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Assessment of Model-Plant Mismatch by the Nominal Sensitivity Function for Unconstrained MPC

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Abstract: Model Predictive Control (MPC) is a class of control systems which use a dynamic process model to predict the best future control actions based on past information. Thus, a representative process model is a key factor for its correct performance. Therefore, the investigation of model-plant-mismatch effect is very important issue for MPC performance assessment, monitoring, and diagnosis. This paper presents a method for model quality evaluation based on the investigation of closed-loop data and the nominal complementary sensitivity function. The proposed approach ensures that the MPC tuning is taken into account in the evaluation of the model quality. A SISO case study is analyzed and the results show the effectiveness of the method.

Keywords: Model Predictive Control, Model Plant Mismatch, Sensitivity Function, Control Performance Assessment

1. INTRODUCTION

The industrial use of Model Predictive Controllers (MPC) has increased significantly in the last years, due to growing requirement of more profitability and safety associated with the reduction of pollution and energy consume. This kind of control systems uses a dynamic process model to predict the behavior of controlled variables along a future horizon based on past control actions and knows disturbances. From this result, an optimization algorithm calculates the control actions that lead the process to the optimal operational condition.

The maintenance of MPC is a challenging problem due to multicausal nature. The performance degradation can be caused by model plant mismatch, bad tuning , bad set of soft and hard constraints, and unmeasured disturbances (Sun *et al.*, 2013). Among these many sources, the poor model quality is the most frequent and impactful one. Several methods are focused on model quality investigation. A class of the methods such as: Huang *et al.* (2003), Conner *et al.* (2005), Jiang et *al.* (2012) are based on investigate the need of system re-identification. Other approaches (e.g., Badwe et *al.* (2009), Kano *et al.* (2010), Ji *et al.* (2012) and Sun *et al.* (2013)) are looking for the locations in the model (i.e. the pairs controlled-manipulated variables) responsible for the performance degradation.

A quite common temptation is simply to simulate the model using the control action as inputs and compare the simulated results with the real plant outputs. However, in the context of process control this is not a good approach, since it does not take into account the feedback effect produced by the controller, which can compensate part of the model mismatch that is not critical for closed loop control performance.

A good model should represent the real system in the frequency where the MPC works. Therefore, the metric to quantify the model plant mismatch (MPM) should consider the feedback effect, so that the effective impact on the closed loop performance can be correctly quantified. Here, we propose a new metric for MPM based on the nominal complementary sensitivity function. Although this approach can be applied for any controller type, it is particularly useful for MPC, since the kernel of this control strategy is a model. Based on the proposed approach it is possible to quantify the MPM for MPC.

The first work that uses the sensitivity function for model quality assessment was presented by Badwe *et al.* (2010), where an identified sensitivity function was used to quantify the impact of model-plant mismatch in MPC performance. Although the results demonstrate its efficiency, this method have a narrow applicability, since it requires the identification of a sequence of models and, for that, requires performing several perturbations in the setpoints, which is not always trivial to be obtained in practical terms. The method proposed

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here circumvents such limitations through determining of a benchmark response of the controlled variable without any data-based model identification.

Section 2 introduces the proposed approach. Section 3 suggests some diagnosis rules for model inconsistency detection. The method is tested using an SISO MPC and the results are discussed in section 4. Section 5 compares our approach with Badwe *et al.* (2010) method. Section 6 finalizes the paper with conclusions and final remarks.

2. PROPOSED METHOD

Consider a control loop free of disturbances illustrated in Figure 1 where $C(s)$ is the controller, $G_0(s)$ is the nominal model, $\Delta G(s)$ is the model-plant-mismatch, i.e. the difference between $G_0(s)$ and the real plant model $G(s)$, $r(s)$ is the set-point, $u(s)$ is the manipulated variable, $y(s)$ is the measured output and $\hat{y}(s)$ is the simulated output.

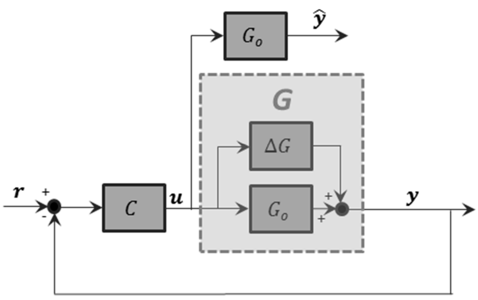

Fig. 1. Schematic diagram of closed-loop system.

The measured output can be calculated by the closed loop transfer function $T(s)$ defined by:

$$T(s) = \frac{GC}{1+GC} = \frac{y(s)}{r(s)} \quad \textbf{(1)}$$

Being the nominal closed loop transfer function, $T_0(s)$, given by:

$$T_0(s) = \frac{G_0 C}{1+G_0 C} = \frac{y_0(s)}{r(s)} \quad \textbf{(2)}$$

The model-plant mismatches, $\Delta G = G - G_0$, produces a corresponding closed loop plant mismatch ΔT given by:

$$\Delta T = T - T_0 \quad \textbf{(3)}$$

<u>Theorem:</u> The closed-loop output free from model-plant-mismatch, called nominal output (y_0), can be estimated from the difference between measured and simulated output by filtering with the nominal complementary sensitivity function T_0, according to following equation:

$$\boldsymbol{y_0(s) = y(s) - [1 - T_o(s)][y(s) - \hat{y}(s)]} \quad \textbf{(4)}$$

Proof:

Substituting (1) and (2) in (3), after some simple manipulations we get:

$$\frac{\Delta T}{T} = \frac{GC - T_0(1+GC)}{GC} \quad \textbf{(5)}$$

Replacing G by

$$G = G_0 + \Delta G \quad \textbf{(6)}$$

in (5) it is possible to rewrite this equation as follows:

$$\frac{\Delta T}{T} = \frac{(G_0+\Delta G)C - T_o[1+(G_0+\Delta G)C]}{(G_0+\Delta G)C} = \frac{\Delta G - \Delta G T_o}{(G_0+\Delta G)} = (1-T_0)\frac{\Delta G}{G} \quad \textbf{(7)}$$

Now, it is possible to apply the following equivalent expressions:

$$\frac{y(s) - \hat{y}(s)}{y(s)} = \frac{Gu(s) - G_0 u(s)}{Gu(s)} = \frac{\Delta G}{G} \quad \textbf{(8)}$$

to convert (7) into a more appropriated formulation, which can be applied directly to the measured (y) and simulated ($\hat{y}$) outputs, i.e.,

$$\frac{\Delta T}{T} = (1-T_0)\left[\frac{y-\hat{y}}{y}\right] \quad \textbf{(9)}$$

In a similar way, it is possible base on (1) to substitute $T(s)$ by $y(s)/r(s)$ what transform (9) into:

$$(1-T_0)\left[\frac{y-\hat{y}}{y}\right] = \frac{\Delta T\, r}{y} \quad \textbf{(10)}$$

With these further simplifications steps:

$$(1-T_0)[y - \hat{y}] = \Delta T\, r \quad \textbf{(11)}$$

$$(1-T_0)[y - \hat{y}] = y - y_0 \quad \textbf{(12)}$$

We finally arrive to (4):

$$\boldsymbol{y_0(s) = y(s) - [1 - T_0(s)][y(s) - \hat{y}(s)]} \quad \textbf{(13)}$$

Q.E.D

Equation (4) presents a useful tool to work directly with the measured (y) and simulated ($\hat{y}$) data. It shows that it is possible to estimate the behavior that would be happen in closed loop in case of no model plant mismatch at current tuning. y_0 may be considered as a benchmark for the model quality for control, so that, for good model in closed loop the difference between y and y_0 should be small. Note that the model can produce a good result in closed loop even if his quality in open loop is quite different, i.e., even in the case where the difference between y and $\hat{y}$ is large.

The nominal closed loop response (T_0) can be obtained analytically from the nominal process model (G_0) and controller model (C), as showed in eq. (2). However, considering the complexity of MPC formulation it is more simple and practical to identify it using simulation data in closed loop with the MPC considering no model-plant-mismatch. Note that it is only necessary to apply this procedure again if the MPC tuning or nominal model is changed.

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3. MODEL QUALITY ASESSMENT

Since y_0 is an estimation of the output process in case of an inexistence of model-plant-mismatch and unmeasured disturbance, it could be considered as a model output response benchmark. From this benchmark, any output performance indicator can be applied. The diagnosis procedure is represented by Figure 2.

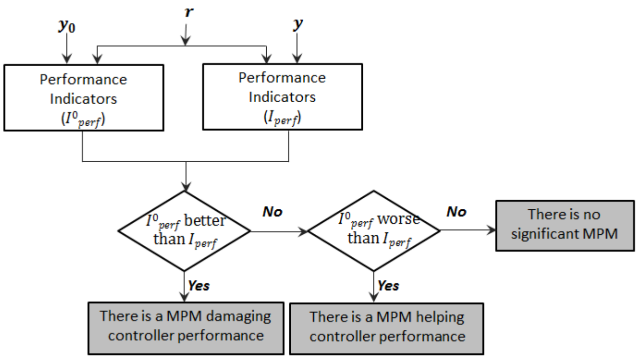

Fig 2. Diagnosis procedure of proposed methodology.

For example, a useful indicator is the comparison of control errors variances, as suggest by Badwe *et al.* (2010):

$$I_{var} = \frac{var(y-r)}{var(y_0-r)} \qquad \textbf{(14)}$$

Another possibility is the analysis of autocorrelation function (ACF) of control errors (i.e., $y-r$ and y_0-r) . The ACF is an indicator of the correlation of a temporal series with itself. A high value of ACF means that the current control error is strongly correlated with past errors. High ACF is an undesirable behavior for a control systems. It can be also used to identify oscillatory behavior of control loops (Kempf,2003).

4. APPLICATION OF PROPOSED METHOD

To illustrate the application of the proposed approach, a SISO MPC was configured in MATLAB. For this controller, a slow and a fast tuning were considered, by changing the Move Suppression parameter. The tuning parameters are summarized in Table 2 and the corresponding nominal sensitivity function ($S_0 = 1 - T_0$) are presented in Figure 3. Table 1 shows the three different models that will be considered as real plant. Note that these models have model-plant mismatch in different dynamic regions, as illustrated in Figure 4.

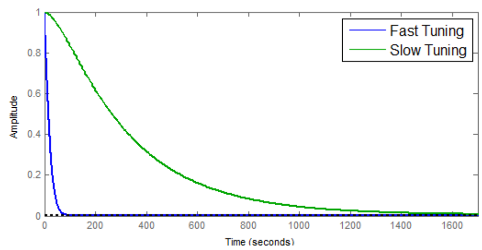

Fig 3. The nominal sensitivity response ($S_0 = 1 - T_0$) for Fast and Slow Tuning.

Table 1: Plant and Controller Models

Model 1	**Model 2**	**Model 3**
$\frac{3.04}{63.1s+1}$	$\frac{3.04}{960s^2+33s+1}$	$\frac{4}{80s+1}$

Table 2. Tuning parameters of MPC

Tuning Parameter	**Fast Tuning**	**Slow Tuning**
Sample Time	15	15
Prediction Horizon	32	32
Control Horizon	8	8
CV Weight	550	550
Move Suppression	100	50000

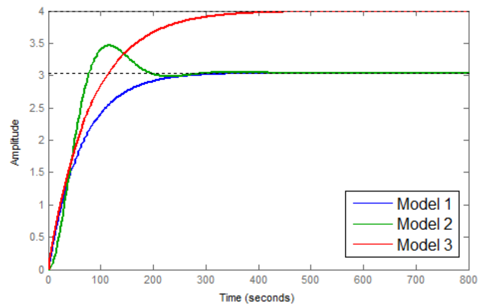

Fig 4. Step Response of Plant and Controller Models

Combining the tunings with the models, different scenarios are configured according to Table 3. The controller is configured using Model 1 in all scenarios. The basis cases are Scenarios BF and BS, corresponding to situations without model-plant-mismatches. Simulations are performed considering a series of step changes in the set-point. No disturbance or noise were added. The comparative results of data with and without model-plant-mismatch and the resultant $\hat{y}$ are presented on Figures 5 to 8.

Table 3 : MPC SISO: Scenario Configuration

Scenario Name	**Controller Model**	**Controller Tuning**	**Plant Model**
Base case Slow (BS)	Model 1	Slow	Model 1
Base case Fast (BF)	Model 1	Fast	Model 1
Model 2 Slow (M2S)	Model 1	Slow	Model 2
Model 2 Fast (M2F)	Model 1	Fast	Model 2
Model 3 Slow (M3S)	Model 1	Slow	Model 3
Model 3 Fast (M3F)	Model 1	Fast	Model 3

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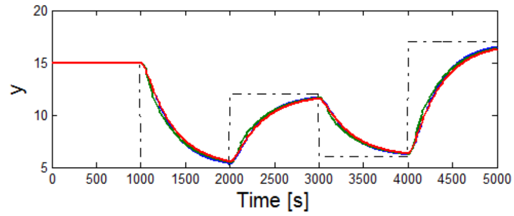
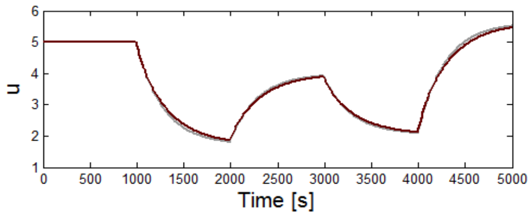
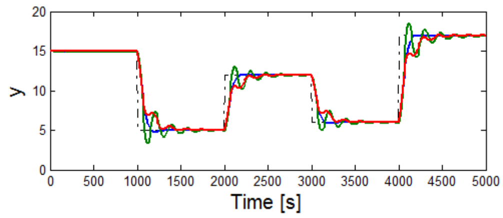


(a)

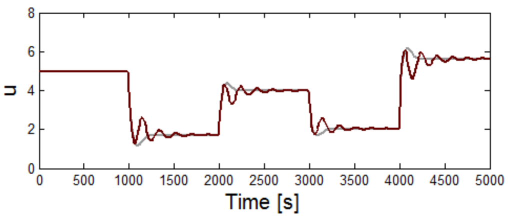


(b)

Fig 5. Comparison of M2S and BS outputs (a) and inputs (b). The green line is the M2S output measurement, the red line is the corresponding output simulation, the blue line is the BS output measurement, the brown and grey lines are the input measurements of M2S and BS, respectively.

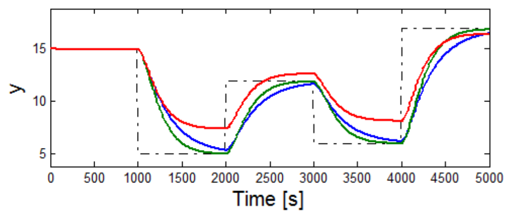


(a)

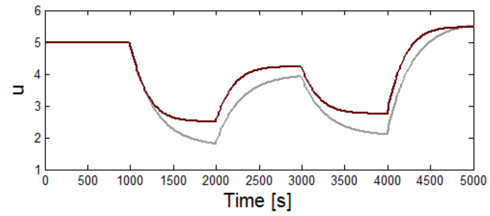


(b)

Fig 6. Comparison of M2F and BF outputs (a) and inputs (b). The green line is the M2F output measurement, the red line is the corresponding output simulation, the blue line is the BF output measurement, the brown and grey lines are the input measurements of M2F and BF, respectively.

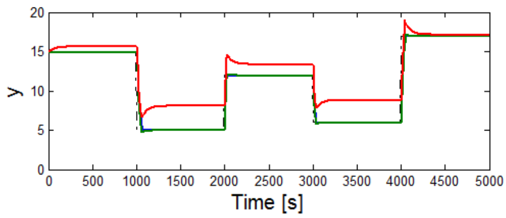


(a)

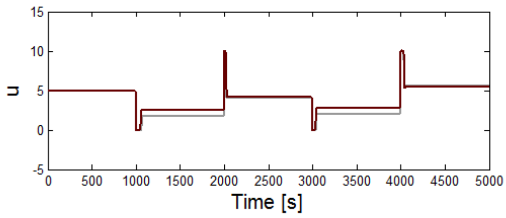


(b)

Fig 7. Comparison of M3S and BS outputs (a) and inputs (b). The green line is the M3S output measurement, the red line is the corresponding output simulation, the blue line is the BS output measurement, the brown and grey lines are the input measurements of M3S and BS, respectively.



(a)



(b)

Fig 8. Comparison of M3F and BF outputs (a) and inputs (b). The green line is the M3F output measurement, the red line is the corresponding output simulation, the blue line is the BF output measurement, the brown and grey lines are the input measurements of M3F and BF, respectively.

The results above show that the model-plant-mismatches cause different effects, depending on the MPC tuning. In Figure 5, there is no significant effect of model-plant-mismatch. This is consistent with our expectations because the tuning is slow and Model 2 and Model 1 have the same steady state behavior, making the result very similar to BS. Similarly, it is expected that M3F and BF present similar results because Model 3 and Model 1 have the same initial dynamic behavior, as showed by Figure 8. Scenarios M2F and M3S are most sensitive to the model-plant-mismatches, as demonstrated by Figure 6 and 7. A relative evaluation of results is performed by the investigation of variance index (eq. 14) and comparative ACF. The result is presented on Table 4 and Figure 9. The results demonstrate that, although the mismatches are not very intensive in any scenario, Scenarios M2S and M3F present the most similar Auto-Correlation Function (ACF) and I_{var} nearest to 1. The ACF indicates that a mismatch in scenario M2F causes an oscillatory behavior in the system. The mismatch in scenario

M3S has a helpful effect on performance, because I_{var} is lower than 1. All these results are consistent with showed on Figures 5 to 8.

Table 4: Relative Variance Index (I_{var})

Scenario	I_{var}
M2S	0.94
M2F	1.12
M3S	0.86
M3F	0.96

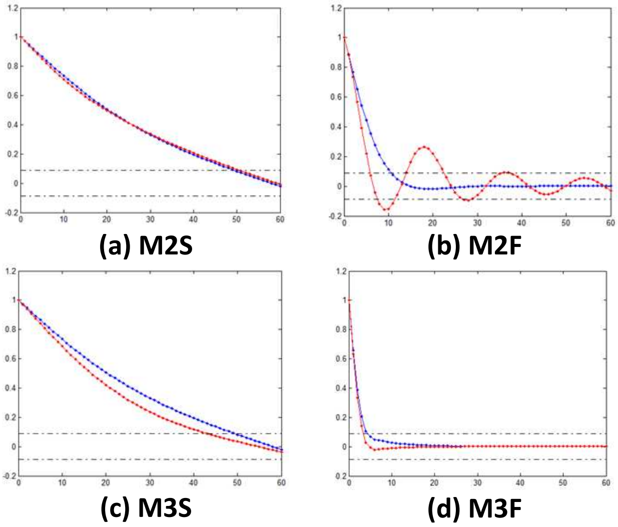


Fig 9. Comparative Autocorrelation Function (ACF) of $y - r$ (red line) and $y_0 - r$ (blue line): Scenario M2S (a), M2F (b), M3S (c) and M3F (d).

5. ESTIMATION OF y_0: COMPARISION WITH THE METHOD OF BADWE ET AL. (2010)

Badwe *et al.* (2010) proposed also method based on system sensitivity to quantify the impact of model-plant mismatch for MPC performance. The method consists in the identification of an Output-Error (OE) model for the design sensitivity (S_d) (eq. 15). Then another OE model is identified to quantify ΔGC (eq. 16) and finally the signal is reconstructed according to eq. 17.

$$(r - y) = S_d(r - y + \hat{y}) \quad \textbf{(15)}$$

$$(y - \hat{y}) = \Delta GC(r - y + \hat{y}) \quad \textbf{(16)}$$

$$r - y_0 = S_d(1 + \Delta GC)\,(r - y + \hat{y}) \quad \textbf{(17)}$$

The Badwe *et al.* (2010) nominal output was estimated for all scenarios of section 4, considering estimation of third order OE model. Results are compared with our proposed method as well as the output obtained using the base case in Figures 10 to 13. A quantitative comparison is presented in Table 5, performed according to eq. 18, where z is the approach used for y_0 (i.e., proposed or Badwe *et al.* 2010) estimation and y_{base} is the data of corresponding basis case, and $nsample$ is the number of sampled data.

$$SQR_z = \sum_{i=1}^{nsample} |y_{base} - y_{0_z}| \quad \textbf{(18)}$$

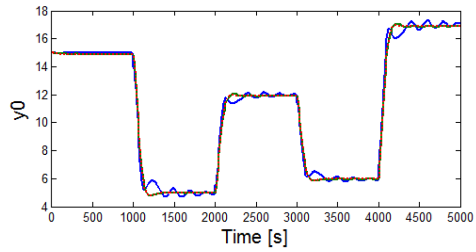


Fig. 10. Comparative result of y_0 for M2F: Badwe *et al.* (2010) approach (blue line), proposed method (green line) and BF output (red line).

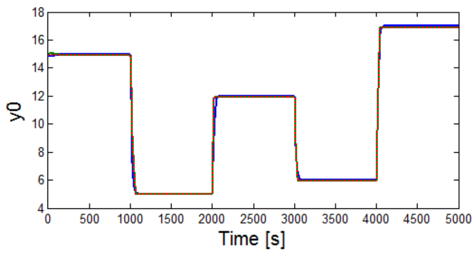


Fig. 11. Comparative result of y_0 for M3F: Badwe *et al.* (2010) approach (blue line), proposed method (green line) and BF output (red line).

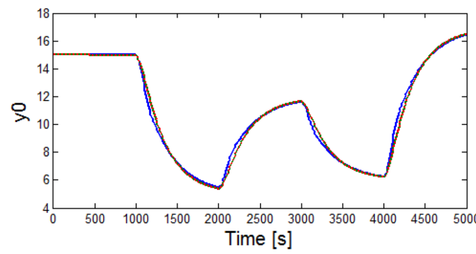


Fig. 12. Comparative result of y_0 for M2S: Badwe *et al.* (2010) approach (blue line), proposed method (green line) and BF output (red line).

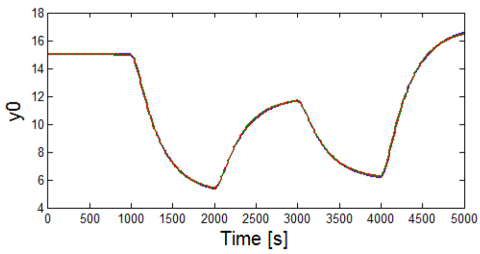


Fig. 13. Comparative result of y_0 for M3S: Badwe *et al.* (2010) approach (blue line), proposed method (green line) and BF output (red line).

Table 5: SQR for Badwe et a. (2010) and proposed method.

Scenario	y_{0_z}	SQR
M2S	**Badwe *et al.* (2010)**	83.53
	Proposed Method	0.8163
M2F	**Badwe *et al.* (2010)**	118.5
	Proposed Method	9.582
M3S	**Badwe *et al.* (2010)**	9.302
	Proposed Method	3.011
M3F	**Badwe *et al.* (2010)**	57.92
	Proposed Method	19.06

Figures 10 to 13 and Table 5 show that both approaches are capable to provide y_0, however, the proposed method has superior results when compared with Badwe *et al*. (2010), since the results are nearest to the base case. It occurs because the existence of two model identifications steps in Badwe *et al.* (2010) method, what inserts uncertainties in the results and is strongly dependent of the data quality and input excitation. Furthermore, the best model order must be determined, what can be considered an additional drawback of the Badwe *et al.* (2010) method. Ultimately, our approach has the advantage to be independent of setpoint, which makes it flexible to be used in several industrial applications where variables are controlled by operating intervals instead of a single setpoint.

6. CONCLUSIONS

A methodology for quantifying the model-plant-mismatch (MPM) was developed and applied for MPC performance monitoring. The proposed approach takes into account the closed loop effect produced by different controller tunings. Based on this nominal behavior it is possible to define an output in closed loop that would be expected in case of no MPM, i.e., y_0. Two different indices have been proposed considering a comparison between the nominal and measured outputs.

The proposed indices have been tested using a simple and illustrative case study. The results show that the MPM is directly impacted by controller tuning, since a same MPM causes different effects in the process, depending on the MPC moving suppression. The indices based on y_0 are capable to detect significant effects of MPM in MPC performance.

The estimation of the nominal output by the proposed approach has superior quality in relation to Badwe *et al.* (2010) method. It occurs because the existence of two model identifications steps in Badwe *et al.* (2010), what inserts uncertainties in the results and is strongly dependent of the data quality, input excitation and model order selection. Furthermore, the proposed method is independent of setpoints, which makes it flexible to be used in several industrial applications where variables are controlled by operating intervals.

7. ACKNOWLEDGEMENTS

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