcal{S}_{\rm com}^T)H_{\rm c}).$$

Proof. The proof is similar to Theorem 2, but using the communication network $\hat{H}_{\rm c} = \begin{pmatrix} H_{\rm c} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ H_{\rm c} & \mathbf{0} & \dots & \mathbf{0} \end{pmatrix}$. By using this switching network, the lifted signal $(\hat{u}_{\rm r}^T(k), \dots, \hat{u}_{\rm r}^T(k+\tilde{\tau}))^T$ is equal to $(H_{\rm c}\hat{y}_{\rm r}^T(k), \dots, H_{\rm c}\hat{y}_{\rm r}^T(k))^T$. Then, the induced polynomial matrix $\tilde{M}$ becomes $\tilde{M}'$. □

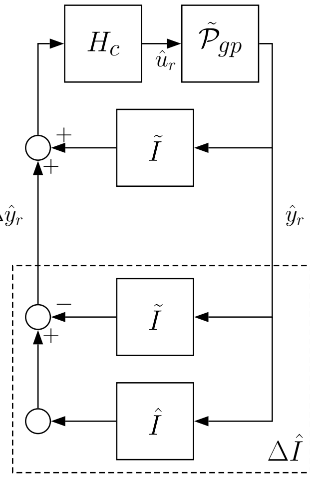

Fig. 4. Virtual partitioning of the lifted plant-wide system with reduced-rate communication

Remark 3. Theorem 2 and Corollary 1 can be interpreted by employing an error gain from $\hat{y}_{\rm r}$ to $\Delta\hat{y}_{\rm r}$, which is the sensitivity of the control system to the changes in exchanged information. For the purpose of illustration, the plant-wide system may be virtually partitioned as shown in Fig. 4. It is reasonable that $\tilde{\mathcal{P}}_{\rm gp}$ is stable with regular communication. That is to say that the distributed control system can stabilize the plant-wide system if it communicates every sampling instant. The communication network is the series connection of the controller topology $H_{\rm c}$ and the lifted identity matrix $\tilde{I} = \mathtt{diag}_{\tilde{\tau}+1}(I)$ (i.e. distributed MPC which communicates every sampling instant). The lower block in the dashed box represents the difference between regular communication and reduced-rate communication, which is formulated as $\Delta\hat{I} = \hat{I} - \tilde{I} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ I & -I & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ I & \mathbf{0} & \dots & -I \end{pmatrix}$. This has a maximum singular value of $\sqrt{\tilde{\tau}+1}$. This implies that longer communication periods yield larger upper bounds on the error gain γ_Q. Using small gain arguments, this in turn requires the gain of the upper loop in Fig. 4 (the sensitivity of the control system to communication) to be smaller.

The following result ensures a minimum plant-wide performance of the system with communication period $(\tilde{\tau}+1)$.

Theorem 4. Consider a lifted plant-wide system, as described in Theorem 2. If the supply rate of the lifted plant-wide system, from external disturbances $\hat{d}$ to the plant-wide outputs $\hat{y}_{\rm pw} = (\hat{y}^T, \ \hat{y}_1^T, \ \hat{y}_{\rm r}^T)^T$, is induced by $\tilde{M} = \begin{pmatrix} \tilde{\mathbb{X}}_{11} & \tilde{\mathbb{X}}_{13} \\ \tilde{\mathbb{X}}_{13}^T & \tilde{\mathbb{X}}_{33} \end{pmatrix}$ with $\tilde{\mathbb{X}}_{11} < 0$, then the plant-wide performance level

$$\|\mathcal{W}\tilde{\hat{y}}_{\rm pw}\|_2 \leq \|\tilde{\hat{d}}\|_2, \tag{31}$$

is guaranteed with $\mathcal{W}(\eta) = \dfrac{1}{p(\eta)}\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}}$, where $\underline{\sigma}(\mathcal{W}(j\omega)) \geq \dfrac{1}{\gamma}$ $\forall\omega \in [0, 2\pi]$ and a scalar polynomial $p(\eta)$ such that its coefficient column vector, p, satisfies $p^T p \geq \max(\overline{\sigma}(\tilde{\mathbb{X}}_{33} + \tilde{\mathbb{X}}_{13}^T \hat{\tilde{\mathbb{X}}}_{11} \tilde{\mathbb{X}}_{13}), \overline{\sigma}(\tilde{\mathbb{X}}_{13}^T \hat{\tilde{\mathbb{X}}}_{11} \tilde{\mathbb{X}}_{13}))$.

Proof. According to Proposition 1, the plant-wide system traces a dissipative trajectory, which implies

$$\sum_{t=0}^{k} \tilde{y}_{\mathrm{pw}}^T(t)\tilde{\mathbb{X}}_{11}\tilde{y}_{\mathrm{pw}}(t) + 2\tilde{y}_{\mathrm{pw}}^T(t)\tilde{\mathbb{X}}_{13}\tilde{\hat{d}}(t) + \tilde{\hat{d}}^T(t)\tilde{\mathbb{X}}_{33}\tilde{\hat{d}}(t) \geq Q_\Psi(k+1) - Q_\Psi(0), \tag{32}$$

where the order of the extended signals is equal to $\tilde{n}\tilde{\tau}$. For convenience, the time dependence is dropped in the following inequalities. Assuming $Q_\Psi(0) = 0$. we have

$$\sum_{t=0}^{k} \tilde{\hat{d}}^T \tilde{\mathbb{X}}_{33} \tilde{\hat{d}} \geq \sum_{t=0}^{k} \tilde{y}_{\mathrm{pw}}^T \hat{\tilde{\mathbb{X}}}_{11} \tilde{y}_{\mathrm{pw}} - 2\tilde{y}_{\mathrm{pw}}^T \tilde{\mathbb{X}}_{13} \tilde{\hat{d}}, \tag{33}$$

where $\hat{\tilde{\mathbb{X}}}_{11} = -\tilde{\mathbb{X}}_{11}$. Completing the square leads to

$$\sum_{t=0}^{k} [\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}} \tilde{y}_{\mathrm{pw}} - \hat{\tilde{\mathbb{X}}}_{11}^{-\frac{1}{2}} \tilde{\mathbb{X}}_{13}]^T [\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}} \tilde{y}_{\mathrm{pw}} - \hat{\tilde{\mathbb{X}}}_{11}^{-\frac{1}{2}} \tilde{\mathbb{X}}_{13}] \leq \tag{34}$$

$$\sum_{t=0}^{k} \tilde{\hat{d}}^T [\tilde{\mathbb{X}}_{33} + \tilde{\mathbb{X}}_{13}^T \hat{\tilde{\mathbb{X}}}_{11} \tilde{\mathbb{X}}_{13}^T] \tilde{\hat{d}}. \tag{35}$$

Let p be a row vector with appropriate dimension such that $p^T p \geq \max(\tilde{\mathbb{X}}_{33} + \tilde{\mathbb{X}}_{13}^T \hat{\tilde{\mathbb{X}}}_{11} \tilde{\mathbb{X}}_{13}^T, \tilde{\mathbb{X}}_{13}^T \hat{\tilde{\mathbb{X}}}_{11} \tilde{\mathbb{X}}_{13}^T)$. The following can be obtained using the reverse triangle inequality.

$$\|\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}} \tilde{y}_{\mathrm{pw}}\| - \|\hat{\tilde{\mathbb{X}}}_{11}^{-\frac{1}{2}} \tilde{\mathbb{X}}_{13} \hat{d}\| \leq \|p\tilde{\hat{d}}\|,$$
$$\|\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}} \tilde{y}_{\mathrm{pw}}\| \leq \|\hat{\tilde{\mathbb{X}}}_{11}^{-\frac{1}{2}} \tilde{\mathbb{X}}_{13} \hat{d}\| + \|p\tilde{\hat{d}}\|,$$
$$\|\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}} \tilde{y}_{\mathrm{pw}}\| \leq 2\|p\tilde{\hat{d}}\|.$$

It is clear that $\|\tilde{y}_{\mathrm{pw}}\| \leq \gamma \|\tilde{\hat{d}}\|$, if $\frac{\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}}}{2p} \geq \frac{1}{\gamma}$. □

4. CONCLUSIONS

A dissipativity-based framework for studying the effects of reduced communication rates on the plant-wide stability with distributed MPC has been developed. This allows for a reduction in network traffic in the controller network, and may find application in processes where the plant-wide dynamics occur on a time-scale slower than that of the individual processes. The dissipativity conditions capture the interactions between the processes and controllers. This approach provides new insights into the analysis of plant-wide systems and the effect of communication rate from the perspective of plant-wide dissipativity. Further work may include determination of an optimal communication rate and network topology. In addition to distributed MPC as discussed in the current paper, the proposed analysis approach may also be applied to closed-form controllers.

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