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Drill-down diagnosis of deficient models in MPC

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Abstract: Model maintenance is the most time-consuming and cost-intensive in industrial model predictive control. In this paper, a drill-down diagnosis algorithm for deficient models of industrial MPC via a model quality index (MQI) is proposed. The CVs with poor models can be detected first by MQI values with all controlled variables. Then, a leave-one-out algorithm is proposed to further diagnose which sub-models are deficient for the CVs with poor model performance. Thus, the effort and cost of model maintenance can be reduced. The application result to the Wood-Berry distillation column process indicates the effectiveness of the proposed assessment method.

Keywords: Drill-down diagnosis, Model quality index, Predictive control, Wood-Berry distillation column

1. INTRODUCTION

Industrial model predictive control (MPC) usually needs maintenance in less than a year after it has been put into operation due to process and disturbance dynamics changes over time. MPC controller maintenance usually leads to remodeling of the plant and disturbance dynamics and retuning of the controller accordingly, which is a costly practice and may require interrupting the process operation. Furthermore, there is a lack of analytic tools to determine whether it is necessary to remodel the whole plant and whether remodeling will definitely improve the overall control performance. To avoid unnecessary cost in remodeling and controller maintenance, it is desirable to study which part of the model is deficient and to what extent the controller or model is deficient for industrial MPC.

Control performance monitoring is concerned with advising the engineers whether the MPC needs maintenance at some point. A popular approach is the comparison between the actual output variance estimated by routine closed-loop data and theoretical minimum variance control (MVC) benchmark. The method was firstly developed by Harris(1989) and many extensions were studied subsequently (Harris et al,1996; Huang,2002; Harrison and Qin,2009a; Yu and Qin, 2009). More recent work focuses on diagnosing the cause of control performance deterioration. Tian et al. (2011) presented a data-driven classier method to diagnose the root cause of performance deterioration. The detection and diagnosis methods of control valves stiction were presented in (Yu et al.,2009; Thornhill and Horch, 2007). Integral square error (ISE) performance for multiplexed MPC (MMPC) was proposed to reflect the performance of controller itself which helps to select suitable controller parameters in the design of MMPC controller (Ling et al., 2011).

Model performance diagnosis is a key focus recently in the field of control performance assessment. The method in (Abhijit et al, 2010) has the ability to assess impact of model-plant mismatch on control performance by the designed control error benchmark. The work of Ji et al. (2012) presents a model error detection method by comparing the tested process frequency responses and frequency responses of the current MPC model. Sun and Qin (2013) proposed a model quality index (MQI) to evaluate the performance of MPC by comparing the MPC model residual to process innovations, which is a theoretical minimum. The contribution of Harrison and Qin (2009b) is the discrimination between the disturbance model mismatch and process model mismatch by the order of innovations.

Our proposed work in this paper focuses on the MPC model performance assessment and model deficiency diagnosis. The MQI method shows great ability to assess the model mismatch of MPC by simply using the routine setpoint data, output data and prediction model (Sun and Qin, 2013). It can infer whether the poor MPC performance is caused by a poor prediction model. In practice, there are usually many controlled variables (CV) in a predictive control system and many sub-models corresponding to each CV. Usually, not all the sub-models are deficient when the MQI of the overall predictive control system is poor. Hence, it is desirable to determine which CV has a poor model and further which sub-model of the CV is deficient, which can save cost by updating only the deficient sub-model.

We obtain separate MQIs of each CV from the setpoint and output closed-loop data, which can be obtained easily from the control system. Therefore, we can infer which CV model is poor according to the values of separate MQIs. However, the effect of all sub-models of each CV is considered as a whole in the computation of prediction errors and the MQI.

To isolate the response of a manipulated variable (MV) from the output data, we present a drill-down diagnosis algorithm to give elaborate advice to engineers about the necessity of model maintenance. Firstly, we present an MQI computation method by the MV and CV closed-loop data and it gives the information on which CV model is poor. Then, the effect of each sub-model is considered and corresponding MQI is computed for those CVs with poor models assessed before. Because the CV data is the overall result of the input response and disturbance response, and the effect of each MV cannot be isolated from closed-loop CV data, MQI corresponding to each sub-model cannot be obtained directly by the MQI method. Therefore, we present a leave-one-out method in this paper to compute MQIs corresponding to sub-models by leaving out one MV at a time. By comparing the overall MQI to MQI that one MV is removed, we can infer the performance of corresponding sub-model.

2. AN ALTERNATIVE MQI ASSESSMENT ALGORITHM

Consider a MIMO predictive control system in Fig.1, where $\boldsymbol{u}(k)$ is N_u*1 dimensional control input vector(MV, manipulate variable), $\boldsymbol{y}(k)$ is N_y*1 dimensional output vector (CV, controlled variable), $\hat{\boldsymbol{y}}(k)$ is N_y*1 dimensional prediction output vector, N_y* N_u dimensional transfer function $\boldsymbol{G}^o(q)$ and N_y* N_y dimensional diagonal transfer function $\boldsymbol{H}^o(q)$ are the true process model and disturbance model, $\boldsymbol{G}_m(q)$ and $\boldsymbol{H}(q)$ are corresponding control model and disturbance model of prediction, N_u* N_y dimensional transfer function $\boldsymbol{G}_c(q)$ is the prediction controller, $\boldsymbol{e}^o(k)$ and $\boldsymbol{d}^o(k)$ are N_y*1 dimensional process innovation and disturbance, respectively, $\boldsymbol{e}(k)$ and $\boldsymbol{d}(k)$ are N_y*1 dimensional prediction error and control model error, respectively, $\boldsymbol{r}(k)$ is N_y*1 dimensional reference trajectory.

Assuming that there is no mismatch in the control model and disturbance model, it is obvious that the prediction error

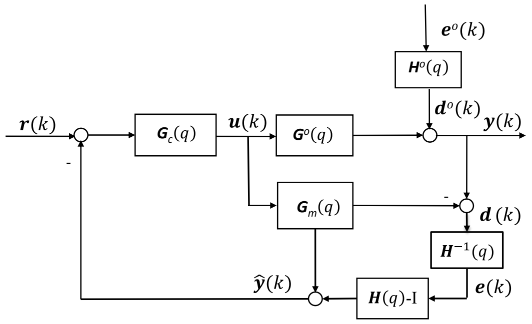

Fig.1. Schematic diagram of model predictions and residuals

$\boldsymbol{e}(k)$ equals to the process innovation $\boldsymbol{e}^o(k)$. Otherwise, if there is mismatch either in control model or in disturbance model, prediction error $\boldsymbol{e}(k)$ includes some model mismatch information and the innovation $\boldsymbol{e}^o(k)$ is the projection component of $\boldsymbol{e}(k)$ to the space spanned by previous $\boldsymbol{y}(k)$ and $\boldsymbol{u}(k)$ (Sun and Qin, 2013). So we define model quality index (MQI) as

$$\eta = \frac{\sum_{k=1}^{N} \boldsymbol{e}^o(k)^T \boldsymbol{Q}(k) \boldsymbol{e}^o(k)}{\sum_{k=1}^{N} \boldsymbol{e}(k)^T \boldsymbol{Q}(k) \boldsymbol{e}(k)} \tag{1}$$

where N_y*N_y dimensional matrix $\boldsymbol{Q}(k)$ is the weight matrix for outputs at time k that are properly chosen at the MPC design stage, N is the data length of assessment period. $\boldsymbol{e}^o(k) = [e_1^o(k) \quad \dots \quad e_{N_y}^o(k)]^T$ can be obtained by routine CV data $\boldsymbol{y}(k)$ and MV data $\boldsymbol{u}(k)$, and $\boldsymbol{e}(k) = [e_1(k) \quad \dots \quad e_{N_y}(k)]^T$ can be computed by the control model coefficients of prediction model and process output $\boldsymbol{y}(k)$. The range of MQI η is in (0, 1]. An η approaching 1 indicates an accurate prediction model and a small value implies a poor control model or disturbance model.

In addition, MQI corresponding to lth CV can be defined as

$$\eta_l = \frac{\sum_{k=1}^{N} e_l^o(k) Q_l(k) e_l^o(k)}{\sum_{k=1}^{N} e_l(k) Q_l(k) e_l(k)} \tag{2}$$

By $\eta_l (l = 1, \cdots, N_y)$, the model quality of each CV can be evaluated.

2.1 Estimating Disturbance Innovations From Input-output Data

We summarize an algorithm to estimate disturbance innovations from closed loop input-output data in the following theorem that is similar to Sun and Qin (2013).

Theorem 1 Consider a multi-input-multi-output (MIMO) process under linear-time-invariant (LTI) control in Fig.2. Assume N_y*1 dimensional output vector $\boldsymbol{y}(k)$ (CV) corresponds to N_u*1 dimensional input vector $\boldsymbol{u}(k)$ (MV).

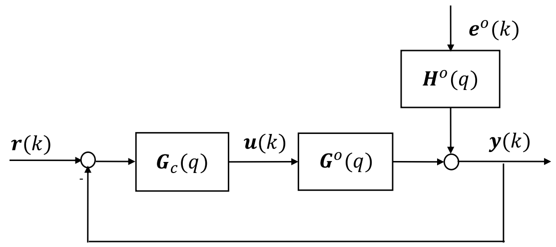

Fig.2 Schematic diagram of a closed-loop controlled process

Define

$$\boldsymbol{y}_p(k) = [\boldsymbol{y}(k) \quad \boldsymbol{y}(k-1) \quad \dots \quad \boldsymbol{y}(k-p)]$$
$$\boldsymbol{e}_p^o(k) = [\boldsymbol{e}^o(k) \quad \boldsymbol{e}^o(k-1) \quad \dots \quad \boldsymbol{e}^o(k-p)]$$

$$\boldsymbol{u}_p(k) = [\boldsymbol{u}(k) \quad \boldsymbol{u}(k-1) \quad \dots \quad \boldsymbol{u}(k-p)]$$
$$\boldsymbol{Y}_M(k-1) = [\boldsymbol{y}_p(k-1) \quad \boldsymbol{y}_p(k-2) \quad \dots \quad \boldsymbol{y}_p(k-M)]^{\mathrm{T}},$$
$$\boldsymbol{U}_N(k-1) = [\boldsymbol{u}_p(k-1) \quad \boldsymbol{u}_p(k-2) \quad \dots \quad \boldsymbol{u}_p(k-N)]^{\mathrm{T}},$$
$$\overline{\boldsymbol{Z}}_p(k) = [\boldsymbol{Y}_M(k-1) \quad \boldsymbol{U}_N(k-1)]^T \tag{3}$$

where p is the time window, the dimension of $\boldsymbol{y}_p(k)$ and $\boldsymbol{e}_p^o(k)$ are N_y*p, and the dimension of $\boldsymbol{u}_p(k)$ is N_u*p, the dimension of $\boldsymbol{Y}_M(k-1)$, $\boldsymbol{U}_N(k-1)$ and $\overline{\boldsymbol{Z}}_p(k)$ are $(N_y*M)*p$, $(N_u*N)*p$ and $(N_y*M+N_u*N)*p$, respectively.

Define the projection to the orthogonal complement of the row space of $\overline{\boldsymbol{Z}}_p(k)$

$$\boldsymbol{\Pi}^{\perp}_{\overline{\boldsymbol{Z}}_p(k)} = \mathbf{I} - \overline{\boldsymbol{Z}}_p(k)^T \left[\overline{\boldsymbol{Z}}_p(k)\overline{\boldsymbol{Z}}_p(k)^T \right]^{-1} \overline{\boldsymbol{Z}}_p(k) \tag{4}$$

For a linear process controlled by an LTI controller, the sequence of disturbance innovations $\boldsymbol{e}_p^o(k)$ is obtained by performing the orthogonal projection

$$\boldsymbol{e}_p^o(k) = \boldsymbol{y}_p(k)\boldsymbol{\Pi}^{\perp}_{\overline{\boldsymbol{Z}}_p(k)} \quad \text{as } p \to \infty. \tag{5}$$

The proof of Theorem 1 is given in the appendix. To implement the projection in Theorem 1 efficiently and robustly, LQ decomposition can be performed (Sun and Qin, 2013)

$$\begin{bmatrix} \overline{\boldsymbol{Z}}_p(k) \\ \boldsymbol{y}_p(k) \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{11} & \\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q}_1 \\ \boldsymbol{Q}_2 \end{bmatrix} \tag{6}$$

Thus

$$\boldsymbol{e}_p^o(k) = \boldsymbol{y}_p(k) - \boldsymbol{L}_{21}\boldsymbol{L}_{11}^{\dagger 1}\overline{\boldsymbol{Z}}_p(k) \tag{7}$$

where the dimensions of $\boldsymbol{L}_{21}$ and $\boldsymbol{L}_{11}$ are independent of the number of the data samples for they are only required to calculate the innovations sequence.

2.2 Estimating Prediction Errors From Mismatched Model Residuals

In MPC algorithms, one-step ahead predictive output is computed by N_y*N_u dimensional control model $\boldsymbol{G}_m(q)$ and N_y*N_y dimensional diagonal disturbance model $\boldsymbol{H}(q)$ that are different from the actual plant transfer functions.

$$\boldsymbol{y}(k) = \boldsymbol{G}_m(q)\boldsymbol{u}(k) + \boldsymbol{H}(q)\boldsymbol{e}(k) \tag{8}$$

gives the prediction and prediction errors as follows

$$\begin{aligned} \hat{\boldsymbol{y}}(k \mid k-1) &= (\mathbf{I} - \boldsymbol{H}^{-1})\boldsymbol{y}(k) + \boldsymbol{H}^{-1}\boldsymbol{G}_m\boldsymbol{u}(k) \\ \boldsymbol{e}(k) &= \boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k \mid k-1) \\ &= \boldsymbol{H}^{-1}(\boldsymbol{y}(k) - \boldsymbol{y}_m(k)) \end{aligned} \tag{9}$$

where $\boldsymbol{y}_m(k) = \boldsymbol{G}_m(q)\boldsymbol{u}(k)$ is the MPC control model output at time k. In DMC it is often obtained by finite step response model

$$\boldsymbol{y}_m(k) = \overline{\boldsymbol{y}}(k) + \sum_{i=1}^{N_0} \boldsymbol{a}_i \Delta\boldsymbol{u}(k-i) \tag{10}$$

where $\overline{\boldsymbol{y}}(k)$ is the initial state value of $\boldsymbol{y}(k)$, $\boldsymbol{a}_i\ (i = 1, \dots, N_0)$ is the ith N_y*N_u dimensional step response coefficients matrix of the process

$$\boldsymbol{a}_i = \begin{bmatrix} a_{11i} & a_{12i} & \cdots & a_{1n_u i} \\ a_{21i} & a_{22i} & \cdots & a_{2n_u i} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n_y 1i} & a_{n_y 2i} & \cdots & a_{n_y n_u i} \end{bmatrix},$$

$\Delta\boldsymbol{u}(k) = [\Delta u_1(k) \quad \Delta u_2(k) \quad \dots \quad \Delta u_{N_u}(k)]^T$, N_u is the input number, and N_0 is the model length. Thus

$$\begin{aligned} \boldsymbol{y}(k) &= \boldsymbol{y}_m(k) + \boldsymbol{H}(q)\boldsymbol{e}(k) \\ &= \overline{\boldsymbol{y}}(k) + \sum_{i=1}^{N_0} \boldsymbol{a}_i \Delta\boldsymbol{u}(k-i) + \boldsymbol{H}(q)\boldsymbol{e}(k) \end{aligned} \tag{11}$$

In industrial MPC, the general practice is using a step disturbance model, which is equivalent to a random walk disturbance model, hence

$$\boldsymbol{H}(q) = \mathrm{diag}\{\frac{1}{1-q^{-1}}, \dots, \frac{1}{1-q^{-1}}\}$$

Prediction error can be expressed by

$$\begin{aligned} \boldsymbol{e}(k) &= (1-q^{-1})\left(\boldsymbol{y}(k) - \boldsymbol{y}_m(k)\right) \\ &= (1-q^{-1})\boldsymbol{d}(k) \end{aligned} \tag{12}$$

where $\boldsymbol{d}(k) = \boldsymbol{y}(k) - \boldsymbol{y}_m(k)$ is the disturbance estimate.

3. DRILL-DOWN DIAGNOSIS OF DEFICIENT SUB-MODELS

By MQI method in Section 2, we can infer which CV has poor prediction model by routine CV data, MV data and model coefficients. However, for a multi-input CV, usually not all the sub-models corresponding to its MVs are deficient. Engineers usually hope to further know which sub-model has mismatch in order to reduce the work of model maintenance. So the model performance diagnosis is essential in practice. Because we cannot tell the root of deficiency by the overall model performance index, the isolated performance index of each sub-model is required to detect the deficient models.

Consider the MIMO control system in Fig.2. From appendix, the lth output $y_l(k)$ can be expressed as

$$y_l(k) = \sum_{i=1}^{M} H_i^l y_l(k-i) + \sum_{j=1}^{N_u}\sum_{i=1}^{N} G_{ji}^l u_j(k-i) + e_l^o(k) \quad (13)$$

where $u_j(k)$ is the jth input, H_i^l and G_{ji}^l are the corresponding coefficients to the lth output $y_l(k)$, and N_u is the number of input. We wish to assess the model quality of the jth sub-model. Unfortunately, it is difficult to extract the response of jth MV on lth CV, since only the response data $y_l(k)$ of all MVs are available from routine closed-loop data. Therefore, we propose a leave-one-out method to compute the performance of sub-models indirectly.

3.1 MQI of Sub-models by the Leave-one-out Method

The leave-one-out method keeps all the sub-models but one MV to assess the contribution of the sub-model of the specific MV. For example, for lth output, jth MV is removed for the evaluation of the sub-model denoting jth input to lth output. $e_{l(j)}^o(k)$ is the corresponding disturbance innovation. In the computation of $e_{l(j)}^o(k)$, the contribution of jth MV cannot be isolated from $y_l(k)$ and so we use the original output $\boldsymbol{Y}_M(k\text{-}1)$ and input $\boldsymbol{U}_{N(j)}(k-1)$ (removing $u_j(k)$)in (3) to construct $\bar{\boldsymbol{Z}}_p(k)$. Thus, the obtained disturbance innovations $e_{l(j)}^o(k)$ include the effect of the removed MV. That is, the contribution of jth MV is considered as disturbance. Similarly, $e_{l(j)}(k)$ can be computed by (12) where the jth MV is removed in the computation of $y_m(k)$ by (10).

Define the MQI for lth output removing jth input

$$\eta_{l(j)} = \frac{\sum_{k=1}^{N} e_{l(j)}^o(k)Q(k)e_{l(j)}^o(k)}{\sum_{k=1}^{N} e_{l(j)}(k)Q(k)e_{l(j)}(k)} \quad (14)$$

If isolated MQI $\eta_{l(j)}$ improves after removing the jth MV, the corresponding sub-model is deficient. So we can infer the sub-model performance by the comparison of the isolated MQI and overall MQI. Hence, we define

$$\kappa_{l(j)} = \frac{\eta_{l(j)}}{\eta_l} \quad (15)$$

which measures the influence on MQI by removing an input u_j. $\kappa_{l(j)} > 1$ indicates an improvement of model performance by removing u_j, which implies that the corresponding sub-model is deficient. Conversely, $\kappa_{l(j)} < 1$ shows deterioration of model performance after leaving input u_j out, which implies that the removal of sub-model is detrimental.

3.2 Procedure of Drill-down Diagnosis of Model Performance

The procedure to diagnose a CV prediction model is summarized as follows.

1) Collect routine closed-loop data $\boldsymbol{y}(k)$ and $\boldsymbol{u}(k)$, construct related vectors in (3), and compute disturbance innovations $\boldsymbol{e}^o(k)$ by (5) or (7);

2) Compute model output $\boldsymbol{y}_m(k)$ by (10) using routine data $\boldsymbol{y}(k)$, $\boldsymbol{u}(k)$ and model coefficients $\boldsymbol{a}_i$, then further calculate $\boldsymbol{e}(k)$ by (12);

3) Calculate η by (1) using $\boldsymbol{e}^o(k)$ and $\boldsymbol{e}(k)$ obtained in step 1) and step 2), assess the model performance by the value η. Compute η_l one by one by (2) and evaluate the model of each CV. If η_l is below a threshold, go to step 4, else a good prediction model is concluded;

4) Remove a MV, reconstruct vectors in (3) and compute corresponding $e_{l(j)}^o(k)$ by (5) or (7);

5) Compute model output $y_m(k)$ by (10) removing the same MV as that in step 4, then further calculate $e_{l(j)}(k)$ by (12);

6) Calculate $\eta_{l(j)}$ by (14) and $\kappa_{l(j)}$ by (15), assess the corresponding sub-model performance by the value of $\kappa_{l(j)}$;

7) Repeat step 4 to step 6 until all the related MVs are treated.

4. CASE STUDY OF THE WOOD-BERRY DISTILLATION COLUMN PROCESS

The presented drill-down model performance diagnosis and improvement via IMA(1,1) disturbance model are demonstrated by the Wood-Berry distillation column process. The process transfer function matrix(Wood and Berry, 1973) is

$$\boldsymbol{G}(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21.0s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}. \quad (16)$$

Reflux rate and steam flow rate are inputs (manipulated variables) of the process in lb/min; composition of top product and bottom product of column in mol% are two outputs (controlled variables), respectively. The process transfer function matrix is discretized to

$$\boldsymbol{G}^o(q)=\begin{bmatrix} q^{-2}\dfrac{0.744}{1-0.9419q^{-1}} & q^{-4}\dfrac{-0.8789}{1-0.9535q^{-1}} \\ q^{-8}\dfrac{0.5786}{1-0.9123q^{-1}} & q^{-4}\dfrac{-1.302}{1-0.9329q^{-1}} \end{bmatrix} \quad (17)$$

with the sampling time of 1 min. The actual disturbance model is chosen as

$$\boldsymbol{H}^o(q)=\begin{bmatrix} \dfrac{1-0.5q^{-1}}{1-q^{-1}} & \\ & \dfrac{1-0.7q^{-1}}{1-q^{-1}} \end{bmatrix} \quad (18)$$

$\boldsymbol{e}^o(k)$ is independent white noise with covariance of $\mathrm{diag}\{0.7442,\ 0.132\}$.The MPC prediction and control horizons are chosen to be 100 and 10, respectively. The weight matrices are $Q=\mathrm{diag}\{1,10\}$ and $S=\mathrm{diag}\{1,10\}$. For simplicity, no constraint is added. The setpoints are set to $\overline{y}_1=90\%$ and $\overline{y}_2=5\%$. The model length M=N=30, time window p=3940 in the experiment.

To verify the effectiveness of proposed diagnosis and the improvement method, MQI values in different model cases are calculated and the results are listed in Table 1. Corresponding comparison is shown in Fig.3. Case 1: accurate control model and disturbance model; Case 2: DMC with accurate control model and random walk disturbance model. Case 3: DMC with the mismatch control model

$$G^o(q)=\begin{bmatrix} 0.8*q^{-2}\dfrac{0.744}{1-0.9419q^{-1}} & 0.6*q^{-4}\dfrac{-0.8789}{1-0.9535q^{-1}} \\ q^{-8}\dfrac{0.5786}{1-0.9123q^{-1}} & 1.2*q^{-4}\dfrac{-1.302}{1-0.9329q^{-1}} \end{bmatrix} \quad (19)$$

and random walk disturbance model; Case 4: DMC with the mismatch control model

$$G^o(q)=\begin{bmatrix} q^{-2}\dfrac{0.744}{1-0.9419q^{-1}} & q^{-4}\dfrac{-0.8789}{1-0.9535q^{-1}} \\ q^{-8}\dfrac{0.5786}{1-0.9123q^{-1}} & 3.0*q^{-4}\dfrac{-1.302}{1-0.65q^{-1}} \end{bmatrix} \quad (20)$$

and random walk disturbance model; Case 5: Diagnosis of CV2 model removing the first MV in Case 4; Case 6: Diagnosis of CV2 model removing the second MV in case 4.

From Table 1 and Fig.3, we have these discussions:

1) Both MQI_1 and MQI_2 approach to 1 in the case of accurate control model and disturbance model (Case 1), which demonstrates that MQI can precisely express the model performance.

2) Once there is mismatch either in control model or in disturbance model (case 2, case 3 and case 4), the MQIs drop to some extent and indicate corresponding mismatch.

3) There is severe mismatch in the 2th sub-model of CV2 while other three sub-models are accurate in case 4. So the MQI_2 related to the deficient sub-model drops greatly to 0.119. This more less than 1 MQI value implies the significant mismatch exists in the second CV model. This is the assessment result of our first diagnosis stage.

4) Case 5 and Case 6 belong to the second stage. To further diagnose which sub-model of CV2 is poor, corresponding two MVs are removed in Case 5 and Case 6, respectively. Compared MQI_2 in Case 5 with that in Case 4, it decreases greatly and corresponding $\kappa_{2(1)}$ is 0.559. This result implies that the sub-model corresponding to the first removing MV is not bad, which is well consistent with the experiment condition. Differently, MQI_2 in Case 6 increases remarkably compared to Case 4 and corresponding $\kappa_{2(2)}$ is 3.137, which indicates the removing second sub-model is a deficient one and it is also reflects the truth.

Table 1 Assessment results of Wood-Berry process

	Case1	Case2	Case3	Case4	Case5	Case6
MQI_1	0.99	0.784	0.768	0.792	0.798	0.793
MQI_2	1.00	0.664	0.609	0.119	0.066	0.373
$\kappa_{2(1)}$	-	-	-	-	0.559	-
$\kappa_{2(2)}$	-	-	-	-	-	3.137

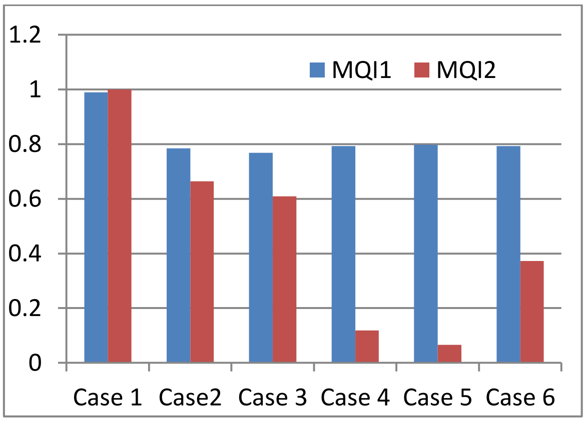

Fig.3. Diagram of assessment results of Wood-Berry process

5. CONCLUSIONS

Timely monitoring of model performance without interfering with the normal operation of MPC is of great importance to the maintenance of MPC. The proposed model performance drill-down diagnosis algorithm via MQI provides an effective and practical method to the monitoring and diagnosis of the control model. Moreover, the index is easily obtained by using routine closed-loop data and the MPC control model. From the application in the Wood-Berry distillation column, all poor models and deficient sub-models can be correctly detected by the diagnosis algorithm. The diagnosis results can provide model performance information. More importantly, it can give suggestions on which models need to be maintained and thus saving work and cost to the maintenance of all models.

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Appendix. PROOF OF THEOREM 1

It can be seen from Fig.2 that the closed-loop output $\boldsymbol{y}(k)$ can be express as the sum of response caused by input $\boldsymbol{u}(k)$ and disturbance $\boldsymbol{e}^o(k)$, that is

$$\boldsymbol{y}(k) = \boldsymbol{G}^o \boldsymbol{u}(k) + \boldsymbol{H}^o \boldsymbol{e}^o(k) \ . \tag{A-1}$$

From Ljung(1999), it is straightforward that one-step ahead prediction of the output is

$$\begin{aligned} \hat{\boldsymbol{y}}(k|k-1) &= (\mathbf{I} - \boldsymbol{H}_o^{-1})\boldsymbol{y}(k) + \boldsymbol{H}_o^{-1}\boldsymbol{G}_o \boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \hat{\boldsymbol{y}}(k|k-1) + \boldsymbol{e}^o(k) \end{aligned} \tag{A-2}$$

where $\boldsymbol{H}^o$ and $\boldsymbol{G}^o$ are strictly causal. Denoting

$$\begin{aligned} \mathbf{I} - \boldsymbol{H}_o^{-1} &= \sum_{i=1}^{\infty} \boldsymbol{H}_i q^{-i}, \\ \boldsymbol{H}_o^{-1}\boldsymbol{G}_o &= \sum_{i=1}^{\infty} \boldsymbol{G}_i q^{-i} \end{aligned} ,$$

where $\boldsymbol{H}_i$ and $\boldsymbol{G}_i$ are the coefficients. Thus (A-2) becomes

$$\begin{aligned} \boldsymbol{y}(k) &= \sum_{i=1}^{\infty} \boldsymbol{H}_i \boldsymbol{y}(k-i) + \sum_{i=1}^{\infty} \boldsymbol{G}_i \boldsymbol{u}(k-i) + \boldsymbol{e}^o(k) \\ &\approx \sum_{i=1}^{M} \boldsymbol{H}_i \boldsymbol{y}(k-i) + \sum_{i=1}^{N} \boldsymbol{G}_i \boldsymbol{u}(k-i) + \boldsymbol{e}^o(k) \end{aligned} \tag{A-3}$$

for sufficiently large M and N. It is clear from (A-3) that $\boldsymbol{e}^o(k)$ can be obtained from closed-loop data $\boldsymbol{y}(k)$ and $\boldsymbol{u}(k)$. Applying vectors expression in (3), (A-3) becomes

$$\boldsymbol{y}_p(k) = \bar{\boldsymbol{L}}_p \bar{\boldsymbol{Z}}_p(k) + \boldsymbol{e}_p^o(k) \tag{A-4}$$

where $\bar{\boldsymbol{L}}_p = [\boldsymbol{H}_1 \ \dots \ \boldsymbol{H}_M \ \boldsymbol{G}_1 \ \dots \ \boldsymbol{G}_N]$.

Post-multiplying (A-4) by $\frac{1}{p}\boldsymbol{\Pi}^{\perp}_{\bar{\boldsymbol{Z}}_p(k)}$ and using $\bar{\boldsymbol{Z}}_p(k)\boldsymbol{\Pi}^{\perp}_{\bar{\boldsymbol{Z}}_p(k)} = 0$ leads to

$$\begin{aligned} &\tfrac{1}{p}\boldsymbol{y}_p(k)\boldsymbol{\Pi}^{\perp}_{\bar{\boldsymbol{Z}}_p(k)} \\ &= \tfrac{1}{p}\bar{\boldsymbol{L}}_p\bar{\boldsymbol{Z}}_p(k)\boldsymbol{\Pi}^{\perp}_{\bar{\boldsymbol{Z}}_p(k)} + \tfrac{1}{p}\boldsymbol{e}_p^o(k)\boldsymbol{\Pi}^{\perp}_{\bar{\boldsymbol{Z}}_p(k)} \\ &= \tfrac{1}{p}\boldsymbol{e}_p^o(k) - \tfrac{1}{p}\boldsymbol{e}_p^o(k)\bar{\boldsymbol{Z}}_p(k)^T[\bar{\boldsymbol{Z}}_p(k)\bar{\boldsymbol{Z}}_p(k)^T]^{-1}\bar{\boldsymbol{Z}}_p(k) \end{aligned} \tag{A-5}$$

where $\boldsymbol{e}_p^o(k)$ is uncorrelated to the past input and output data, *i.e.*,

$$\begin{aligned} &E[\boldsymbol{e}_p^o(k)\boldsymbol{y}_p(k-i)^T] = 0 \\ &E[\boldsymbol{e}_p^o(k)\boldsymbol{u}_p(k-i)^T] = 0, \forall i \ge 1 \end{aligned} \tag{A-6}$$

Hence, $\frac{1}{p}\boldsymbol{e}_p^o(k)\bar{\boldsymbol{Z}}_p(k)^T \to 0 \text{ as } p \to \infty$, that is, $\frac{1}{p}\boldsymbol{e}_p^o(k)\bar{\boldsymbol{Z}}_p(k)^T[\bar{\boldsymbol{Z}}_p(k)\bar{\boldsymbol{Z}}_p(k)^T]^{-1}\bar{\boldsymbol{Z}}_p(k) \to 0 \text{ as } p \to \infty$ in (A-5). Thus we have (5).

