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Well Placement Optimization with Geological Uncertainty Reduction

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Abstract: Well placement optimization aims to determine optimal well locations so that the economic benefit from oil production can be maximized. Geological uncertainty has a significant impact on the optimal well placement plan and therefore has to be considered in the well placement optimization problem. A geological realization reduction framework for well placement under geological uncertainty is proposed in this work. The objective is to optimally select a small subset of realizations and incorporate them into the well placement optimization problem, so as to reduce the computational efforts. A reservoir case study demonstrates that the selected smaller subset of realizations is a very good representation of a larger superset of realizations and can significantly decrease the computational time associated with the well placement optimization problem.

Keywords: geological uncertainty, well placement, optimization, uncertainty reduction

1. INTRODUCTION

For oil reservoir operations, the production amount of oil greatly depends on the well locations and the geological property of the reservoir. To achieve the maximum economic benefit, well placement optimization is necessary for determining the best locations for placing wells in a reservoir. Reservoir flow simulation is commonly used in well placement optimization problems. The well positions are determined by maximizing the output variable of interest such as the cumulative oil production (COP) or net present value (NPV) generated by a reservoir flow simulator. The objective function for the well placement optimization problem is evaluated by running the reservoir flow simulator with given well positions. As a result, the computational time for the flow simulator significantly increases with the size of the reservoir grid and the number of wells to be placed. In the literature, various methods have been used in well placement optimization to determine optimal well positions of a reservoir. In most cases, the objective function for the well placement optimization problem is to maximize the NPV or COP (Nasrabadi et al., 2012). Optimization methods used in well placement include: mixed integer programming (Rosenwald and Green, 1974), gradient-based optimization using finite difference method (Bangerth et al., 2006), genetic algorithms (Bittencourt and Horne, 1997), simulated annealing (Beckner and Song, 1995) and particle swarm optimization (Onwunalu and Durlofsky, 2010), etc.

The complexity of the well placement optimization problem is further increased by incorporating uncertainty associated with geological properties of the reservoir. Geological uncertainty in well placement optimization is generally considered by incorporating multiple geological realizations of the reservoir in the optimization model. Hence, the calculation of COP or NPV is based on the flow simulation on multiple geological realizations. However, since flow simulation for a large number of realizations is a very computationally demanding task and impractical for larger realistic reservoirs with multiple wells, a smaller subset of realizations are generally selected and used in the well placement optimization model to account for geological uncertainty. Thus, reducing the number of geological realizations for flow simulation becomes an important step in well placement optimization. Yeten et al. (2003) used multiple equiprobable geological realizations in the determination of objective function of well placement optimization to account for the geological uncertainty associated in a reservoir. Wang et al. (2012) selected a smaller subset of realization to quantify geological uncertainty in well placement optimization using k-means clustering. K-means clustering uses cumulative field oil production which requires to be calculated for every possible locations of well and therefore is computationally intensive. Yasari et al. (2013) used robust well placement optimization under uncertainty using a risk weighted objective function for multiple realizations. They selected a subset of realization from a superset by calculating the NPV for all the realizations using base case well position and then used ranking to select the small subset of realization. Similarly, Yang et al. (2011) combined Steam Assisted Gravity Drainage (SAGD) well production and placement optimization under uncertainty by selecting a subset of realizations using traditional ranking method based on the NPV of all the realizations for a base case scenario.

In this study, reservoir well placement optimization considering geological uncertainty is studied based on a novel method for geological uncertainty reduction. The well placement optimization problem is solved using derivative free optimization method. Geological uncertainty is considered by using a reduced subset of geological realization from a superset of realization in the well placement optimization model. An optimal realization reduction method using geological property of the reservoir and static measures is used in selecting the subset of

realization. The well placement optimization was applied using the subset of realizations obtained from the optimal realization reduction method on a reservoir with a fixed number of wells. Comparison studies with other geological realization selection method are performed to demonstrate the effectiveness of the proposed method.

The rest of the paper is organized as follows. Section 2 discusses the well placement optimization model. Section 3 provides the model used to select a smaller subset of realizations from a larger superset of realizations. Results and discussions of applying the well placement optimization model under geological uncertainty are provided in Section 4. The paper is concluded in section 5.

2. WELL PLACEMENT OPTIMIZATION UNDER GEOLOGICAL UNCERTAINTY

Well placement optimization is a computational intensive task. To evaluate the performance of a certain well placement plan (i.e., the decision variables), a reservoir flow simulation is performed for multiple geological realizations. So it is a simulation based optimization problem. Since there is no explicit objective function of the decision variables, derivative free optimization method is desired. Specifically, the derivative free optimization solver NOMAD is used in this work. NOMAD implements the Mesh Adaptive Direct Search (MADS) algorithm for constrained blackbox functions. The MADS algorithm is an extension of the pattern search method for nonlinear constrained optimization problems and therefore is a derivative free method (Audet et al., 2009). In this work, the objective function for the well placement optimization problem is designed as maximizing the risk averted expected cumulative oil production from a set of realizations as given by (1).

$$\text{Max } COP_{risk} = COP_{Expected} - \gamma \sqrt{\sum_{i=1}^{N_R} p_i (COP_i - COP_{Expected})^2} \quad (1)$$

where the expected COP is given by

$$COP_{Expected} = \sum_{i=1}^{N_R} p_i COP_i \quad (2)$$

In (1) and (2), N_R is the number of realizations used to determine the geological uncertainty, p_i is the probability of a geological realization i, COP_i is the cumulative oil production of realization i, γ is the risk averted factor (set as 0.1 in this work). The blackbox function is the reservoir simulator which determines the COP value based on the positions of the producer wells.

The robust well placement optimization used in this study is summarized in a flow diagram as given in Fig. 1. The steps in the well placement optimization under uncertainty are:

- Generate a large number of geological realizations using geostatistical method.
- Select a smaller subset of those realizations using realization reduction. The proposed realization reduction model minimizes the probability distance between the discrete distribution represented by the superset of realizations and the reduced discrete distribution represented by the selected realizations.
- Using the selected subset of realizations, perform well placement optimization by maximizing the objective function as given by (1). Each function evaluation calls on the reservoir simulator to calculate the COP.
- The optimal well locations using the subset of realizations are obtained when the stopping criteria for the optimizer are satisfied.

Robust well placement optimization is also performed using all the realizations in the superset to obtain optimal well locations for all the realizations. The well placement plan using the reduced subset of realizations obtained from the proposed method is compared to the well placement plan using all the realizations in the superset.

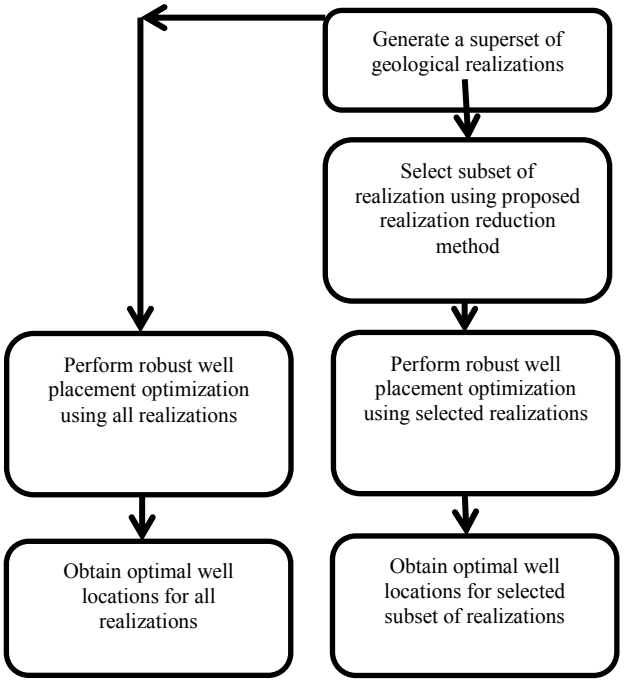

Fig. 1. Workflow for well placement optimization under uncertainty

3. GEOLOGICAL UNCERTAINTY REDUCTION

Geological uncertainty exists because it is not possible to know the exact geological properties of every section of a realistic reservoir. Techniques such as well exploration and core holes can give an idea of the geology property of that particular area of the reservoir. However, the geological parameters of the area between the exploration wells or core holes will still be unknown. As a result, geological uncertainty will always exist for a reservoir. Reservoir performance can be quantified by flow simulation, which provides production parameters of interest such as the COP and the NPV, etc. All the production parameters depend on the geological properties of the reservoir. It is very important to incorporate geological uncertainty in a reservoir model;

otherwise it may result in an incorrect prediction of production parameters. To represent the geological uncertainty, multiple geological realizations are usually generated using geostatistical tool so as to obtain a broad range of possible geological properties for a reservoir. However, reservoir flow simulation cannot be run for all the possible realizations due to significant computer processing times. Therefore, in practice only a small number of geological realizations are chosen to perform reservoir simulation, so as to obtain a reservoir performance model which incorporates geological uncertainty.

An optimal realization reduction model based on mixed integer linear optimization (MILP) technique is used to select a smaller subset of realizations from the superset of realizations (Rahim et al., 2014). The proposed algorithm uses reservoir geological properties and static measures to quantify the dissimilarity between realizations, and uses Kantorovich distance to quantify the probability distance between the superset and the subset of realizations. The objective is to find out the optimal subset which has a similar statistical distribution characteristic to the superset of realizations. The MILP model which selects the subset of realizations from a superset of realization is given by the set of equations below.

The objective function of the realization reduction algorithm is to minimize the Kantorovich distance between the original distribution and the reduced distribution

$$\min D_{Kan} = \sum_{i \in I} p_i^{orig} d_i \tag{3}$$

where d_i represents the cost of removing a realization i (i.e., transporting and distributing its probability mass to preserved realizations). This cost is quantified by a weighted summation of the transported probability mass,

$$d_i = \sum_{i' \in I} c_{i,i'} v_{i,i'} \quad \forall i \in I \tag{4}$$

where the weight is the dissimilarity $c_{i,i'}$ between realizations i and i' given by

$$c_{i,i'} = \sum_k \left| m_{ik} - m_{i'k} \right| + \sum_{c,t} \lambda \left| \theta_{ict} - \theta_{i'ct} \right| \quad \forall i, i' \tag{5}$$

The dissimilarity between realizations is computed using the geological properties and the static measures. m_{ik} is the value of the k type static measure for realization i, θ_{ict} is the t type geological property value of cell c in the reservoir grid for realization i, λ is a weight parameter (set as 0.01) which reflects the contribution of geological property data in the dissimilarity calculation. The static measures used in the calculation of the dissimilarity between realizations include the average net permeability, the average net porosity, the average net irreducible water saturation, the fraction of net cells, the net pore volume, and the original oil in place and net oil in place. For example, the average net permeability is calculated as $K_{net} = \sum_c k_c I_c^{net} \Big/ \sum_c I_c^{net}$, where binary indicator parameter I_c^{net} is used to denote whether a cell c in the reservoir grid is net ($I_c^{net} = 1$) or not ($I_c^{net} = 0$), k_c denotes the permeability of cell c. The idea of net cell stems from the fact that if a section of the reservoir rock has very low porosity and permeability value, then that section of the rock will be unable to carry any oil through it. For a complete definition of the other static measures, the reader is referred to (Rahim et al., 2014). All the static measures used in the realization reduction method are properties of the reservoir and independent of the location of wells within the reservoir.

The following constraints are also included in the proposed MILP model for optimal geological realization reduction/selection. First, if a realization i is removed (y_i=1), then all of its probability mass should be transported ($\sum_{i' \in I} v_{i,i'}$=1). If a realization i is selected/preserved (y_i =0), then its probability mass should not be transported to any realization ($\sum_{i' \in I} v_{i,i'}$=0).

$$\sum_{i' \in I} v_{i,i'} = y_i \quad \forall i \in I \tag{6}$$

Furthermore, if a realization i' is removed ($y_{i'}$=1), then no probability mass can be transported to it ($v_{i,i'}$=0 for any i). If a realization i' is selected ($y_{i'}$=0), then probability mass can be transported to it ($0 \le v_{i,i'} \le 1$).

$$0 \le v_{i,i'} \le 1 - y_{i'} \quad \forall i, i' \in I \tag{7}$$

If the total number of realizations to be removed is given by R then the following equation ensures that R realizations are removed

$$\sum_{i \in I} y_i = R \quad \forall i \in I \tag{8}$$

The next set of equations ensure that at least 2 realizations are selected from the subset I_{SB} and subset I_{SW}

$$\sum_{i \in I_{SB}} (1 - y_i) \ge 2 \quad \forall i \in I_{SB} \tag{9}$$

$$\sum_{i \in I_{SW}} (1 - y_i) \ge 2 \quad \forall i \in I_{SW} \tag{10}$$

where the subset I_{SB} has 2 realizations which is identified using the following steps. For each static measure, the realizations corresponding to the top 3 highest static measure values are identified. Those identified realizations ID are combined into a superset, and from which the 2 most frequent realizations are selected to from the set I_{SB}. Same idea is used to identify the subset I_{SW} which has top 2 most frequent realizations which represent the potential worst performance. The idea behind incorporating these constraints is to ensure the potential worst and best case realizations from the superset of realizations is included in the selected subset of realizations. With the selected realizations (i.e., y_i) and the probability mass transportation plan (i.e., $v_{i,i'}$), the new probability of realizations in the reduced distribution p_i^{new} can be evaluated as follows:

$$p_{i'}^{new} = (1 - y_{i'}) p_{i'}^{orig} + \sum_i v_{i,i'} p_i^{orig} \quad \forall i' \in I \tag{11}$$

Finally, the complete optimization model is composed of (3) to (11) and it is a mixed integer linear optimization problem. This problem can be solved using MILP solver such as CPLEX (IBM, 2010).

4. CASE STUDY

To illustrate the proposed geological realization reduction method and demonstrate its application in well placement optimization, a two dimensional reservoir model with 50 × 50 grid size (2500 cells) and 5m × 5m cell size is investigated in this section. The reservoir has 5 fixed vertical injector well placed at grid positions: [8 45], [16 45], [24 45], [32 45] and [40 45]. The number of vertical producer wells was fixed as 5. The objective function for the well placement optimization was evaluated using Matlab Reservoir Simulation Toolbox (MRST) (Lie et al., 2012) on different geological realizations. MRST provided the COP for each Producer well location plan. The simulation time horizon for the simulator was set as 3000 days divided into 10 equal periods. The case study parameters used by MRST are provided in Table 1.

Table 1. Case study parameters

Parameter	Value
Initial pressure p_o	5080 psi
Oil viscosity μ_o at p_o	1.18 cp
Water viscosity μ_w at p_o	0.325 cp
Oil density ρ_o	865 kg/m^3
Water density ρ_w	929 kg/m^3
Relative permeability exponent for oil n_o	2
Relative permeability exponent for water n_w	2
Residual phase saturation for oil Sr_o	0
Residual phase saturation for water Sr_w	0
Relative permeability for oil kwm_o at Sr_o	1
Relative permeability for water kwm_w at Sr_w	1

In this study, a superset of 100 realizations were generated for realization reduction. For each realization, porosity values of the reservoir grid were generated in MRST using a built-in function '*GaussianField*' with a range parameter of [0.1, 0.5]. The function creates an approximate Gaussian random field by convolving a normal distributed random field with a Gaussian filter with a standard deviation of 2.5 (Lie et al., 2012). Permeability values were further generated from the porosity values using Karmen-Cozeny relationship (Lie et al., 2012)

$$k_c = \frac{1}{2\tau A_v^2}\frac{\phi_c^3}{(1-\phi_c)^2} \tag{12}$$

In (12), k_c is the permeability of cell c, ϕ_c is the porosity of cell c, A_v is the surface area of spherical uniform grains with a constant diameter of 10 and τ is tortuosity with value 0.81 (Lie et al., 2012). In the case study, the well placement optimization results using a subset of realizations from the proposed method are compared to subset of realizations obtained using static measure based ranking method and random selection.10 realizations were selected for the subset of realizations. In random selection, 10 realizations are arbitrarily selected from the superset of realizations.

To evaluate the static measures for different geological realizations, the threshold porosity is set as $\phi_o = 0.3$ and threshold permeability is set as $k_o = 3\times10^{-13}\ m^2$ to determine whether a cell is a net or non-net cell. Static measure based ranking method was applied next to obtain a subset of 10 selected realizations from the superset of 100 realizations. In the ranking based methods, all the 100 realizations in the superset are sorted in ascending order based on the static measure values. 10 realizations are evenly selected from the sorted list with ranks 1, 12, 23, 34, 45, 56, 67, 78, 89, 100. In this study, static measures of Net Pore Volume (PV_{net}) and Original Oil in Place (OOIP) were used to perform realization reduction using the ranking based method. Equations for PV_{net} and OOIP are provided as follows:

$$PV_{net} = \sum_c V_c \phi_c I_c^{net} \tag{13}$$

$$OOIP = \sum_c V_c \phi_c (1 - S_c) \tag{14}$$

where, V_c is the volume of reservoir cell c, ϕ_c is the porosity of cell c, S_c is the irreducible water saturation of cell c and I_c^{net} is an indicator to see if cell c is a net cell (I_c^{net}=1) or non-net cell (I_c^{net}=0).

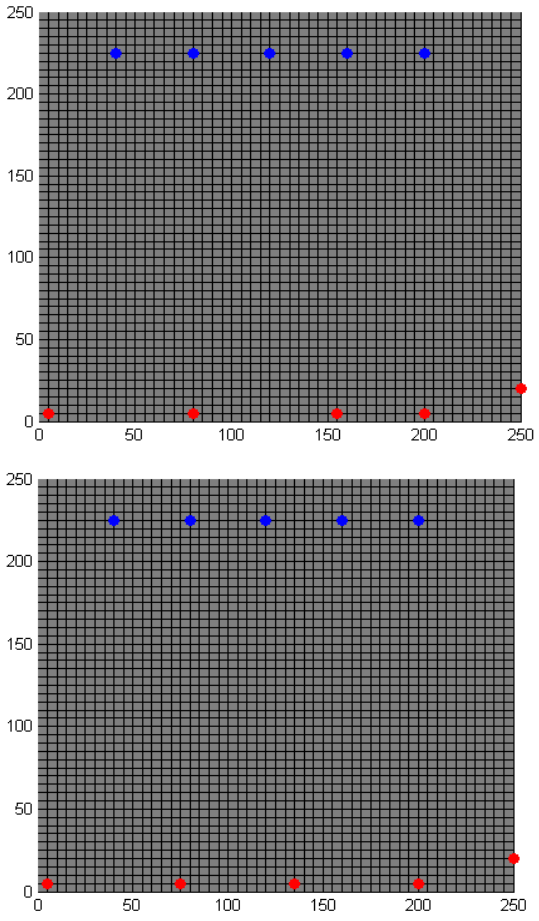

Fig. 2. (top) Well placement plan using selected realizations from proposed method; (bottom) Well placement plan using full set of realizations

The decision variables for the case study were the X and Y locations of the producer wells to be placed. The objective was to maximize the risk averted expected cumulative oil production after 3000 days of the simulation period. The well

placement optimization problem was simulated using a system with 3.2GHz Intel Core i5 processor and 8 GB memory. Well placement plans obtained from the optimization on the case study using the subset of realizations and the superset of realizations are provided in Fig. 2. In Fig. 2, the fixed injector wells are denoted by blue dot and the producer well locations are denoted by red dot.

It is evident from Fig. 2 that the producer well placement plan using a subset of realizations from the proposed realization reduction method is very similar to the producer locations used from the well placement optimization using all the realizations. The computational time for the well placement optimization and the mean and variance of the COP from the final prodcuer well location of using the subset of realizations using different methods and the original superset of all the realizations are given in Table 2.

Table 2. Case study results

	Mean COP (m^3/day)	COP standard deviation (m^3/day)	Simulation time (hours)
All realizations	11529	1000.4	25
Selected realizations from proposed method	11537	940.3	2.5
Selected realizations from PV_{net} ranking	11554	1402.5	2.5
Selected realizations from OOIP ranking	11472	1474.9	2.5
Selected realizations from random selection	11310	1235.4	2.5

Table 2 shows that the well placement optimization results using the subset of realizations selected by the proposed realization reduction method has the closest mean and variance values of COP when compared to the mean and variance COP obtained using all the realizations in the well placement optimization method. More importanlty, the well placement optimization problem using a subset of realization takes one-tenth of the computational time with the same 1000 iterations of optimization.

Fig. 3 provides the expected COP versus the number of iterations used by the NOMAD optimizer for the different realization reduction methods. It is clear that the expected COP of the well placement plan using the subset of realizations comes closest to the expected COP of the well placement plan using all the realizations as the number of iterations increases. The expected COP error is the absolute difference between the expected COP of the well plans using different realizations reduction methods with the expected COP of the well plan using all the realizations in the superset. It is clear from Fig. 3 that as the number of iterations increases, the expected COP of the proposed method becomes very close to the expected COP from the full set of realizations.

Similarly, Fig. 4 provides the plot of standard deviation of COP and error in the standard deviation of COP versus the number of iterations used by the NOMAD optimizer for the different realization reduction methods. Fig. 4 further confirms that the amongst the realization reduction methods, the proposed method has a standard deviation of the COP closest to the standard deviation of the COP calculated with all the realizations in the superset.

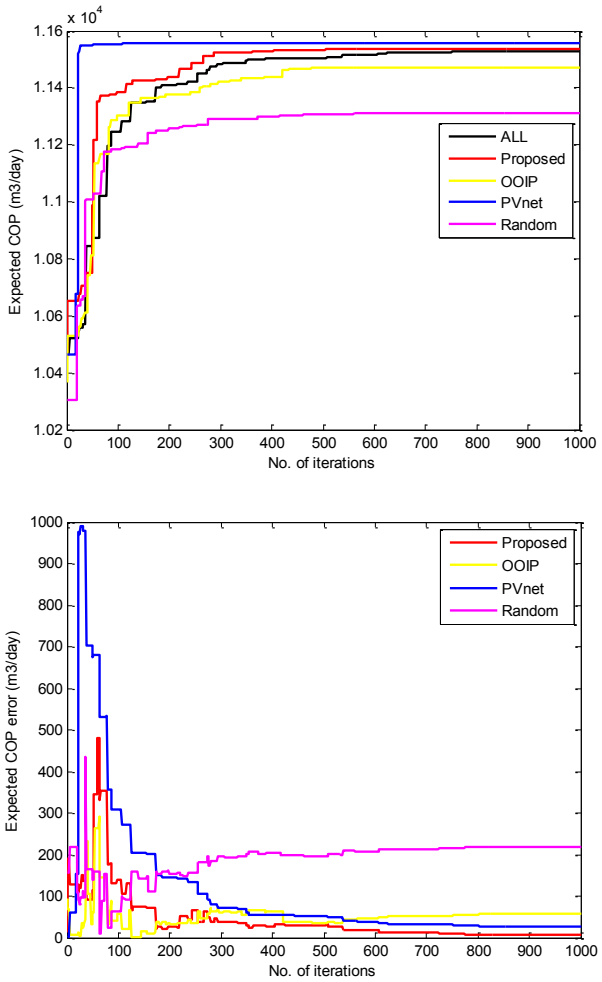

Fig. 3. (top) Expected COP versus number of iterations using different realization reduction methods; (bottom) Error in expected COP versus number of iterations using different realization reduction methods

5. CONCLUSION

In this study, a framework for well placement optimization with geological uncertainty reduction was proposed. The well placement optimization was formulated as a risk averted optimization problem by considering geological uncertainty. The optimization problem is solved using the derivative free optimization method. Geological uncertainty was incorporated into the robust optimization model which is formulated based on a set of optimally selected realizations. The subset of realizations was selected from a superset of

realizations using a mixed integer linear optimization model with an objective of finding out the optimal subset which has a similar statistical distribution characteristic to the superset of realizations. The realization reduction model is independent of well positions and depends on the reservoir geology. Results from case studies show that the well placement optimization problem using the proposed realization reduction method is very efficient. The well placement plan obtained using the small subset of realizations and the well placement plan for the superset of realizations are very similar and have similar mean COP values. Significant reduction in the computational time was achieved by using a subset of realizations in the well placement optimization problem. Comparison studies show that the results of the proposed method are also superior to the traditional ranking method and the random selection method.

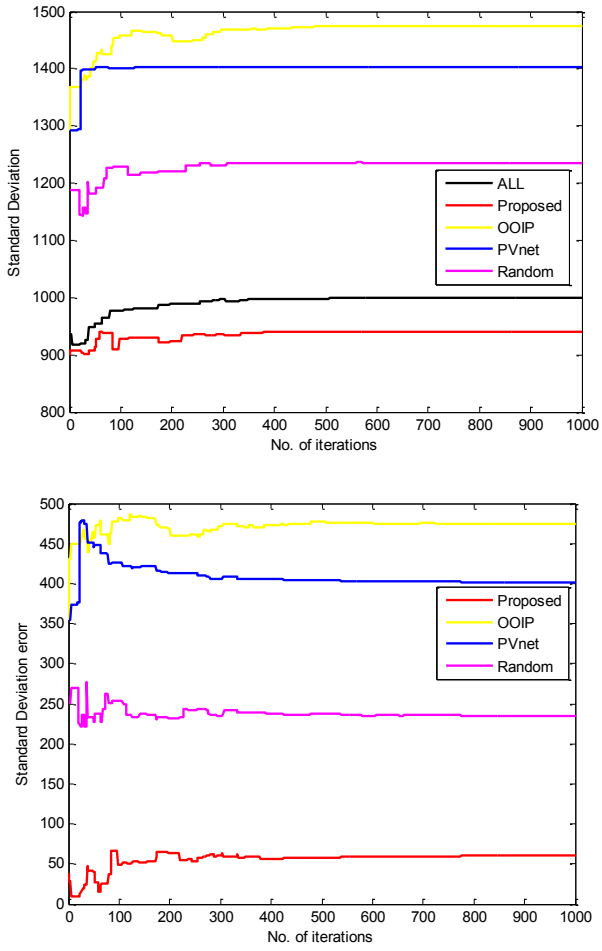

Fig. 4. (top) Standard deviation of COP versus number of iterations using different realization reduction methods; (bottom) Error in standard deviation of COP versus number of iterations using different realization reduction methods

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