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Stochastic proxy modelling for coalbed methane production using orthogonal polynomials ⋆

Gouthami Senthamaraikkannan * **Vinay Prasad** * **Ian Gates** **

* *University of Alberta, Department of Chemical and Materials Engineering, Edmonton T6G2V4, AB, Canada*
** *University of Calgary, Dept. Chemical and Petroleum Engineering, Calgary T2N1N4, AB, Canada*

Abstract: Uncertainty in data or in the parameters of models occurs in many real world applications. Quantifying this uncertainty and its effects is required for robust design, control and optimization. In this paper, we attempt to build a proxy model for the stochastic solutions of coupled governing equations describing coalbed methane (CBM) production at different well bottomhole pressures. To achieve this, monthly production from wells (output) is expanded as a linear combination of Legendre orthogonal polynomials in the input (well bottomhole pressure) and the Wiener-Askey polynomial chaos is used to propagate the uncertainty of the model parameters. A Gaussian quadrature technique is then employed to solve for the coefficients of the basis functions in the proxy model. Alternatively, nonlinear least squares curve fitting using the Levenberg-Marquardt algorithm (LMA) is also used with polynomial chaos expansion to generate the stochastic proxy model. The proxy model now enables robust optimization using statistical metrics of CBM production calculated over the entire parameter space. In the case of multiple decision variables, the appropriate proxy model built using these techniques will allow for robust optimization without the use of any search algorithms.

Keywords: Uncertainty, stochastic modelling, polynomial chaos expansion, least-squares approximation, regression, robust estimation

1. INTRODUCTION

Many applications in science and engineering require mathematical models which can simulate solutions for a physical variable of interest along spatial and temporal dimensions. The simulations are usually not fully deterministic due to the presence of uncertain parameters/input random variables (Fagiano and Khammash (2012)). Different methods are available for the propagation of uncertainty. The Monte Carlo method is a popular technique where simulations are performed for a large number of values sampled from a known distribution of the random source. Although this method is robust, it requires a large number of simulations, and is therefore computationally expensive. Another method, the power series expansion (PSE), coupled with contour mapping techniques was explored for distributional robustness analysis in Nagy and Braatz (2007). However it is less acurate for lower order expansions and requires confirmation *a posteriori* using Monte Carlo simulations. The same work establishes that the polynomial chaos expansion (PCE) method for uncertainty propagation usually gives better results even with relatively lower order approximations.

PCE is a method used for uncertainty propagation in nonlinear dynamic systems and was introduced by Wiener as homogeneous chaos. It is derived from the Cameron Martin theorem which states that an expansion in Hermite polynomials in Gaussian random variables converges in the L_2 sense for any arbitrary stochastic process with finite second moment (Dutta and Bhattacharya (2010)). Xiu and Karniadakis (2002) extended these results to represent stochastic processes with an optimum trial basis from the Askey family of orthogonal polynomials, that reduces dimensionality of the polynomial chaos expansions and leads to exponential convergence of error. This came to be known as the Wiener-Askey polynomial chaos. It expands a stochastic output X as,

$$X(\theta) = \sum_{j=0}^{p} a_j \Psi_j(\xi)$$

where Ψ_j is the polynomial basis (in the random variable ξ), belonging to a complete orthogonal basis. The best choice of orthogonal polynomials for a PCE is related to the probability distribution of the random source. Projection of the PCE onto each polynomial basis Ψ_j will result in estimation of expansion coefficients a_j and the expansion error is orthogonal to the functional space spanned by the basis functions (Ghanem and Dham (1998)).

PCE for an output variable is thus an effective way to track uncertainty evolution. In this paper, we introduce the input variable also into the PCE, thus developing a robust proxy model that can significantly reduce compu-

⋆ Corresponding author, email:vprasad@ualberta.ca. Financial support from Carbon Management Canada and the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

tational time in optimization studies. The main concern in building the proxy model, however, is obtaining the underlying functional relation between the input and output in the presence of uncertainty. When the analytical structure of the relation is explicitly known, the Levenberg-Marquardt algorithm (LMA) is used with PCE to generate the stochastic proxy model.When this is not the case, we suggest the use of orthogonal polynomials to weave the input variable(s) into the PCE. Orthogonal polynomials in the input variable will allow accurate evaluation of coefficients of the proxy model (by Gaussian quadrature rules) using only a few simulations according to the dimensionality. Thus, a robust proxy model can be built for a certain output variable by expansion with two orthogonal basis sets, the Legendre orthogonal polynomials corresponding to the input (assuming uniform distribution for the input to avoid any preferences while choosing collocation points) and the Hermite orthogonal polynomials corresponding to the random source from a known/assumed distribution or a Bayesian estimate as suggested by Mandur and Budman (2012). This technique would be computationally superior to other robust optimization methods presented Mandur and Budman (2012), Xiong et al. (2011) and Molina-Cristobal et al. (2006), which evaluate metrics of the objective function (by constructing a PCE) at each decision variable (input) value in the search space.

The approach to proxy model development is demonstrated on coalbed methane production. Coalbed methane is the gas naturally occuring in coalbeds due to thermogenic or biogenic processes, and is an important unconventional source of natural gas. Geologic heterogeneity, the existence of multiple porosity scales, coal matrix shrinkage/swelling, varying pressure-temperature conditions and many other phenomena lead to significant uncertainty in assessing CBM production. For simplicity, a proxy model for monthly coalbed methane gas production is built with respect to the input parameter (well bottomhole pressure), while considering uncertainty only in the micropore diffusion time constant (τ).

2. CBM MODEL

Coalbed seams are highly heterogeneous with a wide range in the scale of pore spaces occurring within coal. For simulation purposes, coal seams are broadly assumed to have two levels of porosity - micropores and macropores. In this study, the production of gas from a hortizontal well drilled into a coalbed seam containing only gas phase is simulated by solving a 1D radial equation representing the multi-step transport process described using a pseudo steady-state sorption model for gas desorption, Fick's law for diffusion through micropores, Darcy's law for gas flow through open fractures (i.e., the macroporous spaces) and gas slippage factor for surface diffusion through the surface of solid coal (Wei et al. (2007), Jalal and Mohaghegh (2004)). A cylindrical volume of coal reservoir, considered for simulations with a horizontal well drilled at its center is shown in Fig. 1. The other assumptions made in the model are:

(1) Gas permeability through macropores, the gas diffusion constant and geometry dependent factor for diffusion through the micropore matrix are constant throughout the spatial volume and over time.

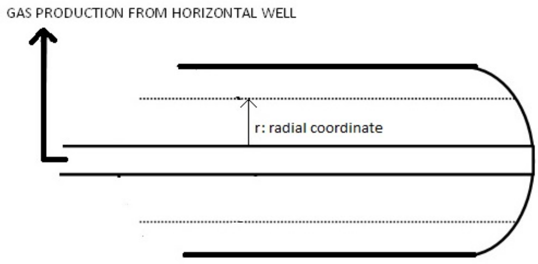

Fig. 1. Gas production from horizontally drilled well in a cylindrical reservoir volume

(2) Flow velocity is assumed to have only a radial component.
(3) There is no change of temperature in the coal seam during degasification. The gas compressibility factor Z and viscosity μ are considered constant.

Consider a small cylindrical control volume dV in the macroporous space consisting of micropores. The following coupled governing equations are derived for radial gas flow into a producer well operating at a constant pressure of P_{wf}:

$$\int_{V_{bma}} \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{rP_g}{Z}\left(\frac{k}{\mu}+\frac{D_{ma}}{P_g}\right)\frac{\partial P_g}{\partial r}\right)dV + \int_{V_{bma}} \frac{P_{sc}T}{T_{sc}}\frac{q_t}{V_{bma}}dV = \int_{V_{bma}}\frac{\partial}{\partial t}\left(\frac{P_g\phi_{ma}}{Z}\right)dV \qquad (1a)$$

$$\frac{q_t}{V_{bma}} = \frac{dV_{mi}}{dt} - L_{mi}\tau(V_E(P_g) - V_{mi}); V_{mi}^0 = \frac{V_l P_{g0}}{P_l + P_{g0}} \qquad (1b)$$

$$\text{Production} = 2\pi r_{well}h_{well}\left(\lambda + D_{ma}/P_g)\frac{dP}{dr}\right)_{well} \qquad (1c)$$

The partial differential equations were discretised at equally separated time intervals and logarithmically spaced spatial co-ordinates, and was solved using the IMPLICIT method. The model variables and parameters are defined in Table 1. The nominal values of all the parameters were obtained by history matching the model against existing CBM production data from Manville wells.

$$\pi_1\left(P\tilde{F}_{ma}\tilde{P}_g^{N+1}\right)_{i+1/2}(\pi_2 + \frac{1}{\tilde{P}_{gi+1/2}^{N+1}})(\tilde{P}_{gi+1}^{N+1} - \tilde{P}_{gi}^{N+1}) - \pi_1\left(P\tilde{F}_{ma}\tilde{P}_g^{N+1}\right)_{i-1/2}(\pi_2 + \frac{1}{\tilde{P}_{gi-1/2}^{N+1}})(\tilde{P}_{gi}^{N+1} - \tilde{P}_{gi-1}^{N+1}) + \pi_3(V_{mi}^N - \frac{P_{gi}^N V_l}{P_l + P_{gi}^N}) = \frac{(\tilde{P}_{gi}^{N+1} - \tilde{P}_{gi}^N)}{\delta T} \qquad (2a)$$

$$V_{mi}{}_i^{N+1} = exp(-\tau\delta t)V_{mi}{}_i^N + (1 - exp(-\tau\delta t))\,V_E(P_g)_i^N \qquad (2b)$$

$$\text{Production} = 2\pi r_{well}h_{well}(\lambda + D_{ma}/P_{gr_{well}}^N)\left(\frac{P_{g(r_{well}-1)}^N - P_{wf}}{r_{well} - r_{well-1}}\right) \qquad (2c)$$

A proxy to the above model is built with P_{wf}, the well bottomhole pressure, as the input (u) and τ, the micropore diffusion time constant, as the only source of uncertainty

Table 1. Notations

$\lambda = k/\mu$	Mobility
D_{ma}	Gas Slippage
τ	Micropore diffusion time constant
L_{mi}	Geometry dependent factor for micropore diffusion
P_l, V_l	Langmuir adsorption constants
ϕ_{ma}	Macropore porosity
V_{bma}	Macropore volume
r_{well}, h_{well}	Dimensions of producing well
r, t	Continuous spatial and temporal coordinate
N	N^{th} discrete spatial co-ordinate
i	i^{th} discrete time step
$PF_{ma\,i+1/2}$	$\frac{2r_{i+1/2}}{(r^2_{i+1/2}-r^2_{i-1/2})(r_{i+1}-r_i)}$
P_g	Gas pressure at r, t
V_{mi}	Volume of gas from micropores diffusing into the macroporous space
V_E	Volume of gas adsorbed in micropore spaces
π_1	$\frac{D_{ma}t_0}{R_0^2\phi}$
π_2	$\frac{\lambda P_0}{D_{ma}}$
π_3	$\frac{P_{sc}TL_{mi}t_0Z}{P_0T_{sc}\tau\phi}$

(ξ). The operating range of P_{wf} is assumed to lie between $1 \times 10^6\ N/m^2$ to $1 \times 10^7\ N/m^2$. τ is assumed to belong to a Gaussian distribution with mean value of 5000 s and standard deviation of 500 s. The output is the CBM gas production in the first month (m^3), denoted as Y. The functional dependence of CBM production on time can be estimated by aggregating proxy models built at different time instants. It is to be noted here that including all the input variables and uncertain sources into the proxy model will increase dimensionality and consequently, the computational expense.

3. PROXY MODEL DEVELOPMENT

The first step is to establish a functional relation between the input (u) and the output (Y). Any given function $f(x)$ can be approximated by minimizing the inner product, $\langle f(x) - p(x), f(x) - p(x)\rangle$ where $p(x)$ is a combination of a sequence of orthogonal polynomials, $p_0(x), ..., p_k(x)$. The chosen family of orthogonal polynomials represent an orthogonal basis for the subspace of polynomial functions of degree $\leq k$. The inner product is defined as

$$\langle g, h\rangle = \int_a^b g(x)h(x)w(x)dx = \sum_{i=1}^{n} g(x_i)h(x_i)w(x_i) \quad (3)$$

where $g(x), h(x)$ belong to the class of orthogonal polynomials and $w(x)$ is the weighting function.

Thus, Y can be expanded as

$$Y = a_0 + a_1L_1(u) + a_2L_2(u) + a_3L_3(u) + ... \quad (4)$$

where u is the input variable and $L_0, L_1, ..$ belong to the sequence of Legendre polynomials that are orthogonal on the interval $[-1, 1]$. Legendre polynomials are chosen because their weighting function is a constant (this will prevent any bias in selection of input variable during estimation or prediction). The range for P_{wf} is projected on to a range of $[-1, 1]$ for u.

Table 2. Orthogonal polynomials

Hermite	Legendre
$H_0(\xi) = 1$	$L_0(u) = 1$
$H_1(\xi) = \xi$	$L_1(u) = u$
$H_2(\xi) = \xi^2 - 1$	$L_2(u) = u^2 - 0.33$
$H_3(\xi) = \xi^3 - 3\xi$	$L_3(u) = u^3 - 0.6u$
$H_4(\xi) = \xi^4 - 6\xi^2 + 3$	$L_4(u) = u^4 - 0.8571u^2 + 0.0848$
$H_5(\xi) = \xi^5 - 10\xi^3 + 15\xi$	$L_5(u) = u^5 - 1.11u^3 + 0.2372u$
$H_6(\xi) = \xi^6 - 15\xi^4 + 45\xi^2 - 15$	$L_6(u) = u^6 - 1.364u^4 + 0.4549u^2 - 0.0214$

The next step is uncertainty propagation on the orthogonal polynomials in the expansion of equation 4 using Wiener-Askey chaos. To illustrate this, let Y be defined as a second-order polynomial (in u) and the stochastic process in each direction (i.e. the orthogonal basis used to define Y) be approximated by second order Hermite polynomials in the standard Gaussian random variable, ξ. Hermite polynomials are orthogonal on $[-\infty, \infty]$ relative to the weight function $e^{\frac{-x^2}{2}}$, which is similar to the probability density function of a Gaussian distribution.

$$\begin{aligned} Y = a_0(b_0^1 + b_1^1H_1(\xi) + b_2^1H_2(\xi))(L_0(u)) \\ +a_1(b_0^2 + b_1^2H_1(\xi) + b_2^2H_2(\xi))(L_1(u)) \\ +a_2(b_0^3 + b_1^3H_1(\xi) + b_2^3H_2(\xi))(L_2(u)) \end{aligned} \quad (5)$$

The random variable τ in the CBM model and ξ are linearly related as $\frac{\tau-5000}{500} = \xi$.

The first few Legendre and Hermite polynomials are shown in Table 2. These are generated using ORTHOPOL by Gautschi (1994).

At certain known values of the input, the expansion in equation 5 reduces to

$$\begin{aligned} Y = (a_0b_0^1L_0(u) + a_1b_0^2L_1(u) + a_2b_0^3L_2(u)) \\ + (a_0b_1^1L_0(u) + a_1b_1^2L_1(u) + a_2b_1^3L_2(u))H_1(\xi) \\ + (a_0b_2^1L_0(u) + a_1b_2^2L_1(u) + a_2b_2^3L_2(u))H_2(\xi)) \end{aligned} \quad (6)$$

The statistical moments (mean and variance) of the output distribution would then be

$$\mu = a_0b_0^1 + a_1b_0^2 + a_2b_0^3 \quad (7a)$$

$$var = (a_0b_1^1 + a_1b_1^2 + a_2b_1^3)^2 + (a_0b_2^1 + a_1b_2^2 + a_2b_2^3)^2 \quad (7b)$$

The accuracy of this proxy model depends on the set of basis polynomials chosen for the expansion and accurate evaluation of the coefficients of the basis polynomials in the proxy model.

4. ESTIMATING COEFFICIENTS OF BASIS FUNCTIONS IN THE PROXY MODEL

The coefficients of the basis functions in the proxy model are estimated using the orthogonal property of the expanding polynomials. A simple model of Y expanded using first order polynomials in u and ξ is considered to illustrate this.

$$Y = a_0(b_0^1 + b_1^1(\xi)) + a_1(b_0^2 + b_1^2(\xi))(u) \quad (8)$$

The inner product of Y with each of the Hermite polynomials and Legendre polynomials (according to the definition in equation 3) give,

$$\int_{-1}^{1}\int_{-\infty}^{\infty} \frac{Y}{5} e^{\frac{-\xi^2}{2}} d\xi du = a_0 b_0^1; \int_{-1}^{1}\int_{-\infty}^{\infty} \frac{Y}{5} \xi e^{\frac{-\xi^2}{2}} d\xi du = a_0 b_1^1 \quad (9a)$$

$$\int_{-1}^{1}\int_{-\infty}^{\infty} \frac{Y}{1.67} e^{\frac{-\xi^2}{2}} d\xi u du = a_1 b_0^2; \int_{-1}^{1}\int_{-\infty}^{\infty} \frac{Y}{1.67} \xi e^{\frac{-\xi^2}{2}} d\xi u du = a_1 b_1^2 \quad (9b)$$

The parameters of the proxy model are estimated by evaluating the above integrals. If an analytical expression was available for Y, the integrals could be evaluated easily. In its absence, non-intrusive methods characterised either as Galerkin projection methods or least squares methods are used. Galerkin projection evaluates the integrals in equation 9 using sampling approaches or Gaussian quadrature rules. The linear least squares method employed is a regression approach, also known as point collocation or the stochastic response surface method (Eldred et al. (2008), Kewlani and Iagnemma (2008)).

The sampling approach evaluates products in the integrals of equation 9 at samples within the density of the weighting function. The quadrature approach evaluates the inner products as a summation of the product of basis functions at the roots of the next higher order polynomials as described by the Gaussian quadrature technique (Webster et al. (1996)). According to Gaussian quadrature rules,

$$\text{if } \int_a^b f(x)dx = \int_a^b w(x)g(x)dx; \text{ then } \int_a^b f(x)dx \simeq \sum_c w(x)g(x)dx$$

where there are c roots of the next higher order orthogonal polynomial $h(x)$ ($\int_a^b w(x)g(x)h(x)dx = 0$). It is a very useful method when the number of basis functions in the proxy model are small, since the number of collocation points exponentially increases with an increase in the dimensionality of the expansion. The regression approach uses a linear squares solution of the form $\Psi\alpha = R$ to solve for expansion coefficients that best matches a set of response values R. However, it requires oversampling, i.e., the number of samples needs to be at least twice the number of parameters. In spite of this, the approach may still be significantly more affordable than quadrature for large problems.

Since there are only two variables in the proxy model developed in this study, the Gaussian quadrature approach is chosen as it provides accurate results with a lower number of samples. Fig. 2 represents the work flow for building the proxy model. Along with the application of this technique, a nonlinear least squares method employing the Levenberg-Marquardt algorithm for parameter estimation is also used to estimate the proxy model, provided that the structure of the functional relation between Y and u is identified. For example,

$$Y = Y_0(u) + Y_1(u)H_1(\xi) + Y_2(u)H_2(\xi) \quad (11a)$$

if known that relation between Y and u is expontential

$$Y = ae^{f(u)} + be^{g(u)}H_1(\xi) + ce^{h(u)}H_2(\xi) + ... \quad (11b)$$

Y_0, Y_1, Y_2 for a set of collocation points in ξ are evaluated at different values of u. These values are regressed against the functions $ae^{f(u)}, be^{g(u)}, ce^{h(u)}$ at the corresponding values of u.

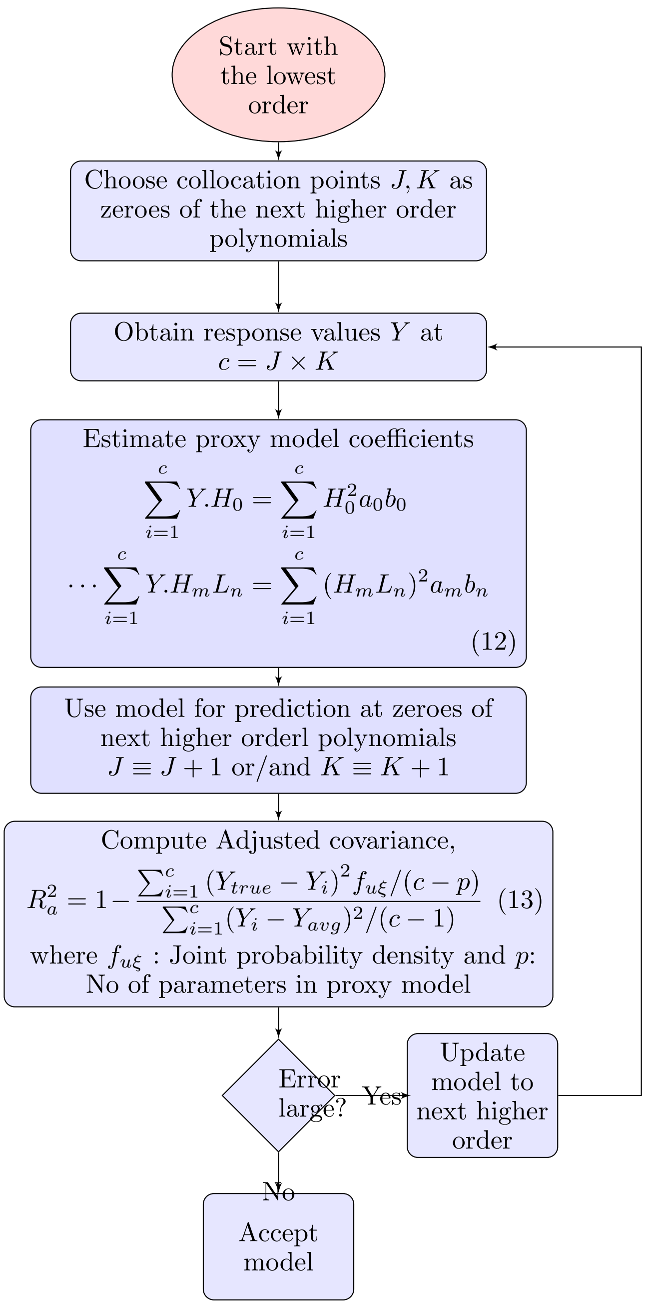

Fig. 2. Work flow for proxy model development

5. RESULTS AND DISCUSSIONS

Four different model orders - $[1, 1]$, $[2, 1]$, $[3, 1]$, $[3, 2]$ (denoting the model order in u and ξ, respectively) were tested. The expansion coefficients of the basis functions for all the four models are shown in Table 5. Collocation points of prediction were chosen to be zeros of polynomials two orders higher than the order of the model. Model predictions were compared against data obtained from simulations of the original model (equation 1), and the results are shown in Fig. 3. The adjusted coefficient of determination (R_a^2) was computed in each case to test the goodness of fit. Fig. 4 shows that, R_a^2 increases with an increase in model order with respect to u but decreases when the order of the polynomial in ξ is increased to 2. The decrease in R_a^2 at this model order could indicate over-fitting. However, computations of the expansion coefficients at this model order ($[u(3), \xi(2)]$) also involved the division of very small

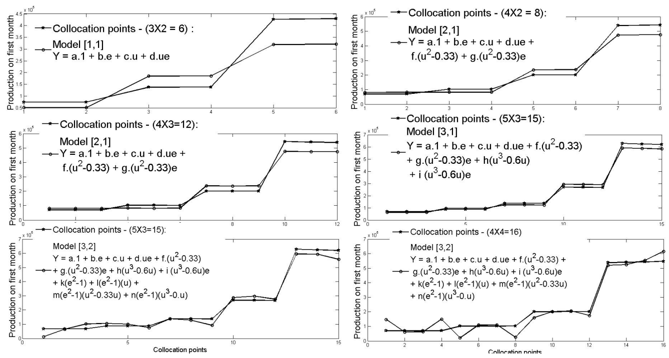

Fig. 3. Comparing performances of models built with Gaussian quadrature with simulation data from original model, equation 1

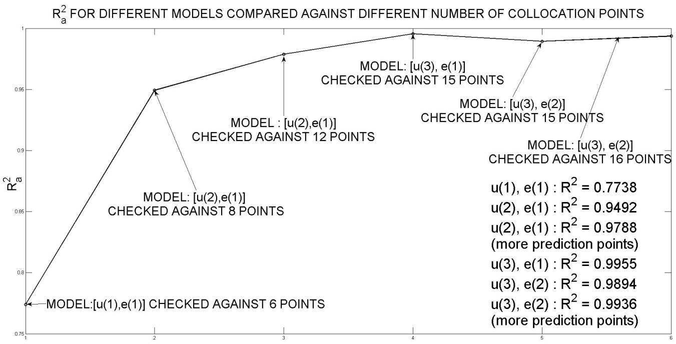

Fig. 4. Adjusted coefficient of determination for different models

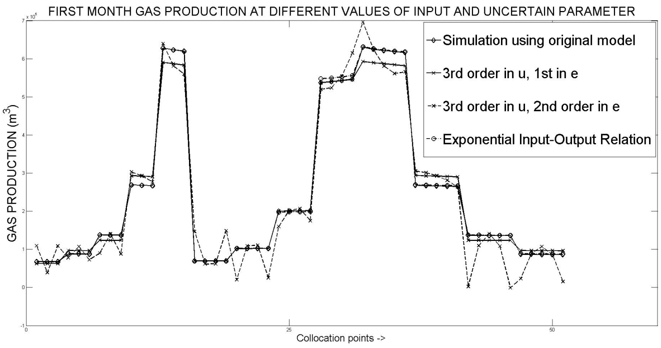

Fig. 5. Comparing third order models built with Gaussian quadrature and the Levenberg-Marquardt algorithm for a large number of collocation points against simulation data

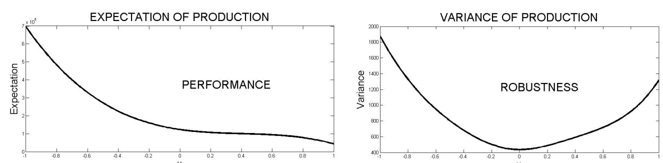

Fig. 6. Expectation & Variance of production

numbers which could have led to numerical issues. It is to be noted that R_a^2 as a criterion discredits increase in number of model parameters. Thus, the model of order $[u(3), \xi(1)]$ has the lowest prediction error. Thus, it is accepted as the proxy model for $Y = F(P_{wf}, \tau(\theta))$ where Y is the CBM production on first month, P_{wf} is the bottomhole pressure and $\tau(\theta)$ represents the uncertain value of the micropore diffusion time constant. Moments of the distribution at a known value of input u obtained from this proxy model are,

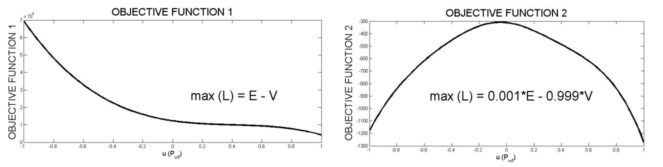

Fig. 7. The two objective function formulations

Table 3. Coefficients corresponding to each polynomial in proxy model

Polynomial	Model order in u and ξ			
	$[1,1]$	$[2,1]$	$[3,1]$	$[3,2]$
1	185215	200030	204450	204490
ξ	303	396	-436	-4976
u	-174081	-228396	-244600	-244830
$u\xi$	422	717	851	9709
$u^2-0.33$		188730	245050	245160
$u^2-0.33.\xi$		-767	-1164	-13267
$u^3-0.6u$			-202320	-202270
$u^3-0.6u.\xi$			1135.4	12991
ξ^2-1				-4350
$\xi^2-1.u$				1820
$\xi^2-1.u^2-0.33u$				38820
$\xi^2-1.u^3-0.6u$				34770

$$\mu = 204450 - 244600u + 245050(u^2 - 0.33) - 202320(u^3 - 0.6u) \tag{14a}$$

$$\sigma = \sqrt{436^2 + 851^2u^2 + 1164^2(u^2 - 0.33u)^2 + 1135^2(u^3 - 0.6u)^2} \tag{14b}$$

Since there is approximation in the way Gaussian quadrature evaluates integrals, some inner products $< \Psi_i \Psi_j >$ with $i \neq j$ are not equal to zero. As a result, the error of the estimated proxy model is not orthogonal to the subspace spanned by the basis functions present in it. Thus, the best values are not obtained for the coefficients in the proxy model resulting in errors when using it for the evaluation of higher order statistical moments of the output distribution. The inaccurate approximation of inner products also increases computational expense.

Nonlinear least squares regression was also applied for the proxy model development. Plotting $\int_{-\infty}^{\infty} Y e^{\frac{-\xi^2}{2}} d\xi$ against u revealed that the underlying functional relation between Y and u is exponential. The proxy model was developed by considering it to be second order in ξ. Model coefficients were evaluated using just 16 collocation points (chosen as roots of 4th order Legendre and Hermite polynomials for a good sample space). The proxy model obtained is

$$\begin{aligned} Y = {} & 7525e^{-4.305u} + 129800e^{-0.7288u} \\ & - 7.648e^{-6.066u}\xi - 141.8e^{-1.398u}\xi \\ & + 3.639e^{-4.5625u}\xi^2 - 1 + 6.604e^{-0.4735u}(\xi^2 - 1) \end{aligned} \tag{15}$$

Fig. 5 compares the prediction (over a large number of simulation points) based on models of order $[u(3)\xi(1)]$, $[u(3), \xi(2)]$ and when the relation between Y and u is considered to be exponential. The model of order $[u(3)\xi(2)]$

does not have adequate predictive capability whereas the model considering an exponential input-output relation displays more accurate prediction with Hermite polynomials in ξ of the order 2. This indicates that the model with order $[u(3), \xi(2)]$ would perform better if interactions between third order terms in u and second order terms in ξ are omitted.

6. ROBUST OPTIMIZATION

Robust optimization is usually performed as a trade-off between maximum performance and robustness (Mandur and Budman (2012)). In this study, we intend to find the optimum bottomhole pressure value for maximum gas production with minimum variability (i.e., low variance in production values in the presence of uncertainty). Therefore the objective function is formulated as $\max_u L = a * E(Y, u, \xi) - b * V(Y, u, \xi)$, where E and V are the expectation and variance of production Y, at any input u and a and b are the respective weights for these functions. Equation 14 provides both E and V as functions of u (P_{wf}). As is seen in Fig 6, the expectation of the production value increases with decreasing u, while variance has a minimum at $u = 0$. Fig. 7 shows the results of two objective function formulations with different values of a & b. For $a = 1, b = 1$ (weighing both E and $-V$ equally), $\max_u L$ is at the lowest value in the range of u $(= -1)$, whereas for $a = 0.001, b = 0.999$ (weighing $-V$ heavily), $\max_u L$ is at $u = -0.05$, i.e., $P_{wf} = 5.3 \times 10^6 N/m^2$.

7. CONCLUSIONS

We have developed a stochastic proxy model that propagates uncertainty in the micropore diffusion time constant (random source) sampled from a Gaussian distribution, to the monthly coalbed methane gas production (output) at different well bottomhole pressures (input) using Legendre polynomials and Hermite polynomial chaos. The coefficients of the basis functions in the proxy model are estimated by Galerkin projection using the Gaussian quadrature technique. Trial and error evaluation of model structures in increasing order shows that a model that is third order in the input variable and first order in the random source has the lowest relative sum square-root error of prediction. Although Gaussian quadrature is an efficient non-intrusive method of evaluating coefficients of basis functions, the computational expense increases with increasing dimension of the proxy model. The error of approximation of inner products occurring from use of Gaussian quadrature reduces accuracy of the higher order statistical moments of distribution of the gas production obtained from a proxy model that appears to predict well at collocation points. Nonlinear least squares regression was also tested for developing a proxy model, and can be employed if the underlying functional relation between the input and output variables is identifiable. The results indicate that the least squares method gives better predictions as compared to Galerkin projection of the expansion of output in an orthogonal polynomial basis. However, the input-output relation is not always easily identifiable. The stochastic proxy model developed was then used for robust optimization of gas production. It is seen that the optimal point varies depending on whether performance or robustness is weighed more in the objective function.

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