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An Observer-based Model Predictive Control Strategy for Distributed Parameter System ⋆

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Abstract: In this paper, an observer-based model predictive control (MPC) strategy is presented for distributed parameter systems (DPSs). First, principal component analysis (PCA) is used for dimension reduction by transforming the high-dimensional spatio-temporal data into a low-dimensional time domain. Then an observer is builded to estimate the low-dimensional temporal output using the real-time measurable spatiotemporal output. Finally, the MPC strategy is proposed based on the low-dimensional estimation models. Simulations demonstrate the accuracy and efficiency of the proposed methodologies.

Keywords: Distributed parameter system, model predictive control, PCA, time/space separation, observer.

1. INTRODUCTION

Model predictive control (MPC) strategy has been used very successfully in industrial process. As many industrial process, such as flexible manipulator, fluid flow process, thermal process and convection diffusion reaction process exhibit strong spatiotemporal characteristic, nowadays more MPC researches focus on these processes (Armaou and Christofides (2002), Dufour and Toure (2004)). The mathematical description of these systems usually consists of systems of PDEs with boundary constraints (Gay and Ray (1995), Christofides (2001)). Such PDE models can accurately predict nonlinear and distributed dynamic behavior of these processes also called distributed parameter system (DPS), but the infinite-dimensional nature leads to the fact that they cannot be used due to the limited sensors and computing powers (S.Dubljevic and Christofides (2006)).

When the PDEs of the process are known, lumped parameter models are derived by traditional time/space discretization methods and the model-based control structure is adopted. The mode of the derived ordinary differential equation (ODE) system that yields the desired degree of approximation may be very large (Baker and Christofides (2000)). It leads to a complex controller design and high dimensionality of the resulting controllers for industrial processes. As it is a general difficulty associated with the control strategies for DPSs, the model-reduction-based MPC strategies are proposed to solve this problem.

Singular value decomposition (SVD) and Karhunen-Loeve (KL) decomposition are examples of popular model reduction methods for DPSs, where the system's spatial information is represented with the derived spatial basis functions (Gay and Ray (1995)). For these methods, an infinite number of basis functions can be found to represent the spatial frequencies of the system (Wang et al. (2011)). After truncation, a finite-dimensional coefficients modeling problem is raised. In the reference from Zheng and Hoo (2004), finite spatial basis functions were identified using a combination of singular value decomposition (SVD) and the KL expansion. Then a low-order linear model was established for dynamic modeling. The low-dimensional model based MPC control strategy was proposed for the DPS.

However, the model reduction method based on KL decomposition method is a linear projection/recontruction process, the modeling error occurs while the real time low-dimensional temporal data cannot be obtained directly. Although the feedback in MPC can reduce the impact of the discrepancy between the process and the predictive behavior, MPC is not designed to explicitly handle model mismatch. Thus, in this paper, an observer based MPC strategy for distributed parameter systems is presented using an identified low-dimensional model, where the s-patiotemporal outputs are used to estimate the state of the observer model. First, spatiotemporal training data is generated from the PDE, from which, the spatial information at the discrete node locations can be obtained. Then, principal component analysis (PCA) is applied to determine the dominant spatial basis functions of the PDE system. High-dimensional spatiotemporal data can then be projected onto the spatial basis functions to generate corresponding temporal data. From this temporal data, a low-dimensional state space model can be identified for control design. Then the observer is builded to approximate the temporal state based on the measurable spatiotemporal information. Finally, the proposed MPC strategy is ap-

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plied to a standard reaction-diffusion-convection process. The simulations demonstrate the accuracy and efficiency of the proposed methodologies.

The paper is organized as follows. In Section 2, the problem and the philosophy are presented. In Section 3, the model reduction process is proposed. Section 4 introduces the observer-based MPC strategy. Section 5 shows the application to a reaction-diffusion-convection process, in which we will also analyze the results and discuss the advantages of the proposed method. In the last section the conclusion is drawn.

2. PROBLEM FORMULATION

Consider a standard reaction-diffusion-convection process, we can obtain the following PDE to represent the system:

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} - k_r C;\ 0 < z < L,\ 0 < t \le t_f \quad (1)$$

where C is the specie concentration,z is the spatial coordinate, v is the known velocity field,L is the domain length, D is the specie diffusion coefficient, and k_r is a first-order rate constant.

Typical initial and boundary conditions are:

$$C(z,0) = 0,\quad C(0,t) = u(t),\quad D\left.\frac{\partial C}{\partial z}\right|_{z=L} = 0 \qquad (2)$$

Discretizing the spatial variable in (1)and (2)by the finite difference method gives:

$$\frac{dC_i}{dt} = D\frac{C_{i+1} - 2C_i + C_{i-1}}{\Delta z^2} - v\frac{C_{i+1} - C_{i-1}}{2\Delta z} - k_r C_i \qquad (3)$$

where C_i denotes the concentration at spatial location $i = 0, \ldots, N+1$ with $C_0 = C(0,t)$ and $C_{N+1} = C(L,t)$ specified from the boundary condition functions.

Under discretization schemes such as (3), the infinite-dimensional state is approximated as finite so that the number of nodes N represents the state dimension (i.e., $[C_1\ C_2\ \cdots\ C_N]^T$). Depending on the PDE, accurate solutions may require a large number of nodes. Furthermore, the total number of nodes scales polynomially with the number of dimensions (i.e., $O(N^{n_d})$ for N nodes in each of the n_d dimensions) which quickly makes these models computationally intractable for real-time control.

To avoid these high-dimensional models, a principal component analysis (PCA) based technique is introduced in the next section, which converts the spatiotemporal data to a low-dimensional time series by identifying the dominant spatial basis functions of the PDEs. From this time series, an state space model can be identified and used as a low-dimensional predictive model for MPC. As the low-dimensional temporal data cannot be obtained directly, an observer is builded to predict the state of low-dimensional model using the real-time spatiotemporal output. This methodology can improve the robustness of the control system and reduce the computational cost of MPC because the spatial evolution of the state is stored in the dominant basis functions (computed offline with PCA) so that the spatial dimensions can be bypassed in the algorithm.

3. MODEL REDUCTION

3.1 Time-space separation

Time-space separation methods are widely used for model identification from input-output data . The time-space separation approach assumes that the spatial dynamics of the system can be captured with an infinite number of basis functions

$$Y(z,t) = \sum_{i=1}^{\infty} \varphi_i(z) y_i(t) \qquad (4)$$

where $Y(z,t)$ denotes the spatiotemporal output of the system, z denotes the spatial variable over domain Ω, t denotes time, $\varphi_i(z)$, $i = 1, \ldots, \infty$ denotes the infinite number of spatial basis functions (frequencies) in ascending order, and $y_i(t)$ are the corresponding temporal coefficients. This expansion is motivated by the Fourier series implying the basis functions must be orthogonal. Orthonormality is further required to ensure each $\varphi_i(z)$ is unique i.e.,

$$\langle \varphi_i(z), \varphi_j(z) \rangle = \delta_{ij} \qquad (5)$$

where $\langle g(z), h(z) \rangle = \int_\Omega g(z)h(z)dz$ denotes the inner product and δ_{ij} is the Kronecker delta. The orthonormality condition in (7) implies that the temporal coefficients in (6) can be computed by

$$y_i(t) = \langle Y(z,t), \varphi_i(z) \rangle \qquad (6)$$

Karhunen-Loeve (KL) decomposition can be used to compute a finite number of spatial basis functions from $Y(z,t)$. The problem is posed as an optimization that minimizes the error between the truncated expansion and $Y(z,t)$. Using the method of “snapshots” (i.e., finite data $\{Y(z_j,t_k)\}_{j=1,k=1}^{Z,T}$ is known at Z spatial locations and T time points), it has been shown that the necessary condition for optimality can be stated as the following eigenvalue problem (EVP)(Li et al. (2009))

$$\int_\Omega R(z,\eta)\varphi_i(\eta)d\eta = \lambda_i \varphi_i(z) \qquad (7)$$

where $R(z,\eta) = \frac{1}{T}\sum_{k=1}^{T} Y(z,t_k)Y(\eta,t_k)$ is the spatial two-point correlation function. Because only a finite amount of data is known in the spatial dimension (positions $z_1, \cdots, z_Z$), the integral (7) must be solved numerically via discretization. This results in an $Z \times Z$ matrix EVP. This estimation procedure is known as the “spatial correlation method” and estimates Z eigenfunctions at the Z sampled spatial locations.

As T is typically much smaller than Z, a more computationally efficient method for solving (7) has been developed known as the “temporal correlation method” (Li et al. (2009)). The main assumption is that a general eigenfunction $\varphi(z)$ can be expressed as a linear combination of the snapshots

$$\varphi(z) = \sum_{k=1}^{T} \gamma_k Y(z,t) \qquad (8)$$

Substituting (8) into (7) produces the following $T \times T$ EVP

$$\mathbf{S}\boldsymbol{\gamma} = \lambda\boldsymbol{\gamma} \qquad (9)$$

where $\boldsymbol{\gamma} = [\gamma_1\ \gamma_2\ \cdots\ \gamma_T]^T$ denotes the eigenvector with corresponding eigenvalue λ and $\mathbf{S}$ is a positive semidefinite matrix defined as

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$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1T} \\ S_{12} & S_{22} & \cdots & S_{1T} \\ \vdots & \vdots & \ddots & \vdots \\ S_{1T} & S_{2T} & \cdots & S_{TT} \end{bmatrix} \quad (10)$$

with $S_{ij} = \frac{1}{L}\int_{\Omega} Y(z,i)Y(z,j)dz$. The maximum number of nonzero eigenvalues is $K = \min(Z,T)$. The n^{th}-order approximation for the output $Y_n(z,t)$ can be expressed as

$$Y_n(z,t) = \sum_{i=1}^{n} \varphi_i(z) y_i(t) \quad (11)$$

where n is the number of terms kept in the truncated expansion. Arranging the eigenvalues in decreasing order (i.e., $\lambda_1 > \lambda_2 > \cdots > \lambda_K$), PCA states that the fraction of variance (or "energy") retained in (11) can be computed by

$$\rho = \frac{\sum_{i=1}^{n} \lambda_i}{\sum_{i=1}^{K} \lambda_i} \quad (12)$$

After solving either (7) or (9) for $\{\varphi_i(z)\}_{i=1}^{n}$, the temporal coefficients $\{y_i(t)\}_{i=1}^{n}$ can be computed numerically from (6). Practically speaking, the basis functions and coefficients are only known at finite points, however, interpolation can easily be used to compute $Y_n(z,t)$ at any z and t values.

3.2 Predictive Model

The spatial basis functions provide a simple mapping from high-dimensional spatiotemporal data to lower-dimensional temporal data according to (6). This implies that a control strategy in terms of $\{y_i(t)\}_{i=1}^{n}$ can be used in place of spatially distributed control. With slight abuse of notation, the argument $k \in \mathbb{N}$ will represent discrete time in the rest of this work (related to continuous time according to $t = kT_s$ where T_s denotes the sampling time).

A low-dimensional temporal model (see e.g., Wang et al. (2011)) can be defined to predict the expansion coefficients

$$A_i(q^{-1})\widehat{y}_i(k+1) = B_i(q^{-1})\Delta u(k); \quad \forall i = 1, \cdots, n \quad (13)$$

where q^{-1} denotes the backward shift operator, $\widehat{y}_i(k+1)$ denotes the i^{th} predicted temporal coefficient at discrete time $k+1$, and $\Delta u(k) = u(k) - u(k-1)$ denotes the input change at k. The parameters $A_i(q^{-1})$ and $B_i(q^{-1})$ can be estimated offline from input-output data.

From the identified low-order temporal models (13), the spatiotemporal output $\widehat{Y}(z,k)$ can be reconstructed pointwise using (11)

$$\widehat{Y}(z,k) = \boldsymbol{\varphi}(z)\widehat{\mathbf{y}}(k) \quad (14)$$

where $\boldsymbol{\varphi}(z) = [\varphi_1(z)\ \varphi_2(z)\ \cdots\ \varphi_n(z)]$ denotes the spatial basis functions stacked into a row vector evaluated at spatial point z and $\widehat{\mathbf{y}}(k) = [\widehat{y}_1(k)\ \widehat{y}_2(k)\ \cdots\ \widehat{y}_n(k)]^T$ denotes the predicted temporal coefficients stacked into a column vector evaluated at discrete time k. Let $\mathbf{z} = [0\ \cdots\ L]^T$ denote a finite reconstruction of the spatial domain Ω, then the reconstructed output over $\mathbf{z}$ is computed as follows

$$\widehat{Y}(\mathbf{z},k) = \boldsymbol{\Psi}\widehat{\mathbf{y}}(k) \quad (15)$$

where

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\varphi}(0) \\ \vdots \\ \boldsymbol{\varphi}(L) \end{bmatrix} \quad (16)$$

which follows straightforwardly from (14).

4. OBSERVER-BASED MPC STRATEGY

Based on the low-dimensional model, the MPC strategy can be designed to achieve the control aim. However, PCA method is a linear projection/recontruction process, the modeling error occurs while the low-dimensional temporal data cannot be obtained directly. Thus, in this paper, an observer model is designed, where the states of the estimation model are estimated using the spatiotemporal measurement information. Next, the MPC strategy is derived using observer-based low-order state space models to predict predict the values of the controlled variables over a finite prediction horizon.

To simplify control design, the spatiotemporal control objective is transformed, using $\boldsymbol{\Psi}$, to be in terms of $\widehat{\mathbf{c}}$

$$\min_{\mathbf{u}(k)} \sum_{i=1}^{p} \|\mathbf{r}(k+i) - \widehat{\mathbf{c}}(k+i)\|^2 + w \sum_{i=0}^{m-1} \|\Delta u(k+i)\|^2 \quad (17)$$

$$\begin{aligned} \text{s.t.: } & u_{\min} \le u(k+i) \le u_{\max}; \quad i = 0, \cdots, m-1 \\ & \Delta u_{\min} \le \Delta u(k+i) \le \Delta u_{\max}; \quad i = 0, \cdots, m-1 \\ & \mathbf{c}_{\text{low}}(k+i) \le \widehat{\mathbf{c}}(k+i) \le \mathbf{c}_{\text{up}}(k+i); \quad i = 1, \cdots, p \end{aligned}$$

where p is the prediction horizon; m is the control horizon; w is the input change weighting; $\mathbf{u}(k) = [u(k)\ \cdots\ u(k+m-1]^T$ is the vector of input changes over the control horizon; $\mathbf{r}(k) = \boldsymbol{\Psi}^{-1}C(\mathbf{z},k)$ is the transformed reference; $u_{\min}$ and $u_{\max}$ are the minimum and maximum allowable supplied input, respectively; $\Delta u_{\min}$ and $\Delta u_{\max}$ are the minimum and maximum allowable input change, respectively; $\mathbf{c}_{\text{low}}(k) = \boldsymbol{\Psi}^{-1}C_{\text{low}}(\mathbf{z},k)$ denotes the lower bound on the temporal coefficients of the concentration; and $\mathbf{c}_{\text{up}}(k) = \boldsymbol{\Psi}^{-1}C_{\text{up}}(\mathbf{z},k)$ denotes the upper bound on the temporal coefficients of the concentration.

The results presented in Section 3 are valid for any PDE model. Thus, reaction-diffusion-convection PDE model of (1) can be represented using (14)

$$\widehat{C}(\mathbf{z},k) = \boldsymbol{\Psi}\widehat{\mathbf{c}}(k) \quad (18)$$

where $\widehat{\mathbf{c}}(k)$ denotes the predicted temporal coefficients corresponding to the growth factor concentration (defined similarly to $\widehat{\mathbf{y}}(k)$).

As the parameter $\boldsymbol{\Psi}$ and the model parameters for the elements of (13) can be obtained by the detailed steps in Section 3, the multi-input multi-output ARX model can be obtained. Considering that the low-dimensional temporal data cannot be obtained directly, the control quality based on the temporal models may be affected by the model mismatch and system noise. Thus, transforming the ARX model into the state space model,an observer is builded to estimate the temporal output using the on-line measured spatiotemporal output. The state space model can be defined as

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{A}\mathbf{x}(k-1) + \mathbf{B}u(k-1) + w(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) \end{aligned} \quad (19)$$

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where $\mathbf{x}(k) = [\mathbf{c}(k)^T\ \mathbf{c}(k-1)^T\ \cdots\ \mathbf{c}(k-d_y)^T\mathbf{u}(k)^T\ \mathbf{u}(k-1)^T\ \cdots\ \mathbf{u}(k-d_u+1)^T]$ denotes the state of the low-dimensional model, and $\mathbf{x}(k) = [\mathbf{c}(k)$ denotes the output of the low-dimensional model.

According to the low-dimensional state space model, the observer model can be described as:

$$\begin{aligned}\widehat{\mathbf{x}}(k|k-1) &= \mathbf{A}\widehat{\mathbf{x}}(k-1) + \mathbf{B}u(k-1)\\ \widehat{\mathbf{x}}(k|k) &= \mathbf{A}\widehat{\mathbf{x}}(k|k-1) + K(z,k)(C(z,k)\\ &\quad - \varphi\mathbf{C}\widehat{\mathbf{x}}(k|k-1))\end{aligned} \tag{20}$$

where $\widehat{\mathbf{x}}(k|k-1)$ represents the estimate state at $k-1$, $\widehat{\mathbf{x}}(k|k)$ represents the state of the observer at time k, $K(z,t)$ represents the Kalman filter gain and $C(z,t)$ represents the measurable spatiotemporal concentration.

Let $P(k-|k-1)$ represent the covariance matrix of state estimation error at time k-1,

$$\begin{aligned}P(k-1|k-1) =& E((\mathbf{x}(k-1) + \widehat{\mathbf{x}}(k-1|k-1))\\ &(\mathbf{x}(k-1) + \widehat{\mathbf{x}}(k-1|k-1))^T)\end{aligned} \tag{21}$$

The covariance matrix of state estimation error at time k can be described as:

$$P(k|k) = (I - K(z,t)\varphi\mathbf{C})P(k-1|k-1) \tag{22}$$

then the Kalman filter gain can be obtained by minimize the the covariance matrix of state estimation error

$$K(z,t) = P(k|k-1)\mathbf{C}^T\varphi^T(\varphi\mathbf{C}P(k|k-1)\mathbf{C}^T\varphi^T)^{-1} \tag{23}$$

where $P(k|k-1) = AP(k-1)A^T$.

Using the predictive control strategy, the predicted temporal coefficients over the prediction horizon can be obtained according to the observer model. The main steps of the entire proposed control method can be summarized as follows

(1) Use the input-output data (obtained by solving the governing PDE for the specie concentration at desired spatial locations) to compute the spatial basis functions using (7) or (9).
(2) Compute a time series for the temporal coefficients using (8) and identify a low-dimensional ARX model for each of these coefficients according to (15).
(3) Transform the spatially distributed reference and concentration bounds to a simple time-series using $\mathbf{\Psi}$ as detailed in (21).
(4) Using the spatiotemporal output $C(z,t)$ to calculate the Kalman gain as detailed in (23).
(5) Compute the optimal input sequence $\Delta\mathbf{u}(k)^*$ by solving the optimal problem in (17). Apply the first element of this optimal input sequence $\Delta u(k)^*$ to the system. Set $k := k+1$ and repeat this process until the final time (i.e,. in a receding horizon manner).

5. SIMULATION STUDY

The proposed observer-based MPC strategy is applied to an example of (1),with $L = 1$ m, $D = 1$ m^2/s, $v = 1$ m/s, and $k_r = 0.1$ 1/s. To generate the model training data, (1) was solved in time increments of $\Delta t = 0.05$ s and spatial increments $\Delta z = 0.01$ m until time 10 s with input trials

$$u_k = 3U\sin(k\Delta t/50 + U) + 0.5e^{-1/20}\sin(k\Delta t/10) \tag{24}$$

where $k = 1, \ldots, 10/\Delta t$ and U is a uniform random number between 0 and 1 (i.e., `rand`). Using this 100×200 spatiotemporal training data, we can compute the dominant spatial basis functions using the PCA method (Section 2). Figure 1 shows the $n = 3$ dominant spatial basis functions used to identify a low-dimensional ARX model. For the control problem, define the input constraints to be

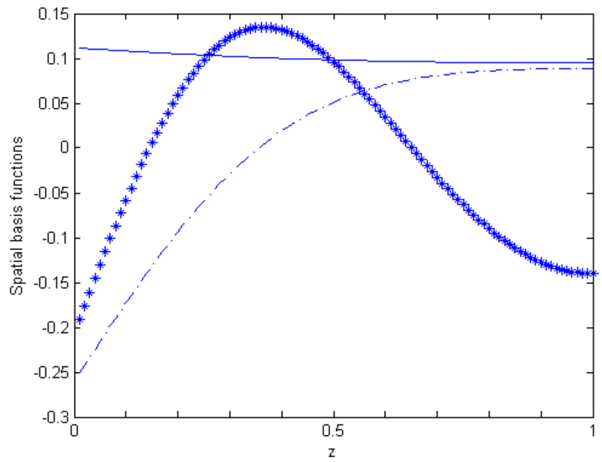

Fig. 1. Spatial basis functions

$u_{\min} = 0.1$, $u_{\max} = 1.2$, $\Delta u_{\min} = -0.1$, and $\Delta u_{\max} = 0.1$, the controller parameters to be $w = 0.9$, $p = 3$, and $m = 3$, and the spatially distributed reference C_{des} to correspond to that shown in Figure 2. Using the proposed control strategy in Section 3, the closed-loop response of the specie concentration can be obtained (see Figure 3), which closely matches the desired spatially-varying reference C_{des}.

To verify the controller's ability to reject disturbances, one case was considered: a step output disturbance with amplitude -0.3 were added to the process. The closed-loop behavior of the concentration at the boundary $C(1,t)$ under the output disturbance is shown in Figure 4.

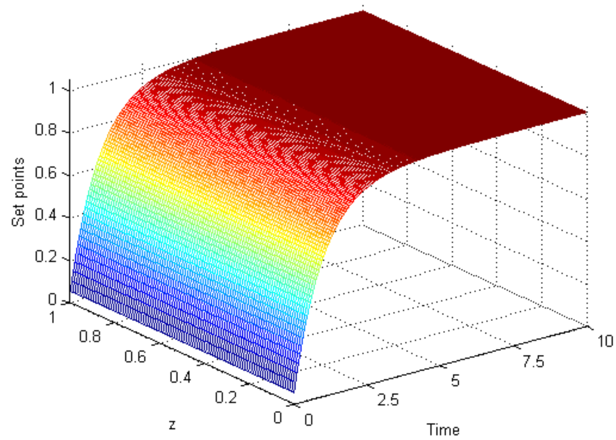

Fig. 2. Spatially distributed reference

Table 1 compares the floating point operations (FLOPs) for the typical discretization based MPC method and the proposed low-dimensional MPC strategy in Problem 2.

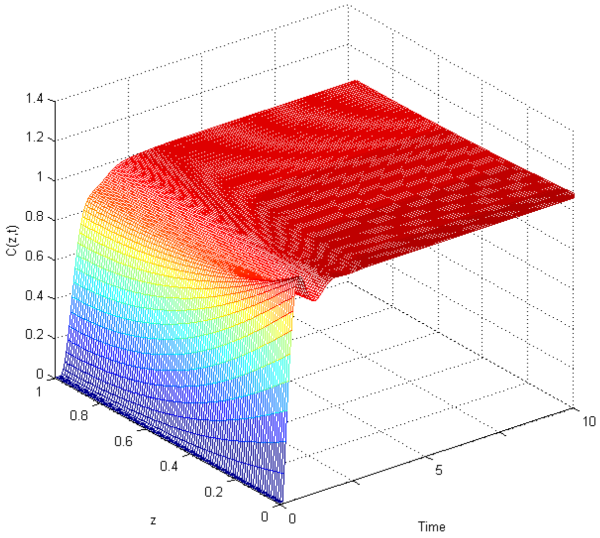

Fig. 3. Spatially distributed concentration $C(z, t)$

The main steps are a prediction step (solving the optimization) and a model update step. The fact that the state-space methods scale with the number of discretization nodes $O(N^2)$ indicates that this will quickly become intractable. However, the proposed method scales as $O(n^2)$ due to the low-order model implying that it is an efficient alternative for the DPS control design while still achieving good performance.

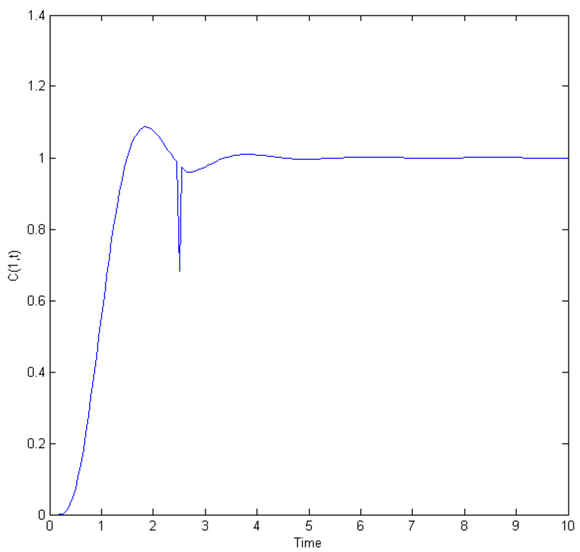

Fig. 4. Closed-loop response under output disturbance

Table 1. FLOP comparison

	Discretization-based MPC	Proposed MPC strategy
Predictive Step	$6N + 9$	$2n(6n - 1)$
Update Step	$N(2N + 1)$	$3n(6n + 1)$

6. CONCLUSION

An observer-based MPC strategy is presented for distributed parameter systems in this paper. First, PCA is used to transform the high-dimensional spatiotemporal data into a low-dimensional time domain. To simplify control design, based on the low-dimensional models,the spatiotemporal control objective is transformed into the temporal control objective. Thus, MPC strategy is proposed based on the low-dimensional models, where an observer is builded to approximate the temporal state based on the measurable spatiotemporal output. Simulations demonstrate the accuracy and efficiency of the proposed methodologies.

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