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Adaptive Control of Chemical Distributed Parameter Systems ⋆

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Abstract: The adaptive output feedback control problem of chemical distributed parameter systems is investigated while the process parameters are unknown. Such systems can be usually modeled by semi-linear partial differential equations (PDEs). A combination of Galerkin's method and proper orthogonal decomposition is applied to generate a reduced order model which captures the dominant dynamic behavior of the system and can be used as the basis for Lyapunov-based adaptive controller design. The proposed control method is illustrated on thermal dynamics regulation in a tubular chemical reactor where the temperature spatiotemporal dynamic behavior is modeled in the form of a semi-linear PDE.

Keywords: Distributed parameter systems, adaptive control, model reduction, output feedback, partial differential equations, Lyapunov stability, process control

1. INTRODUCTION

Distributed parameter systems frequently appear in a wide range of chemical processes where the transport phenomena and reactions take key roles [Adomaitis (2003); Christofides (2000); Lin and Adomaitis (2001); Ray (1981)]. Such systems can be usually described by a set of semi-linear partial differential equations (PDEs) whose infinite dimensional representation can be decomposed to two subsystems; slow and fast, where a time scale separation can be recognized between these subsystems. The slow subsystem contains slow modes of the original system that can possibly be unstable and the fast subsystem includes fast modes of the original system that are stable. Using such decomposition most of the dissipative distributed parameter systems in the chemical process industries where the transport-reaction mechanisms play the essential role can be approximated by a low-dimensional reduced order model (ROM) which contains the dominant dynamics [Adomaitis (2003); Balas (1991); Bentsman and Orlov (2001); Christofides (2000); El-Farra et al. (2003)]. Then the ROM can be applied as the basis for the controller design [Balas (1991); Christofides (2000); Lao et al. (2014); Liu et al. (2014); Pourkargar and Armaou (2013a,b, 2014a,b,c, 2015a,b)].

A practical approach to obtain such ROMs is via weighted residual methods like Galerkin projection, however the bottleneck of such methods' applications is the computations of basis functions required to discretize the governing PDEs. Analytical techniques have been widely applied to compute the basis functions of linear and semi-linear DPSs by solving the eigenproblem of linear part of spatial differential operators [Christofides (2000); El-Farra et al. (2003); Lao et al. (2014); Liu et al. (2014)]. However the analytical approaches can not be used directly to compute the set of basis functions when (a) the spatial differential operator is nonlinear, (b) the process is defined over irregular domains and (c) there are unknown parameters in the spatial differential operator. Such restrictions can be circumvented by employing statistical techniques to compute the empirical basis functions from an ensemble of the system solution data instead of solving the eigenproblem of the spatial operator.

Proper orthogonal decomposition (POD) is one of the industrially used statistical methods to find the dominant components of the system dynamic behavior. POD-type approaches have been widely applied in model order reduction (MOR) based monitoring, optimization and control of DPSs [Armaou and Christofides (1999); Izadi and Dubljevic (2013); Sirovich (1987)]. All POD-based algorithms employ an ensemble of spatially distributed system solutions (snapshots) to construct the basis functions. Such snapshots can be computed using either off-line (before the process starts) or on-line (in parallel to when the process operates) high fidelity simulators when all of the process parameters are known. Such limiting condition is not satisfied for a large class of transport-reaction processes in the chemical industries when the diffusion-dispersion-convection parameters and reaction mechanisms are unknown. In such case the snapshots can be measured directly by a set of sensors which are spatially distributed in the process domain for a rich set of system inputs and initial conditions before closed-loop process operations. After basis function computation by applying POD to the ensemble of collected snapshots, the ROM that contains the unknown parameters can be obtained by employing Galerkin's projection to the PDE model. Such ROM then may be used as the basis for traditional adaptive controller designs tailored for the systems governed by ordinary differential equations (ODEs) [Âström (1983); Bentsman and Orlov (2001); Khalil (1996); Sastry and Isidori (1989)].

In this paper we focus on the POD-based low-dimensional adaptive output feedback control of transport reaction processes to bypass the limitations of direct control of PDE systems [Demetriou and Rosen (2001); Hong and Bentsman (1994); Smyshlyaev and Krstic (2010)] and to extend the MOR-based controller designs [Armaou and Christofides (1999); Balas (1991); Christofides (2000); El-Farra et al. (2003); Lao et al.

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(2014); Liu et al. (2014)] in the presence of unknown system parameters. The mathematical model of semi-linear DPSs with unknown parameters and their mathematical properties is presented in Section 2. We briefly discuss the MOR steps in Section 3; it includes short descriptions for Galerkin's method and POD. An adaptive output feedback control structure is formulated in Section 4 which is a combination of a stable Lyapunov-based adaptive controller and a static observer to estimate the system modes. The proposed low-dimensional adaptive output feedback controller is finally applied to regulate the thermal spatiotemporal dynamics in a tubular chemical reactor in Section 5.

2. SEMI-LINEAR DISTRIBUTED PARAMETER SYSTEMS AND THEIR PROPERTIES

We consider chemical distributed parameter systems with unknown transport-reaction parameters which can be described by the following set of semi-linear PDEs,

$$\begin{aligned}\frac{\partial x(z,t)}{\partial t} &= \mathcal{A}(z,\vartheta)\,x(z,t) + \mathcal{F}\left(z,x(z,t)\right) + \mathcal{G}\left(z,x(z,t)\right)\Theta \\ &\quad + \mathcal{B}(z)\,u(t),\\ y(t) &= \int_\Omega \delta(z-L_s)\,x(z,t)\,dz,\\ \Gamma\Big(t,x(z,t),\frac{\partial x(z,t)}{\partial z}\Big)_{\partial\Omega} &= 0, \quad x(z,0)=x_0(z),\end{aligned} \tag{1}$$

where $x\in\mathbb{R}^n$ denotes the vector of spatiotemporal states, t is time and $z\in\Omega$ is the vector of spatial coordinates. The process domain is presented by $\Omega\subset\mathbb{R}^3$ with the process boundary of $\partial\Omega$ and $\mathcal{A}$ denotes the second order linear spatial differential operator. The terms $\mathcal{F}^{n\times 1}$ and $\mathcal{G}^{n\times q}$ present the smooth nonlinear vector and matrix functions, respectively. We also consider $\vartheta\in\mathbb{R}^p$ and $\Theta\in\mathbb{R}^q$ as the vectors of unknown transport and reaction parameters, respectively. The vector of manipulated inputs is denoted by $u\in\mathbb{R}^l$ where $\mathcal{B}\in\mathbb{R}^{n\times l}$ describes their spatial distributions, $y\in\mathbb{R}^r$ is the vector of point measurements where the vector of L_s presents location of the sensors and δ is standard Dirac function. The initial spatial profile of the system state is denoted by x_0 and Γ is the vector of homogeneous boundary conditions.

In the PDE system of (1), the term $\mathcal{A}(z,\vartheta)\,x(z,t)$ presents transport phenomena where the diffusion, dispersion and convection parameters may be unknown. $\mathcal{F}\left(z,x(z,t)\right)$ describes the known part of nonlinear reaction dynamics and $\mathcal{G}\left(z,x(z,t)\right)\Theta$ presents the unknown-affine (linear in terms of unknown parameters) dynamics of the reaction. Note that the boundary conditions don't have to be homogeneous necessarily. To homogenize the non-homogeneous boundary conditions, we can include the non-homogeneous parts in the PDEs using standard Dirac functions.

We can represent the above PDE system as the following infinite-dimensional system [Curtain and Zwart (1995)]

$$\begin{aligned}\dot{\bar{x}}(t) &= \mathbf{A}(\vartheta)\bar{x}(t) + \mathbf{F}\big(\bar{x}(t)\big) + \mathbf{G}\big(\bar{x}(t)\big)\Theta + \mathbf{B}u(t),\\ \bar{x}(0) &= \bar{x}_0,\end{aligned} \tag{2}$$

in the Sobolev subspace of $\mathbb{W}$,

$$\mathbb{W}(\Omega) = \Big\{\mathcal{H},\ \frac{\partial\mathcal{H}}{\partial z}\in L_2(\Omega):\ \Gamma\Big(\mathcal{H},\frac{\partial\mathcal{H}}{\partial z}\Big)_{\partial\Omega}=0\Big\},$$

endowed with inner product

$$(\mathcal{H}_1,\mathcal{H}_2) = \int_\Omega \mathcal{H}_1(z)^*\mathcal{H}_2(z)\,dz$$

by defining the infinite-dimensional state of $\bar{x}(t)\in\mathbb{W}$,

$$\bar{x}(t) = x(z,t),$$

the linear operator with unknown parameters,

$$\mathbf{A}(\vartheta)\bar{x}(t) = \mathcal{A}(z,\vartheta)\,x(z,t),$$

the nonlinear vector and matrix functions,

$$\mathbf{F}\big(\bar{x}(t)\big) = \mathcal{F}\left(z,x(z,t)\right),\quad \mathbf{G}\big(\bar{x}(t)\big)\Theta = \mathcal{G}\left(z,x(z,t)\right)\Theta,$$

and the manipulated input operator,

$$\mathbf{B}u(t) = \mathcal{B}(z)u(t).$$

We can discretize the infinite-dimensional system of (2) to infinite and countable modal set of ODEs by applying Galerkin projection to individual PDEs of the components of the spatiotemporal state vector while we also capture the interactions between the PDEs of such components. To employ such discretization we need the system basis functions which are the solution of the eigenproblem of the linear differential operator,

$$\mathbf{A}(\vartheta)\phi_i = \lambda_i\phi_i, \qquad \forall i=1,\dots,\infty \tag{3}$$

subject to

$$\Gamma\Big(\phi_i,\frac{d\phi_i}{dz}\Big)_{\partial\Omega} = 0,$$

where ϕ_i and λ_i denote the $i\,th$ basis function and eigenvalue, respectively, and $\mathbb{W}\triangleq span\{\phi_i\}_{i=1}^{\infty}$, i.e., the linear operator is a strong generator of the defined Sobolev subspace.

For the majority of chemical DPSs, specifically the transport-reaction processes, we can assume that the eigenspectrum of the linear operator, $\{\lambda_1,\lambda_2,\dots\}$, can be decomposed to the following subsets;

(1) finite subset of first m eigenvalues, $\{\lambda_1,\lambda_2,\dots,\lambda_m\}$, which indicate the slow and possibly unstable dynamics of the infinite-dimensional system,
(2) complement countable subset of the remainder eigenvalues, $\{\lambda_{m+1},\lambda_{m+2},\dots\}$, which indicate the fast and stable dynamics of the infinite-dimensional system,

when we order $Re(\lambda_1)\geq Re(\lambda_2)\geq\cdots\geq Re(\lambda_m)\geq Re(\lambda_{m+1})\geq\cdots$, and require $Re(\lambda_{m+1})<0$; here $Re(\cdot)$ denotes the real part. In addition, a time scale separation can also be recognized between the subsets by $\sigma=\frac{|Re(\lambda_1)|}{|Re(\lambda_{m+1})|}$, where σ is a small positive number.

Note that the eigenproblem of (3) can not be solved due to the unavailability of the unknown parameters vector of ϑ, that clearly shows the limitation of the analytical approach even for the system with linear dominant operator defined over simple domains. To bypass the solution of such problem we can apply POD that will be briefly discussed in the next section. However, to continue the analysis and illustrate the system properties let's assume the basis functions are available for the remainder of this section. This assumption will be lifted when the proposed method will be presented.

By considering the set of basis functions, $\{\phi_i\}_{i=1}^{\infty}$, and eigenspectrum decomposition, the Sobolev subspaces for the subsets can be defined as follows;

(1) slow subspace, $\mathbb{W}_s\triangleq span\{\phi_i\}_{i=1}^{m}$,
(2) complement fast subspace, $\mathbb{W}_f\triangleq span\{\phi_i\}_{i=m+1}^{\infty}$,

where $\mathbb{W}=\mathbb{W}_s\cup\mathbb{W}_f$. Then by considering the vector of slow and fast basis functions as

$$\Phi_s = [\phi_1\ \phi_2\ \cdots\ \phi_m]^T,\quad \Phi_f = [\phi_{m+1}\ \phi_{m+2}\ \cdots]^T,$$

and the corresponding integral Galerkin projectors of

$$\begin{aligned} &\mathcal{P}: \mathbb{W} \to \mathbb{W}_s, \quad \mathcal{P}(\cdot) = \int_\Omega (\cdot)\, \Phi_s^* \, dz, \\ &\mathcal{Q}: \mathbb{W} \to \mathbb{W}_f, \quad \mathcal{Q}(\cdot) = \int_\Omega (\cdot)\, \Phi_f^* \, dz, \end{aligned} \tag{4}$$

where φ^T and φ^* denote the transpose and adjoint of φ, we can derive the modal representation of the system as partitioned sets of ODEs,

$$\begin{aligned} &\dot{x}_s = A_s x_s(t) + F_s\big(x_s(t), x_f(t)\big) + G_s\big(x_s(t), x_f(t)\big)\Theta + B_s u(t), \\ &x_s(0) = \mathcal{P}\bar{x}_0, \\ &\dot{x}_f = A_f x_f(t) + F_f\big(x_s(t), x_f(t)\big) + G_f\big(x_s(t), x_f(t)\big)\Theta + B_f u(t), \\ &x_f(0) = \mathcal{Q}\bar{x}_0, \end{aligned} \tag{5}$$

where $\bar{x} = x_s \oplus x_f$, $A_s = \mathcal{P}\mathbf{A}$, $A_f = \mathcal{Q}\mathbf{A}$, $F_s = \mathcal{P}\mathbf{F}$, $F_f = \mathcal{Q}\mathbf{F}$, $G_s = \mathcal{P}\mathbf{G}$, $G_f = \mathcal{Q}\mathbf{G}$, $B_s = \mathcal{P}\mathbf{B}$, $B_f = \mathcal{Q}\mathbf{B}$.

According to singular perturbation analysis [Christofides (2000)] and considering Tykhonov's theorem for solution convergence of systems include slow and fast subsystems [Lobry et al. (2014)], the partitioned infinite-dimensional system of (5) can be reduced to

$$\begin{aligned} &\dot{x}_s = A_s x_s(t) + F_s\big(x_s(t), 0\big) + G_s\big(x_s(t), 0\big)\Theta + B_s u(t), \\ &x_s(0) = \mathcal{P}\bar{x}_0, \end{aligned} \tag{6}$$

when $x_f \to 0$ after a short period of time, t_b. Such required relaxation time, t_b, can be identified form singular perturbation arguments [Christofides (2000); Pourkargar and Armaou (2013a, 2015a)].

3. MODEL ORDER REDUCTION

In this section, we derive the finite-dimensional approximation of the PDE system of (1) by applying Galerkin's projection while the optimal required empirical basis functions are computed by employing POD to the ensemble of snapshots captured from spatially distributed measurement sensors during the open-loop process evolution.

3.1 Proper orthogonal decomposition

Let $w(z) = [w_1(z)\ w_2(z)\ \cdots\ w_K(z)]^T$ be the ensemble of K snapshots of the system, where $w_i(z) = x(z,t_i)$. The basis functions can be obtained by an optimization problem,

$$\begin{aligned} &max\ \frac{\overline{(\phi, w)^2}}{(\phi,\phi)}, \\ &s.t.\ \ (\phi,\phi) = 1,\ \ \phi \in L_2(\Omega), \end{aligned} \tag{7}$$

where $\overline{(\cdot)}$ denotes the ensemble average. The solution of the minimization problem of (7) takes the following integral eigenproblem form,

$$\int_\Omega \overline{\big(w(z)w(\xi)\big)}\phi(\xi)d\xi = \lambda\phi(z). \tag{8}$$

An efficient approach to solve the above integral eigenproblem is via method of snapshots [Sirovich (1987)] wherein the optimal set of required basis functions are presented by a linear combination of the snapshots,

$$\phi(z) = vw(z). \tag{9}$$

where v is the matrix of eigenvectors obtained from the solution of the following eigenvalue-eigenvector problem,

$$C_K v = \lambda v, \tag{10}$$

where the elements of the positive semidefinite covariance matrix can be constructed by

$$C_K(i,j) := \frac{1}{K}\int_\Omega w_i(\xi)w_j(\xi)d\xi. \tag{11}$$

We obtain K non-negative real eigenvalues and their corresponding eigenvectors by solving the eigenproblem of (10), however we only require the dominant eigenvalues and eigenvectors. To obtain the dominant eigenelements, we order the eigenvalues by size and only consider the first m eigenvalues and their corresponding eigenvectors such that

$$\frac{\lambda_{m+1}}{\sum_{i=1}^m \lambda_i} \geq \varepsilon, \tag{12}$$

where ε denotes the desired fraction of the energy of the ensemble. The key steps of the POD algorithm are presented in Table 1.

Table 1. POD algorithm

1)	**Collect** the available snapshots in an ensemble, w.
2)	**Compute** the covariance matrix of C_K using (11).
3)	**Solve** the eigenvalue-eigenvector problem of (10).
4)	**Order** the eigenvalues and eigenvectors.
5)	**Keep** the dominant eigenvalues and their corresponding eigenvectors.
6)	**Compute** the dominant empirical basis functions using (9).

3.2 Galerkin projection

We consider a general form of a one-dimensional ($z \in \Omega \subset \mathbb{R}$) transport-reaction process with a single state ($n = 1$) described by the following semi-linear PDE that is abstractly represented by (1),

$$\begin{aligned} \frac{\partial x(z,t)}{\partial t} = \vartheta_2 \frac{\partial^2 x(z,t)}{\partial z^2} + \vartheta_1 \frac{\partial x(z,t)}{\partial z} + \mathcal{F}\big(z, x(z,t)\big) \\ + \theta \mathcal{G}\big(z, x(z,t)\big) + \mathcal{B}(z)\, u(t), \end{aligned} \tag{13}$$

in the presence of unknown convection, diffusion and reaction parameters which are denoted by ϑ_1, ϑ_2 and θ, respectively (i.e., $p = 2$ and $q = 1$).

By employing the set of empirical basis functions from POD algorithm described in previous section, we can approximate the spatiotemporal state as

$$x(z,t) \approx \sum_{i=1}^m x_{s,i}(t)\phi_i(z). \tag{14}$$

Then by substituting such state approximation in the PDE system of (13), we obtain

$$\begin{aligned} \sum_{i=1}^m \dot{x}_{s,i}\phi_i = \vartheta_2 \sum_{i=1}^m x_{s,i}\frac{d^2\phi_i}{dz^2} + \vartheta_1 \sum_{i=1}^m x_{s,i}\frac{d\phi_i}{dz} + \mathcal{F}\big(z, \sum_{i=1}^m x_{s,i}\phi_i\big) \\ + \theta \mathcal{G}\big(z, \sum_{i=1}^m x_{s,i}\phi_i\big) + \mathcal{B}(z)\, u(t), \end{aligned} \tag{15}$$

Then we can obtain the ROM by employing Galerkin projection to equation (15) as

$$\dot{x}_s = \big(\vartheta_1 A_{s,1} + \vartheta_2 A_{s,2}\big)x_s + F_s(x_s) + \theta G_s(x_s) + B_s u, \tag{16}$$

where

$$\begin{aligned} &[A_{s,1}]_{j,i} = \int_\Omega \phi_j(z)\frac{d\phi_i(z)}{dz}dz, \quad [A_{s,2}]_{j,i} = \int_\Omega \phi_j(z)\frac{d^2\phi_i(z)}{dz^2}dz, \\ &[F_s]_j = \int_\Omega \phi_j(z)\mathcal{F}\big(z, \sum_{i=1}^m x_{s,i}\phi_i\big)dz, \\ &[G_s]_j = \int_\Omega \phi_j(z)\mathcal{G}\big(z, \sum_{i=1}^m x_{s,i}\phi_i\big)dz, \quad [B_s]_{j,k} = \int_\Omega \phi_j(z)\mathcal{B}_k(z)dz \end{aligned}$$

Note that $x_s \in \mathbb{R}^m$, $A_{s,1}, A_{s,2} \in \mathbb{R}^{m\times m}$, $F_s, G_s : \mathbb{R}^m \to \mathbb{R}^m$, $B_s \in \mathbb{R}^{m\times l}$, $u \in \mathbb{R}^l$ and $\vartheta_1, \vartheta_2, \theta \in \mathbb{R}$. Such ROM can be used as the basis for the adaptive output feedback controller design.

4. ADAPTIVE OUTPUT FEEDBACK CONTROL

In this section we address the synthesis of the low-dimensional adaptive output feedback controller structure which is a combination of a Lyapunov-based adaptive controller and a static observer.

4.1 Controller design

We assume that the identification errors remain bounded under the proposed adaptive controller design,

$$\begin{aligned} e_{\vartheta_1} &= \vartheta_1 - \hat{\vartheta}_1, \quad |e_{\vartheta_1}| < \alpha_1 \\ e_{\vartheta_2} &= \vartheta_2 - \hat{\vartheta}_2, \quad |e_{\vartheta_2}| < \alpha_2 \\ e_\theta &= \theta - \hat{\theta}, \quad\quad |e_\theta| < \beta \end{aligned} \tag{17}$$

where $\hat{\vartheta}_1$, $\hat{\vartheta}_2$ and $\hat{\theta}$ denote the estimated values for unknown transport and reaction parameters. To synthesize the controller we consider the following standard quadratic control Lyapunov function (CLF),

$$V_c(x_s) = \frac{1}{2} x_s^T x_s, \tag{18}$$

where $V_c(x_s) > 0$ and $V_c = 0$ only for $x_s = 0$. Then we can obtain the time derivative of the CLF as

$$\dot{V}_c = x_s^T \dot{x}_s = x_s^T \left(\vartheta_1 A_{s,1} x_s + \vartheta_2 A_{s,2} x_s + F_s(x_s) + \theta G_s(x_s) + B_s u\right).$$

Then by considering

$$\begin{aligned} &\vartheta_1 A_{s,1} x_s + \vartheta_2 A_{s,2} x_s + F_s(x_s) + \theta G_s(x_s) + B_s u \\ &= -C_o x_s - \left(\alpha_1 \|A_{s,1}\|_2 + \alpha_2 \|A_{s,2}\|_2\right) x_s - \beta \|G_s(x_s)\|_2 \, sign(x_s), \end{aligned} \tag{19}$$

where $C_o > 0$ and *sign* denotes the sign function, we conclude

$$\begin{aligned} u = -B_s^\perp \Big[&\left(C_o + \vartheta_1 A_{s,1} + \vartheta_2 A_{s,2} + \alpha_1 \|A_{s,1}\|_2 + \alpha_2 \|A_{s,2}\|_2\right) x_s \\ &+ F_s(x_s) + \theta G_s(x_s) + \beta \|G_s(x_s)\|_2 \, sign(x_s) \Big], \end{aligned} \tag{20}$$

where $B_s^\perp = B_s^T (B_s B_s^T)^{-1}$ is the Moore-Penrose pseudo-inverse [Penrose (1955)]. Note that $B_s^\perp = B_s^{-1}$ for $m = l$ and the controller of (20) is implementable when the parameters are known. Then the time derivative of the CLF can be derived by

$$\begin{aligned} \dot{V}_c = &-\left(C_o + \alpha_1 \|A_{s,1}\|_2 + \alpha_2 \|A_{s,2}\|_2\right) x_s^T x_s \\ &-\beta \|G_s(x_s)\|_2 \, x_s^T \, sign(x_s) \le 0, \end{aligned} \tag{21}$$

which shows the closed-loop stability of the controller in the Lyapunov sense [Khalil (2002)].

We obtain the controller formula in the presence of unknown parameters by the applying certainty equivalence principle Åström (1983),

$$\begin{aligned} u = -B_s^\perp \Big[&\left(C_o + \hat{\vartheta}_1 A_{s,1} + \hat{\vartheta}_2 A_{s,2} + \alpha_1 \|A_{s,1}\|_2 + \alpha_2 \|A_{s,2}\|_2\right) x_s \\ &+ F_s(x_s) + \hat{\theta} G_s(x_s) + \beta \|G_s(x_s)\|_2 \, sign(x_s) \Big]. \end{aligned} \tag{22}$$

By considering the closed-loop Lyapunov function as a combination of the CLF and the identification Lyapunov function (ILF),

$$\begin{aligned} V &= V_c(x_s) + V_i(e_{\vartheta_1}, e_{\vartheta_2}, e_\theta) \\ &= \frac{1}{2} x_s^T x_s + \frac{1}{2P_1} e_{\vartheta_1}^2 + \frac{1}{2P_2} e_{\vartheta_2}^2 + \frac{1}{2Z} e_\theta^2, \end{aligned} \tag{23}$$

where $P_1, P_2, Z > 0$, we conclude

$$\dot{V} = \dot{V}_c + \dot{V}_i = x_s^T \dot{x}_s + \frac{1}{P_1} e_{\vartheta_1} \dot{e}_{\vartheta_1} + \frac{1}{P_2} e_{\vartheta_2} \dot{e}_{\vartheta_2} + \frac{1}{Z} e_\theta \dot{e}_\theta. \tag{24}$$

We assume that the unknown parameters of ϑ_1, ϑ_2 and θ do not change sharply as process evolves however it may change step-wise. According to such assumption,

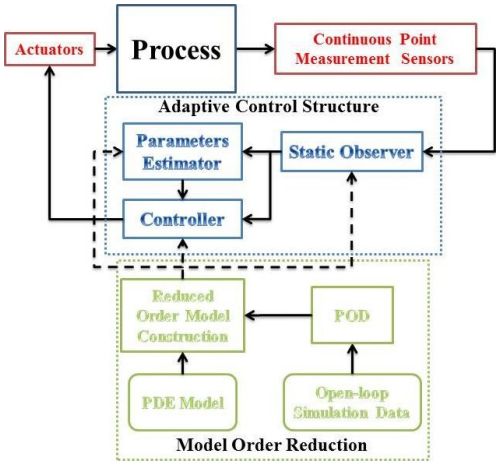

Fig. 1. Closed-loop process.

$$\begin{aligned} &\dot{e}_{\vartheta_1} = \dot{\vartheta}_1 - \dot{\hat{\vartheta}}_1 = -\dot{\hat{\vartheta}}_1, \quad \dot{e}_{\vartheta_2} = \dot{\vartheta}_2 - \dot{\hat{\vartheta}}_2 = -\dot{\hat{\vartheta}}_2, \\ &\dot{e}_\theta = \dot{\theta} - \dot{\hat{\theta}} = -\dot{\hat{\theta}}. \end{aligned} \tag{25}$$

If we employ the following identification laws to estimate the unknown parameters,

$$\dot{\hat{\vartheta}}_1 = P_1 x_s^T A_{s,1} x_s, \quad \dot{\hat{\vartheta}}_2 = P_2 x_s^T A_{s,2} x_s, \quad \dot{\hat{\theta}} = Z x_s^T G_s(x_s), \tag{26}$$

we obtain

$$\begin{aligned} \dot{V} = &-\left(C_0 + \alpha_1 \|A_{s,1}\|_2 + \alpha_2 \|A_{s,2}\|_2\right) x_s^T x_s \\ &-\beta \|G_S(x_s)\|_2 x_s^T \, sign(x_s) \le 0. \end{aligned} \tag{27}$$

Therefore $V = V_c + V_i$ is a decreasing positive definite Lyapunov function which indicates the closed-loop system stability in the Lyapunov sense [Khalil (2002)].

4.2 State estimation

To implement the proposed adaptive control structure we must have access to full measurements of the system slow modes, x_s, to compute the control action of (22) and estimate the parameters from (26). We employ the following static observer design to estimate the slow modes based on continuous point measurements from limited number of sensors,

$$\hat{x}_s(t) = \left(\Phi_s(L_s) \Phi_s^T(L_s)\right)^{-1} \Phi_s(L_s) \, y(t), \tag{28}$$

where $\Phi_s = [\phi_1 \; \phi_2 \; \cdots \; \phi_m]^T$, $y \in \mathbb{R}^r$. Note that the number of required continuous measurement sensors has to be supernumerary to the number of slow modes, i.e. $r \ge m$. Such requirement can be circumvented using dynamic observer design which conceptually needs only one measurement output Pourkargar and Armaou (2013b, 2014a, 2015b).

Figure 1 presents the block diagram of the closed-loop process operation under the proposed adaptive output feedback controller structure.

5. APPLICATION TO THERMAL DYNAMICS REGULATION

5.1 Process description

We consider a tubular flow reactor with a cooling jacket where an irreversible exothermic zero-th order reaction takes place (Figure 2). The reaction rate is considered to be temperature dependent, then it varies with time and location of the reactor. The cooling jacket and feed temperatures are chosen as the manipulated variables for control purposes. The thermal spatiotemporal dynamics can be derived from the energy balance

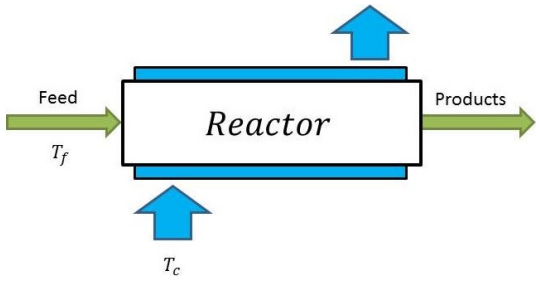

Fig. 2. Tubular flow reactor with a cooling jacket.

inside the reactor and can be presented as the following PDE with initial and boundary conditions,

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2} - v\frac{\partial T}{\partial z} + \frac{(-\Delta H)}{\rho C_p} r_o \exp\left(\frac{-E}{RT}\right) - \frac{hA_c}{\rho C_p}(T - T_c),$$
$$\frac{\partial T}{\partial z}(0,t) = \frac{\rho C_p v}{k}(T(0,t) - T_f), \quad \frac{\partial T}{\partial z}(L,t) = 0,$$
$$T(z,0) = T_0(z), \tag{29}$$

where T is the stream temperature inside the reactor, t is time and $z \in [0, L]$ denotes spatial coordinate where L is the reactor length. The terms k, ρ, C_p, v, $(-\Delta H)$, r_0, E and h are used for thermal conductivity, density, heat capacity, axial velocity, heat of reaction, pre-exponential reaction constant, activation energy, and heat transfer coefficient between reactor and cooling jacket, respectively. The cooling surface area is denoted by A_c and T_c, T_f and T_0 present the cooling jacket temperature, feed temperature and initial temperature profile, respectively. In above PDE system, k, $(-\Delta H)$ and r_0 are considered as unknown transport-reaction parameters of the system.

To generalize the governing equation we reformulate (29) using dimensionless variables and parameters, and homogenize the first boundary condition by inducing the non-homogeneous part in the governing PDE using standard Dirac function. The dimensionless form of the PDE with respect to initial temperature, T_0, takes the following form,

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \vartheta\frac{\partial^2 \bar{T}}{\partial \bar{z}^2} - \frac{\partial \bar{T}}{\partial \bar{z}} + \theta \exp\left(\frac{\gamma \bar{T}}{1+\bar{T}}\right)$$
$$+ B_C(\bar{u}_1 - \bar{T}) + \delta(\bar{z} - 0)\bar{u}_2, \tag{30}$$
$$\frac{\partial \bar{T}}{\partial \bar{z}}(0,\bar{t}) = \frac{1}{\vartheta}\bar{T}(0,\bar{t}), \quad \frac{\partial \bar{T}}{\partial \bar{z}}(1,\bar{t}) = 0, \quad \bar{T}(\bar{z},0) = 0,$$

where $\bar{t} = \frac{tv}{L}$, $\bar{z} = \frac{z}{L}$, $\bar{T} = \frac{T - T_0}{T_0}$, $\vartheta = \frac{k}{\rho C_p v L}$,
$\gamma = \frac{E}{RT_0}$, $\bar{u}_1 = \frac{T_c - T_0}{T_0}$, $\bar{u}_2 = \frac{T_f - T_0}{T_0}$,
$B_C = \frac{hA_c L}{\rho C_p v}$, $\theta = \frac{(-\Delta H) r_o \exp(-\frac{E}{RT_0}) L}{\rho C_p T_0 v}$,

where ϑ and θ are the unknown parameters of the system due to unavailability of thermal conductivity, heat of reaction and pre-exponential reaction constant. We consider the jacket and feed stream temperatures as the manipulated inputs for control purposes.

5.2 Simulation results

The known parameters of the system are set at the following practical values, $\gamma = 7$ and $B_C = 2$, we also consider the following nominal values for unknown transport-reaction parameters of the system, $\vartheta = 0.18$ and $\theta = 0.25$. Note that the controller structure does not have access to such nominal values.

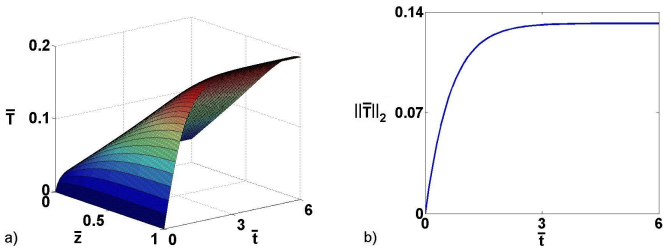

Fig. 3. Open-loop dimensionless temperature (a) spatiotemporal profile and (b) spatial second norm temporal profile.

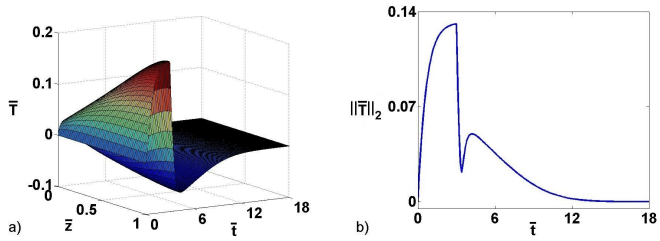

Fig. 4. Closed-loop dimensionless temperature (a) spatiotemporal profile and (b) spatial second norm temporal profile.

Figure 3 presents the open-loop spatiotemporal profile of the stream dimensionless temperature and the temporal profile of its spatial second norm. We can observe that the dimensionless temperature converges to a nonuniform steady state profile. The controller objective is to regulate the dimensionless temperature profile at the origin, i.e., keeping the stream temperature at the uniform reference profile of T_0. To construct the ROM required by the adaptive control structure we collect the snapshots of the system state profile from spatially distributed measurement sensors during the open-loop process operation, $t_{ol} = [0 \;\; 3]$, while the controllers are inactivated ($\bar{u}_1 = \bar{u}_2 = 0$). By applying the POD to the ensemble of open-loop snapshots, we obtain only 1 basis function. The controller actions ($\bar{u}_1$ and $\bar{u}_2 = 0$) and identification laws ($\dot{\hat{\vartheta}}$ and $\dot{\hat{\theta}}$) were derived based on adaptive output feedback control approach which presented in details in Section 4. Note that in this case study we have one unknown reaction parameter and only one unknown transport parameter. To design the control structure we considered the system tuning parameters as follows, $C_o = 2$, $\alpha = 0.5$, $\beta = 0.2$ and $P = Z = 1$. To estimate the system dominant mode using the static observer of (28), we consider $r = 2$ temperature continuous point measurements at $L_s = [0.3 \;\; 0.7]^T$.

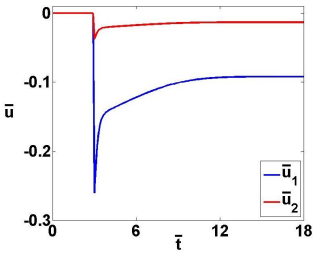

Fig. 5. Required control actions.

After the open-loop process operation time period, $t_{ol} = [0 \;\; 3]$, we activated the controller and identifiers to regulate the system for $t_{cl} > 3$. Figure 4 illustrates the closed-loop spatiotemporal profile of dimensionless temperature and its spatial second norm temporal profile. We observe that the controller successfully stabilizes the dimensionless temperature at the uniform zero profile. The temporal profiles of required control actions are shown in Figure 5. The controller actions converge to the steady state values corresponded to the desired zero dimensionless temperature profile without any chattering. Figure 6 presents the temporal profiles of the estimated unknown transport-reaction parameters. It can be observed that the estimated parameters converge to the final values of $\hat{\vartheta} = 0.485$ and $\hat{\theta} = 0.09$ which indicate the adaptation strategy did not correctly identify the parameters since for that

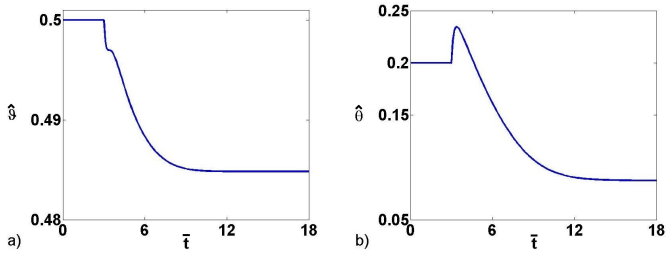

Fig. 6. Estimated (a) transport and (b) reaction parameters.

the required condition of persistent excitation [Âström (1983)] was not satisfied and also because the initial guesses were poor. Note that the objective of the proposed output feedback adaptive control strategy was only regulation not system identification. The open-loop process operation, $t_{ol} = [0\ \ 3]$, can be easily identified in Figures 5-6 while during such time period the controllers and identifiers were inactivated.

6. CONCLUSION

We focused on adaptive output feedback control of chemical distributed parameter systems in the presence of unknown parameters via model order reduction. The reduced order model which used as the basis for the Lyapunov-based adaptive controller design is obtained by applying Galerkin's method to the governing partial differential equations. The basis functions required by the Galerkin projection was computed via employing proper orthogonal decomposition to the set of spatiotemporal profiles of the system states obtained from open-loop process operation. The effectiveness of the proposed control structure was successfully illustrated on thermal dynamics regulation in a flow reactor.

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