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A Bilevel Programming Formulation for Dynamic Real-time Optimization ⋆

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Abstract: Due to ever-challenging global market conditions, plant economic optimization is becoming more critical. Recent advances have transformed the traditional steady-state real-time optimization (RTO) system of plant economic optimization to dynamic real-time optimization (DRTO) based on a dynamic prediction model. DRTO strategies that have been proposed perform economic optimization in an open-loop fashion without taking into account the presence of the plant control system. In this work, we propose a bilevel programming formulation for DRTO that includes effects of the closed-loop dynamics of an underlying constrained model predictive controller (MPC). The bilevel program is subsequently reformulated and solved as a single-level mathematical program with complementarity constraints (MPCC). We investigate the economics and control performance of the proposed strategy under varying MPC controller design parameters, and compare them to open-loop DRTO and rigorous multilevel closed-loop DRTO approaches.

Keywords: real-time optimization, economic optimization, model predictive control, complementarity constraints, back-off mechanism.

1. INTRODUCTION

Real-time optimization (RTO) is a supervisory strategy in the multilevel plant automation hierarchy that computes the best economics of continuous process operations at a time-scale slower than the lower level process automation activities (Marlin and Hrymak, 1997; Darby et al., 2011). RTO interacts with the lower level plant control system in a cascade fashion by providing optimal set-point targets for tracking purposes. The traditional RTO strategy is designed based on a steady-state model, which suffers from a limited execution frequency for processes with frequent transitions and long transient dynamics because the optimizer can only be executed if the process has satisfied the conditional steady-state requirement. Recent advances have transformed the steady-state RTO to dynamic real-time optimization (DRTO) based on a dynamic prediction model, thus substantially increasing the frequency at which economic optimization can be performed.

Proposed DRTO strategies that follow a two-layer architecture (Tosukhowong et al., 2004; Würth et al., 2011) perform economic optimization in an open-loop fashion without taking into account the presence of the plant control system, which we denote here as an open-loop DRTO strategy. In this approach, the set-points prescribed to the underlying control system are based on the optimal open-loop trajectories under an expectation that the closed-loop response dynamics at the plant level will follow the economically optimal trajectories obtained at the DRTO level. An alternative to the multilevel configuration is a single-level, economic model predictive control (EMPC) approach that optimizes the plant economics at the controller sampling frequency. Such a strategy aims to address the issues of model inconsistencies and conflicting objectives between the traditional RTO system and the MPC control layer. In this case, the objective function could be based purely on economics (Amrit et al., 2013), or a hybrid between cost and control performance (Ellis and Christofides, 2014).

In this work, we propose a closed-loop DRTO strategy in the form of a bilevel programming problem with the inclusion of a constrained model predictive control (MPC) optimization model. Therefore, it optimizes the closed-loop response dynamics of the process where the optimal control inputs are computed by the inner MPC optimization subproblem. Specifically, we are computing the MPC set-point trajectories that determine the best economics of the predicted closed-loop response, under the assumption that the process follows the trajectory calculated by MPC until the next DRTO execution. The closed-loop DRTO formulation may be viewed as an EMPC approach due to explicit consideration of control performance while making economic decisions. However, it has the flexibility to be implemented less frequently at the supervisory level because controller set-point trajectories are the primary decision variables of the economic optimization problem. This allows the existing plant automation architecture to be unaltered, and the higher frequency control calculation remains less complex and computationally inexpensive. This paper extends the previous work by Jamaludin and Swartz (2014) that investigates a rigorous inclusion of the

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$$\begin{aligned} &\min_{\hat{\boldsymbol{u}}_{\mathbf{DRTO}}} \; \Phi_{\mathrm{OL}} \\ s.t. \quad &\mathbf{h}_{\mathbf{DRTO}}(\hat{\boldsymbol{x}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{y}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{u}}_{\mathbf{DRTO}}) = 0 \\ &\mathbf{g}_{\mathbf{DRTO}}(\hat{\boldsymbol{x}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{y}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{u}}_{\mathbf{DRTO}}) \leq 0 \end{aligned} \tag{1}$$

$$\begin{aligned} &\min_{\hat{\boldsymbol{y}}_{\mathbf{sp}},\, \hat{\boldsymbol{u}}_{\mathbf{sp}}} \; \Phi_{\mathrm{CL}} \\ s.t. \quad &\mathbf{h}_{\mathbf{DRTO}}(\hat{\boldsymbol{x}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{y}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{u}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{y}}_{\mathbf{sp}},\, \hat{\boldsymbol{u}}_{\mathbf{sp}}) = 0 \\ &\mathbf{g}_{\mathbf{DRTO}}(\hat{\boldsymbol{x}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{y}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{u}}_{\mathbf{DRTO}},\, \hat{\boldsymbol{y}}_{\mathbf{sp}},\, \hat{\boldsymbol{u}}_{\mathbf{sp}}) \leq 0 \end{aligned} \tag{2a}$$

$$\begin{aligned} s.t. \quad &\min_{\hat{\boldsymbol{u}}_{\mathbf{MPC}}} \; \phi \\ &\mathbf{h}_{\mathbf{MPC}}(\hat{\boldsymbol{x}}_{\mathbf{MPC}},\, \hat{\boldsymbol{y}}_{\mathbf{MPC}},\, \hat{\boldsymbol{u}}_{\mathbf{MPC}},\, \hat{\boldsymbol{y}}_{\mathbf{sp}},\, \hat{\boldsymbol{u}}_{\mathbf{sp}}) = 0 \\ &\mathbf{g}_{\mathbf{MPC}}(\hat{\boldsymbol{x}}_{\mathbf{MPC}},\, \hat{\boldsymbol{y}}_{\mathbf{MPC}},\, \hat{\boldsymbol{u}}_{\mathbf{MPC}},\, \hat{\boldsymbol{y}}_{\mathbf{sp}},\, \hat{\boldsymbol{u}}_{\mathbf{sp}}) \leq 0 \end{aligned} \tag{2b}$$

MPC closed-loop dynamics at the DRTO level in the form of a multilevel programming formulation.

In the following sections, general discrete formulations of the open-loop and closed-loop DRTO strategies will be presented, and a reformulation approach to transform the bilevel closed-loop DRTO problem to a single-level, mathematical program complementarity constraints (MPCC) will be described. The effectiveness of the proposed bilevel closed-loop DRTO strategy, in comparison to the open-loop and rigorous multilevel closed-loop DRTO approaches, will be demonstrated through case studies involving variation in the MPC design parameters.

2. PROBLEM FORMULATION

In this study, we utilize the state-space formulation of a standard input-constrained MPC controller with a quadratic objective function, details of which can be found in Maciejowski (2002). In addition to the output tracking and move suppression terms in the regular MPC objective function, we also include control input tracking term that is useful for nonsquare systems with more inputs than outputs. Output constraints are included at the upper level economic optimization in order to avoid control infeasibility or closed-loop instability (Zafiriou, 1990).

2.1 Open-loop DRTO

We formulate the open-loop DRTO problem similar to those found in the literature. The objective is to minimize a prescribed economic criterion, and the control inputs are optimized based on the open-loop response of the process. For a discrete-time dynamic system, the open-loop DRTO formulation may be written as (1), with the corresponding variables defined as,

$$\hat{\boldsymbol{x}}_{\mathbf{DRTO}} = \left[\hat{\boldsymbol{x}}_{\mathbf{DRTO}_1}{}^{\mathrm{T}},\, \hat{\boldsymbol{x}}_{\mathbf{DRTO}_2}{}^{\mathrm{T}},\, \ldots,\, \hat{\boldsymbol{x}}_{\mathbf{DRTO}_J}{}^{\mathrm{T}}\right]^{\mathrm{T}}$$
$$\hat{\boldsymbol{y}}_{\mathbf{DRTO}} = \left[\hat{\boldsymbol{y}}_{\mathbf{DRTO}_1}{}^{\mathrm{T}},\, \hat{\boldsymbol{y}}_{\mathbf{DRTO}_2}{}^{\mathrm{T}},\, \ldots,\, \hat{\boldsymbol{y}}_{\mathbf{DRTO}_J}{}^{\mathrm{T}}\right]^{\mathrm{T}}$$
$$\hat{\boldsymbol{u}}_{\mathbf{DRTO}} = \left[\hat{\boldsymbol{u}}_{\mathbf{DRTO}_0}{}^{\mathrm{T}},\, \hat{\boldsymbol{u}}_{\mathbf{DRTO}_1}{}^{\mathrm{T}},\, \ldots,\, \hat{\boldsymbol{u}}_{\mathbf{DRTO}_{J-1}}{}^{\mathrm{T}}\right]^{\mathrm{T}}$$

Φ_{OL} represents a purely economic objective function. $\mathbf{h}_{\mathbf{DRTO}}$ is an equality constraint set that includes the dynamic prediction model, whereas $\mathbf{g}_{\mathbf{DRTO}}$ is an inequality constraint set that consists of hard constraints on the manipulated inputs and controlled outputs. $\hat{\boldsymbol{x}}_{\mathbf{DRTO}} \in \Re^{n_x \times J}$ is a vector of open-loop DRTO model states and $\hat{\boldsymbol{y}}_{\mathbf{DRTO}} \in \Re^{n_y \times J}$ is a corresponding vector of open-loop DRTO model outputs over the optimization horizon J; $\hat{\boldsymbol{u}}_{\mathbf{DRTO}} \in \Re^{n_u \times J}$ is a vector of DRTO input trajectories. In the open-loop DRTO strategy, the set-points prescribed to the lower-level control system are based on the resulting optimal open-loop trajectories, with the output at the end of each DRTO interval used as the set-point for all MPC sample intervals contained within it.

2.2 Closed-loop DRTO

Closed-loop DRTO takes the form of a multilevel optimization problem, since the control input variables at each time step correspond to the solution of an MPC optimization calculation. In this study, we consider at the DRTO level a single MPC calculation over the DRTO optimization horizon as an approximation of the closed-loop response. Mathematically, we have an outer DRTO optimization problem to predict the closed-loop response (2a), and an inner MPC optimization subproblem (2b) to calculate the optimal control input trajectories.

The controller set-point trajectories, $\hat{\boldsymbol{y}}_{\mathbf{sp}}$ and $\hat{\boldsymbol{u}}_{\mathbf{sp}}$, become the decision variables for the outer problem whereas the controller input trajectories, $\hat{\boldsymbol{u}}_{\mathbf{MPC}}$, become the decision variables for the inner subproblem. The equality constraint, $\mathbf{h}_{\mathbf{DRTO}}$, also enforces the MPC set-point trajectories to be constant over each DRTO sampling interval, while constraints on the outputs are enforced through inequalities, $\mathbf{g}_{\mathbf{DRTO}}$. In this formulation, there is a direct correspondence between the DRTO variables $\hat{\boldsymbol{x}}_{\mathbf{DRTO}}$, $\hat{\boldsymbol{y}}_{\mathbf{DRTO}}$, and $\hat{\boldsymbol{u}}_{\mathbf{DRTO}}$ with the MPC variables $\hat{\boldsymbol{x}}_{\mathbf{MPC}}$, $\hat{\boldsymbol{y}}_{\mathbf{MPC}}$ and $\hat{\boldsymbol{u}}_{\mathbf{MPC}}$, respectively. However, we utilize notation that differentiates between the DRTO variables in the outer problem and the MPC variables in the inner subproblem as the direct correspondence does not carry over to the multilevel formulation.

$\hat{\boldsymbol{x}}_{\mathbf{MPC}} \in \Re^{n_x \times J}$ is a vector of MPC model states and $\hat{\boldsymbol{y}}_{\mathbf{MPC}} \in \Re^{n_y \times J}$ is a corresponding vector of MPC model outputs over the DRTO optimization horizon J; $\hat{\boldsymbol{u}}_{\mathbf{MPC}} \in \Re^{n_u \times J}$ is a vector of MPC inputs over the the DRTO optimization horizon J; $\hat{\boldsymbol{y}}_{\mathbf{sp}} \in \Re^{n_y \times J}$ and $\hat{\boldsymbol{u}}_{\mathbf{sp}} \in \Re^{n_u \times J}$ are vectors of MPC set-point trajectories for the controlled outputs and manipulated inputs, respectively. The vectors of set-point trajectories are defined as follows,

$$\hat{\boldsymbol{y}}_{\mathbf{sp}} = \left[\hat{\boldsymbol{y}}_{\mathbf{sp}_1}{}^{\mathrm{T}},\ \hat{\boldsymbol{y}}_{\mathbf{sp}_2}{}^{\mathrm{T}},\ \ldots,\ \hat{\boldsymbol{y}}_{\mathbf{sp}_J}{}^{\mathrm{T}}\right]^{\mathrm{T}}$$
$$\hat{\boldsymbol{u}}_{\mathbf{sp}} = \left[\hat{\boldsymbol{u}}_{\mathbf{sp}_0}{}^{\mathrm{T}},\ \hat{\boldsymbol{u}}_{\mathbf{sp}_1}{}^{\mathrm{T}},\ \ldots,\ \hat{\boldsymbol{u}}_{\mathbf{sp}_{J-1}}{}^{\mathrm{T}}\right]^{\mathrm{T}}$$

The closed-loop DRTO strategy optimizes the set-point trajectories directly to be prescribed to the MPC controller at the lower level. The set-point trajectories are held constant based on the DRTO sample time, which is an integer multiple of the MPC sampling interval, using equality constraints to provide consistency with the open-loop DRTO formulation in constructing the set-point trajectories. At the lower level, these set-point trajectories are shifted in time to account for the moving horizon of the MPC controller.

We remark that the bilevel programming formulation can be extended to a multilevel approach by repeating the inner MPC subproblem at every DRTO prediction step. In such a formulation, an exact MPC controller model implemented at the lower level can be embedded over the DRTO optimization horizon. However, the computational cost is significantly higher than that of the bilevel approximation.

Problem reformulation In this study, we employ a simultaneous solution approach by transforming the inner MPC subproblem to a constraint set using its first-order, Karush-Kuhn-Tucker (KKT) optimality conditions. For a constrained MPC problem formulated as a convex quadratic programming (QP) problem as considered in this study, such a transformation is valid as the KKT conditions are necessary and sufficient for optimality. The MPC subproblem at each DRTO prediction step may be represented as a QP of the form,

$$\begin{aligned} \min_{\mathbf{u}}\ & \frac{1}{2}\mathbf{u}^T H\mathbf{u} + \mathbf{g}^T\mathbf{u} \\ \text{s.t.}\quad & A\mathbf{u} = \mathbf{b} \\ & \mathbf{u} \geq \mathbf{0} \end{aligned} \tag{3}$$

with the corresponding KKT condition is given by,

$$\begin{aligned} H\mathbf{u} - A^T\boldsymbol{\mu} + \mathbf{g} - \boldsymbol{\eta} &= 0 \\ A\mathbf{u} &= \mathbf{b} \\ u_i\eta_i &= 0 \quad i = 1, ..., n_c \\ (\mathbf{u}, \boldsymbol{\eta}) &\geq \mathbf{0} \end{aligned} \tag{4}$$

Details of the KKT conditions of the MPC (QP) subproblem can be found in Baker and Swartz (2008). Replacement of the inner MPC-QP subproblem gives rise to a single-level, mathematical program with complementarity constraints (MPCC).

Handling complementarity constraints Complementarity constraints, which take the form $u_i\eta_i = 0$, are generally hard to solve due to violation of constraint qualifications in the nonlinear programming (NLP) problem (Baumrucker et al., 2008). Handling complementarity constraints requires reformulation of the MPCC, or an alternative NLP algorithm that internally treats the complementarity constraints.

In this study, the complementarity constraints are handled using an exact penalty formulation (Ralph and Wright, 2004). They are moved from the constraint set of the original MPCC problem represented in (5) to the objective function as an additional penalty term with a penalty parameter ρ, as given in (6), with the resulting problem posed to a standard NLP solver. Tuning of the complementarity penalty parameter ρ starts from a small value, roughly of the same order of magnitude as the decision variables, and increased until it exceeds a critical value, i.e. $\rho > \rho_c$, at which point the original complementarity constraints will be approximately satisfied. However, choosing too large a penalty parameter may lead to scaling issues and longer solution times.

$$\begin{aligned} \min_{\mathbf{u},\mathbf{x},\mathbf{y},\boldsymbol{\mu},\boldsymbol{\eta}}\ & \phi(\mathbf{x}, \mathbf{y}, \mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\eta}) \\ \text{s.t. } & \mathbf{h}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\eta}) = \mathbf{0} \\ & \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\eta}) \geq \mathbf{0} \\ & H\mathbf{u} - A^T\boldsymbol{\mu} + \mathbf{g} - \boldsymbol{\eta} = 0 \\ & A\mathbf{u} = \mathbf{b} \\ & u_i\eta_i = 0 \quad i = 1, ..., n_c \\ & (\mathbf{u}, \boldsymbol{\eta}) \geq \mathbf{0} \end{aligned} \tag{5}$$

$$\begin{aligned} \min_{\mathbf{u},\mathbf{x},\mathbf{y},\boldsymbol{\mu},\boldsymbol{\eta}}\ & \phi(\mathbf{x}, \mathbf{y}, \mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\eta}) + \rho\sum_{i=1}^{n_c} u_i\eta_i \\ \text{s.t. } & \mathbf{h}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\eta}) = \mathbf{0} \\ & \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\eta}) \geq \mathbf{0} \\ & H\mathbf{u} - A^T\boldsymbol{\mu} + \mathbf{g} - \boldsymbol{\eta} = 0 \\ & A\mathbf{u} = \mathbf{b} \\ & (\mathbf{u}, \boldsymbol{\eta}) \geq \mathbf{0} \end{aligned} \tag{6}$$

At the optimum, the value of the complementarity penalty function will be nearly zero, and the optimal solution recovers the original objective function of the MPCC problem due to negligible contribution of the penalty term.

2.3 Economic Objective Function

In general, any appropriate economic objective function suitable for process optimization may be used. However, our case study is specifically motivated by product grade transition problems, such as those arising in the polymer and bioprocess industries. The DRTO objective function is formulated to minimize the input cost while at the same time taking into account the revenue when the product quality is within the desired target tolerance. The revenue is continuously tracked using a hyperbolic tangent function,

$$R(x) = \frac{1}{2}\tanh(\gamma x) + \frac{1}{2} \approx \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases} \tag{7}$$

where γ is a weighting parameter used to define the steepness of the switching function that produces a function value either smoothly or sharply approaching 0 or 1. The function is used as a continuous approximation of a switching function constructed to indicate when the variable enters a specification tolerance band. These may be used in combination to capture specification bands with upper and lower limits around a desired target, and included in the objective function in order for revenue to apply only when the product quality falls within specification limits. Details on the formulation of this construct and its use may be found in Lam et al. (2007).

3. CASE STUDY

The problem considered here is a nonsquare system with one output and two manipulated inputs represented as

$$y(s) = \frac{1}{750s^2 + 65s + 1} u_1(s) + \frac{1}{400s^2 + 40s + 1} u_2(s)$$

Output y responds faster to input u_2, and the cost of input u_2 is higher than input u_1. Since input u_2 has a significant impact on the transition dynamics and the overall process economics, we include it as a manipulated variable and enforce set-points for input u_2, in addition to the set-points for output y. The economic objective is formulated in (8) and the problem constraints are given in (9).

$$\min \Phi = \sum_{j=1}^{J} \left(2u_{1,j} + 10u_{2,j} - 100 R_j^1 R_j^2 \right) \qquad (8)$$

$$\begin{gathered} 0 \le y,\ y_{\rm sp} \le 1.1 \\ 0 \le u_1,\ u_2,\ u_{2,\rm sp} \le 1.5 \end{gathered} \qquad (9)$$

The system is discretized based on an MPC sample time of 2 min. The nominal MPC design parameters are as follows: prediction horizon $p = 30$, control horizon $m = 3$, output tracking weight $Q = 1$, move suppression weight $R = \text{diag}(1,1)$, and control tracking weight $S = \text{diag}(0,1)$. Reoptimization at the DRTO level is carried out for every 20 min of sampling interval and the DRTO optimization horizon $J = 150$ steps. An appropriate penalty parameter for the complementarity constraints is found to be 80. The system is brought from the initial steady-state to the desired target of 1.0 ± 0.1 in output y. R_1 and R_2 are outputs of hyperbolic tangent switching function approximations that indicate satisfaction of the lower and upper output specification tolerances respectively.

In this study, MATLAB R2012b is chosen as the supervisory computational platform to solve the MPC problem using a `quadprog` solver, and also to perform plant simulation. The DRTO problem, which can be significantly larger in size than the MPC problem, is modelled in AMPL and is solved using `IPOPT` (version 3.12.0). Computation is performed using a 3.4GHz INTEL CORE-i7 with 8GB RAM running Windows 7.

3.1 Effect of MPC move suppression weight

A key parameter that significantly affects the MPC closed-loop performance is the weighting parameter for the control move suppression penalty in its algorithm. This parameter controls the speed of the closed-loop response during set-point tracking and disturbance rejection. It also affects the stability properties of the MPC controller (García et al., 1989). We investigate the effect of detuning the MPC controller by increasing the move suppression weight R in the MPC objective function from diag(1, 1) to diag(20, 20). The system closed-loop performance is compared for the open-loop DRTO, bilevel closed-loop DRTO and multilevel closed-loop DRTO strategies.

Closed-loop responses of output y for various values of weighting parameter R are depicted in Fig. 1. Implementation of the open-loop DRTO strategy at the upper level results in a gradual adjustment of the set-point trajectories as they are constructed based on the optimal DRTO open-loop responses. In the open-loop DRTO strategy, we expect that the closed-loop response at the plant level will replicate the optimal trajectories obtained at the upper level but this cannot be achieved due to the presence of the control move suppression penalty that is not being recognized by the open-loop DRTO formulation.

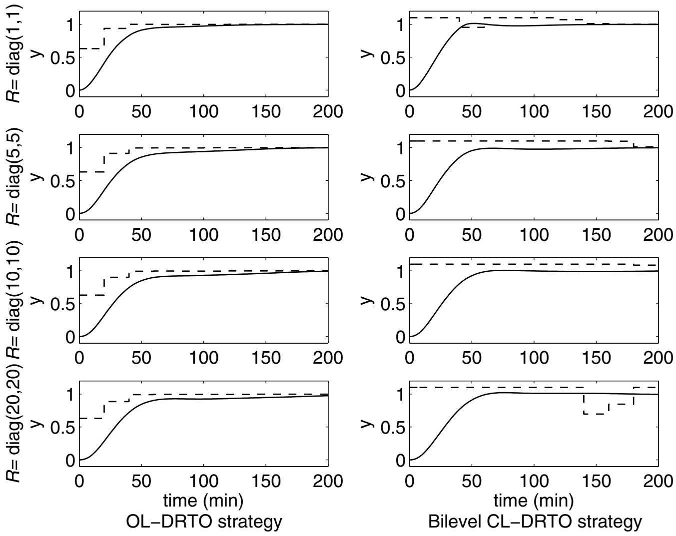

Fig. 1. Closed-loop response of output y (solid lines: output; dashed lines: set-point)

In contrast, the output set-point trajectories based on the closed-loop DRTO strategy are more agile in order to assist a fast transition to the desired target, even under detuned controller settings. The set-point trajectories are driven to the upper bound during the initial transition phase and then enforced to the desired target as the output is approaching the target tolerance. The set-point trajectories are sustained at the upper bound for much a longer period for detuned controller settings, for example with $R = \text{diag}(10,10)$, due to large penalization of control input moves that results in slow closed-loop dynamics. Although the set-point trajectories and the closed-loop dynamics tend to be aggressive when the magnitude of the move suppression weight is smaller, such as when $R = \text{diag}(1,1)$, a back-off mechanism that arises naturally in the closed-loop DRTO formulation prevents output constraint violation at the plant level. This back-off mechanism has the ability to identify at which juncture potential output constraint violation (due to overshoot) might occur under closed-loop implementation and thus moves the set-points away from the constraints.

If we examine the set point trajectories for input u_2 in Fig. 2, the open loop DRTO enforces the input set-point $u_{2,sp}$ to the upper bound for only one time interval, which is insufficient to drive the output to the desired target. The subsequent DRTO executions do not hold this set point at the maximum. On the other hand, the closed-loop DRTO strategy holds the input set-point $u_{2,sp}$ for a longer period in comparison to the open-loop DRTO strategy, which is over two DRTO intervals in this particular example, in order to assist a rapid tradition of the output to the desired target. The closed loop DRTO has the ability to do this because it has information on the MPC properties, specifically the MPC move suppression weight R

that enforces gradual input changes. Therefore, the best economic performance is achieved by making appropriate adjustments to the controller set-point trajectories that accommodates the MPC design parameters.

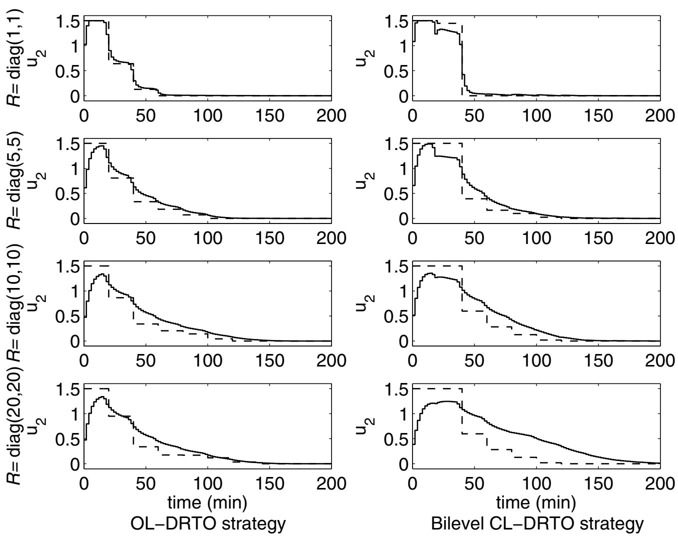

Fig. 2. Closed-loop response of input u_2 (solid lines: input; dashed lines: set-point)

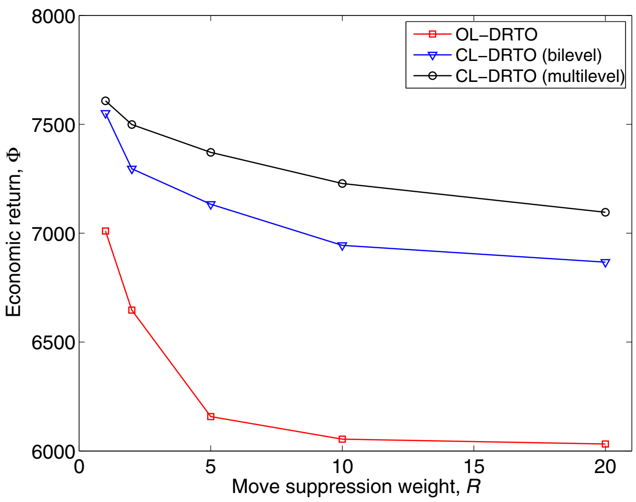

Fig. 3. Effects of MPC move suppression penalty (with $Q = 1,\ S = \mathrm{diag}(0, 1),\ p = 30,\ m = 3$)

The economic return of the closed-loop process response over a 200 min simulation time frame is illustrated in Fig. 3. Economic performance of the closed-loop system regulated by the closed-loop DRTO strategies is remarkably higher than the open-loop approach, even under detuned controller settings. The bilevel closed-loop DRTO formulation gives a good approximation to the rigorous multilevel DRTO formulation as the average economic gap is only 2.7%, as compared to 13.3% for the open-loop DRTO, from the rigorous multilevel DRTO implementation. The proposed bilevel formulation also significantly reduces the problem size and computation time of the rigorous multilevel formulation, from 57,743 decision variables and average 5 CPU(s), to 5,077 decision variables and average 0.9 CPU(s). In comparison, the open-loop DRTO formulation poses an optimization problem with 1,039 decision variables and an average 0.3 CPU(s) solution time.

3.2 Effect of MPC control horizon

In addition to the move suppression weight, the control horizon m also influences the tracking capability of the MPC controller. In general, control becomes aggressive as the horizon m increases. However, the implicit back-off mechanism that arises naturally in the closed-loop DRTO formulation prevents constraint violation of the output, as illustrated in Fig. 4, at the junctures where overshoot might occur. The agility of the output set-point trajectories y_{sp} based on the open-loop DRTO formulation slightly increases as the control horizon m increases. The input set-point $u_{2,sp}$ based on the closed-loop DRTO calculations, on the other hand, are held at the upper bound over two DRTO interval when the control horizon m is 3 or less, but this is not needed when the controller is tuned to become more aggressive, as shown in Fig. 5.

The overall closed-loop responses based on the open-loop and closed-loop DRTO calculations become close to each other as the control horizon m increases while the other MPC design parameters are fixed. This is primarily due to the fact that increasing control horizon m will reduce the discrepancy between the MPC closed-loop dynamics at the plant level and the dynamics at the upper level for both DRTO strategies where optimizations are carried out without limiting the number of input moves, i.e. a full degrees-of-freedom is available to manipulate the input moves based on the length of the optimization horizon. However, the difference remains significant if the MPC controller uses a move suppression weight larger than $R = \mathrm{diag}(2, 2)$.

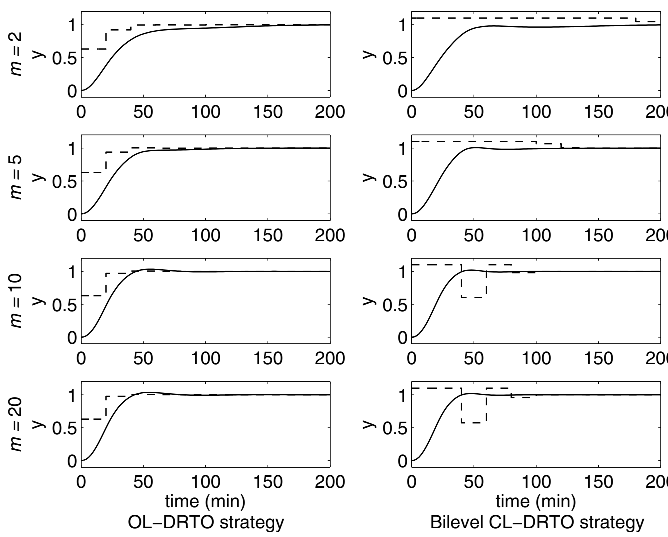

Fig. 4. Closed-loop response of output y (solid lines: output; dashed lines: set-point)

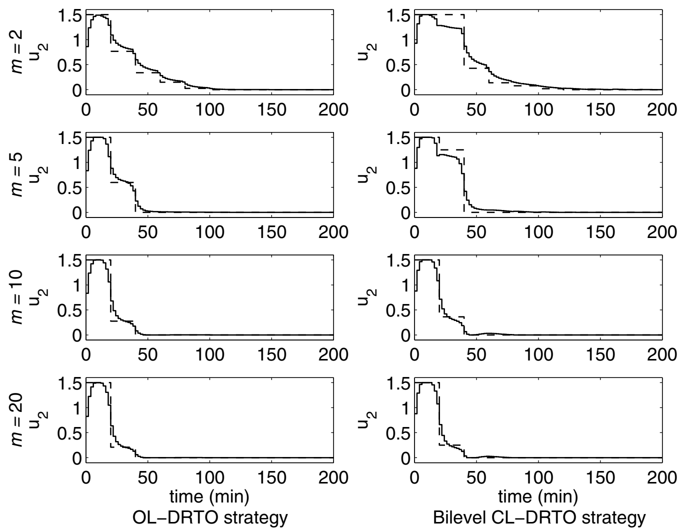

Fig. 5. Closed-loop response of input u_2 (solid lines: input; dashed lines: set-point)

We observe that the economic return for all DRTO strategies improves as the MPC control horizon m increases, as illustrated in Fig. 6. The closed-loop DRTO strategies result in more favorable economics than that of the open-loop approach. A shorter control horizon m results in significant economic differences between the open-loop DRTO, bilevel closed-loop DRTO and the rigorous multilevel closed-loop DRTO strategies. However, these gaps are reduced with longer control horizons. The bilevel closed-loop DRTO strategy presents of an average economic gap of 1.4% relative to the rigorous multilevel DRTO strategy, and is 5.5% for the open-loop DRTO case.

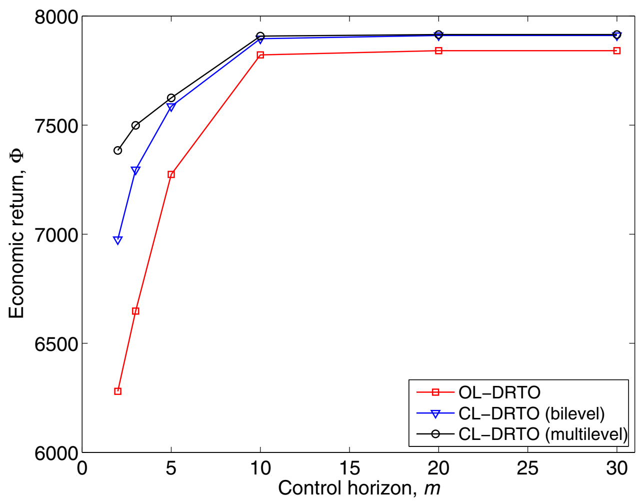

Fig. 6. Effects of of MPC control horizon (with $Q = 1,\ R = \mathrm{diag}(2, 2),\ S = \mathrm{diag}(0, 1),\ p = 30$)

4. CONCLUSION

In this study, the performance of two closed-loop DRTO strategies and an open-loop DRTO approach is compared. The bilevel closed-loop DRTO strategy is shown to offer advantages through assisting a rapid process transition and also keeping the process feasible through a back-off mechanism of the set point trajectories. It also decreases the problem size and solution time of the rigorous multilevel DRTO approach while retaining the economics to some degree. Based on the case studies carried out, the control move suppression weight R affects the economics to a much larger extent than the control horizon m when the controller is detuned. In future work, other techniques for approximating the MPC closed-loop dynamics will be investigated. In addition, the closed-loop DRTO strategy will be applied as a centralized supervisory controller in a distributed MPC environment.

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