

## Energy Demand Response of Process Systems through Production Scheduling and Control

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**Abstract:** Demand response has become a topic of significant research, development, and deployment over the last few years. The energy demand management is a critical task in industrial process systems for the potential benefits to be realized by promoting the interaction and responsiveness of process operation. However, the dynamic behavior, especially transition trajectories, of the underlying process is seldom taken into account during this task. Furthermore, the incorporation of energy constraints related to electricity pricing and availability is one of the key challenges in this process. The purpose of this study is thus to present a novel optimization formulation for energy demand management in dynamic process systems that takes transition behavior and cost explicitly into account, while simultaneously handling time-sensitive electricity prices. This is accomplished by bringing together production scheduling and transition control through a real-time optimization framework. The dynamic formulation is cast as a mixed-integer nonlinear programming problem and demonstrated using a continuous stirred tank reactor example where the energy required is assumed to be roughly proportional to the material flow.

**Keywords:** Demand Response, Energy systems, Production Scheduling, Control, Mixed-integer Programming, Chemical Processes.

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### 1. INTRODUCTION

In recent years, demand-side activities have been in the spot light in all energy policy decisions due to the significant benefits that can be realized - both at the economic and operational levels - through demand-side management. Indeed, the market price of electricity is the dominant incentive and influences heavily the electricity consumption of industrial customers. Typical examples of time-sensitive electricity prices are time-of-use (TOU) rates and real-time prices (RTP). While TOU rates are usually specified in terms of on-peak, mid-peak and off-peak hours, RTP vary every hour and are quoted on a day-ahead or hourly basis [2]. Furthermore, one important component of the current and future power systems is the concept of demand response (DR), which focuses on the operational level. DR is achieved through mechanisms that discourage the energy load when the real-time price is high and vice versa. Although the economic potential of DR for industrial processes has been recognized in a number of recent studies [3-5], it should be noted that since DR requires, by definition, varying production levels, the consideration of transition behaviour between different operating modes is an issue that remains open.

An examination of the existing research on DR for industrial processes shows that the transition behaviour problem has not been fully explored or adequately addressed in the literature. Examples of important contributions in this area include the work of Mitra et al. [2], where an optimal production planning model for continuous power-intensive processes under time-sensitive electricity prices was developed. The focus of that

study was mainly on minimum stay constraints for describing ramp-up transition and rate of change constraints for restricting transitions between operating points. The dynamic profile of the transition behaviour, however, was not taken into account in the optimization formulation. In another study, Mendoza-Serrano and Chmielewski [6] illustrated the potential opportunities of DR for a chemical manufacturing facility, which was assumed to operate at continuously changing production levels. The discrete transitional behaviour between the different operating modes was not considered in the problem formulation or solution.

From the standpoint of DR for industrial processes, the goal lies primarily in determining the optimal production levels at each time instant. The control problem is thus closely related to demand responsiveness given that the major task of controllers is to determine the optimal values of manipulated and controlled variables in order to achieve different production levels. Generally, production scheduling and control problems can be addressed simultaneously or sequentially [7, 8]. Some early attempts to tackle simultaneous scheduling and control problems can be found in the literature [9-11]. However, to the best of our knowledge, energy consumption was not considered in these studies. In addition, one of the most challenging aspects of plant scheduling is undoubtedly the incorporation of energy supply constraints related to electricity pricing and availability.

Motivated by these considerations, the objective of this work is to present a new optimization formulation for energy demand management in process systems that considers

explicitly dynamic transition behaviour and cost, and simultaneously handles time-sensitive electricity prices. The presented study is explorative, focusing on demand response which is realized through a combined production scheduling and control approach which into account the dynamic profile of the transition process as well as time-varying electricity prices. The proposed formulation is illustrated using a conceptual case study involving a continuous stirred-tank reactor (CSTR) process example where the energy required is assumed to be roughly proportional to the material flow and the process has to satisfy an hourly demand for the product.

## 2. FORMULATION OVERVIEW

### 2.1 Problem Definition

The conceptual production system with a set of components considered for demand response in this study, are shown in Figure 1 below.

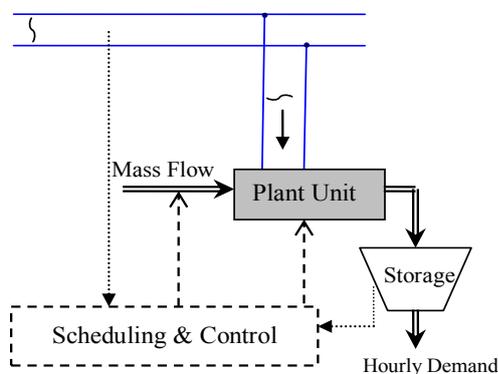


Figure 1. A schematic representation of the problem formulation.

The system has to satisfy an hourly demand of the product, and a particular storage unit is installed to provide flexibility. The electricity prices vary on an hourly basis and the demand response decisions do not influence the electricity prices. A lower bound for the product hourly demand - expressed as a constant rate - is specified. Steady-state operating modes for different production levels are also specified a priori, as well as the price of the inventory and raw material costs. The problem then consists of the simultaneous determination of the operating mode for the plant (i.e., production level) and the control profile for the production level changes. The main objective is to minimize the total production cost, which includes the transition cost (i.e., raw material waste and electricity waste during the transitions), inventory cost, and electricity cost.

### 2.2 A Combined Scheduling and Control Approach

The dynamic process model is incorporated into the constraints of the scheduling problem resulting in a mixed integer dynamic optimization (MIDO) problem. Solution methods of general MIDO problems are presented in a number of papers [12-15]. A general decomposition-based framework was built by Allgor and Barton [13], and Flores-Tlacuahuac and Biegler [14] proposed a methodology to transfer the MIDO problem into an MINLP through the discretization of the dynamic model. Terrazas-Moreno *et al.* [15] extended the work by Flores-Tlacuahuac and Grossmann [9] and proposed a Lagrangean decomposition strategy to simplify the

scheduling and control problem. However, in the current work, to handle the dynamic optimization problem, the steady-state and transition states are treated differently.

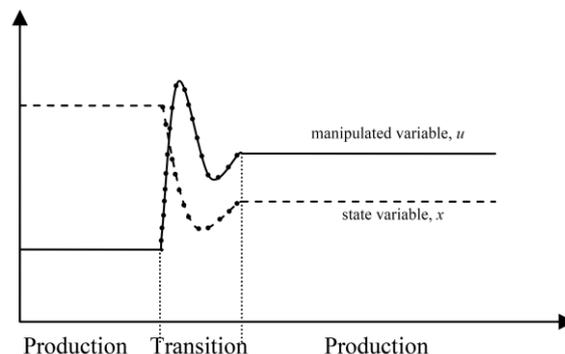


Figure 2. Dynamic system behaviour as production transitions between different operating modes.

As shown in Figure 2, the system states and the manipulated variables remain constant during the production period, while the manipulated variables change within the transition period and so do the system states. Given that the steady-state operating modes corresponding to different production levels are predefined, the steady-state values of the manipulated variables are thus known constants. However, in order to model the transition-state behavior, the transition period is discretized on the basis of the system sampling interval as shown in Figure 2. With a detailed plant model, the approximated transition behavior between different operating modes can then be calculated, and the control actions needed to drive the transitions can be computed off-line. Moreover, in the absence of a mathematical model, the transition profile can still be obtained from historical plant data. Generally, the transition profile between the same pair-wise operating modes should follow a similar trajectory. From this point of view, the key advantages of this method lie in the fact that the incorporated differential equations are transformed into to a set of data and that the transition times between different production levels are also known parameters. These aspects reduce the computational complexity of the combined scheduling and control problem.

## 3. MODEL FORMULATION

### 3.1 Scheduling Constraints

The plant unit has a set of discrete operating modes corresponding to different production levels,  $m \in M$ . For every time period (hour  $h$ ), we introduce two binary variables,  $y_m^h$  and  $z_{m,m'}^h$ , and one continuous variable  $s^h$ , which define the mode assignment, transition status, and storage level, respectively.

**Mode assignment constraints:** The constraints for the mode assignment are given by the following:

$$\sum_{m \in M} y_m^h = 1 \quad \forall h \quad (1)$$

$$\tilde{y}_m^h = y_m^{h-1} \quad \forall m, h \neq 1 \quad (2)$$

$$\tilde{y}_m^1 = y_m^0 \quad \forall m \quad (3)$$

Eq. (1) indicates that, within each time period  $h$ , only one mode can be active. Eq. (2) defines a backward binary variable  $\tilde{y}_m^h$ , which takes the value assigned to the same binary variable but one time step backward. Eq. (3) assigns the backward binary variable with the value of the same variable at the initial time step,  $y_m^0$ . The backward variable will be used later to determine the production transitions.

**Logic constraints for transition:** The logic relationship that couples binary transitional variable  $z_{m,m'}^h$  with the binary variable  $y_m^h$  can be modelled with the following set of equality constraints:

$$\sum_{m' \in M} z_{m',m}^h = y_m^h \quad \forall h \quad (4)$$

$$\sum_{m' \in M} z_{m,m'}^h = \tilde{y}_m^h \quad \forall h \quad (5)$$

where  $z_{m,m'}^h$  is true if and only if a transition from mode  $m$  to mode  $m'$  occurs from time step  $h-1$  to  $h$ .

### 3.3 Storage Balance

The storage unit is supplied by the product from the plant and accommodates the hourly demand,  $d^h$ . Again, similar to the backward binary variable defined in Eq. (2), a backward variable ( $\tilde{s}^h$ ) is defined for the storage level variable ( $s^h$ ) as follows:

$$\tilde{s}^h = s^{h-1} \quad \forall h \neq 1 \quad (6)$$

$$\tilde{s}^1 = s^0 \quad (7)$$

The initial storage level ( $s^0$ ) is assigned to  $\tilde{s}^h$  at time index  $h=1$ . The storage level in the storage unit at the end of each time step is determined by the following equation:

$$s^h = \tilde{s}^h + \sum_{m \in M} (y_m^h \cdot p_m) \cdot \left[ 1 - \sum_{m',m \in M} (z_{m',m}^h \cdot t_{m',m}) \right] - d^h \quad (8)$$

where  $p_m$  is the production level associated with operating mode  $m$ , and  $t_{m',m}$  is the transition time from mode  $m'$  to  $m$ . For a self-transition,  $t_{m,m} = 0, \forall m \in M$ . Note that a certain amount of off-spec product is generated and sent to waste when the process changes production levels because of the existence of transitions.

The storage unit has to satisfy an hourly demand for the product in each time step and has a maximum storage level, which is specified by:

$$0 \leq s^h \leq s_{\max} \quad \forall h \quad (9)$$

Additionally, to avoid depleting the product in the storage unit, the storage level at the last time step ( $s^H$ ) should not be less than the initial value of the storage level  $s^0$ , which is represented by the following constraint:

$$s^H \geq s^0 \quad (10)$$

### 3.3 Electricity Cost

The electricity consumption can be divided into two parts: one part associated with the production periods, and the other with the transition periods. Here, two auxiliary variables  $\Delta E^h$  and  $\Delta e_i^h$  are defined to represent the energy demand within hour  $h$  during the production and transition times, respectively.

**Production period:** Given that the manipulated variables remain constant during the production period, the electricity required for the current operating mode also remains constant, and is determined by:

$$\Delta E^h = \alpha \cdot \sum_{m \in M} \left[ y_m^h \cdot (\bar{u}_m^1 + \bar{u}_m^2 + \dots + \bar{u}_m^n) \right] \cdot \left[ 1 - \sum_{m',m \in M} (z_{m',m}^h \cdot t_{m',m}) \right] \quad (11)$$

where  $\alpha$  is ratio of energy required to material flow rates,  $\bar{u}_m^1, \bar{u}_m^2, \dots, \bar{u}_m^n$  represent the steady-state values of the manipulated variables, and  $n$  is the number of manipulated variables.

**Transition period:** As a measure of the electricity consumption during production transition, the transition profile obtained by the dynamic model (or historical plant data) is then taken into account. Suppose there are total  $N_{m',m}$  sampling points during the transition period from mode  $m'$  to mode  $m$ , the electricity cost  $\Delta e_i^h$  (where  $i=1,2,\dots,N_{m',m}$ ) in each sampling interval ( $\Delta t$ ) is thus given by:

$$\Delta e_i^h = \alpha \cdot \sum_{m',m \in M} \left[ z_{m',m}^h \cdot (u_{m',m}^{i,1} + u_{m',m}^{i,2} + \dots + u_{m',m}^{i,n}) \right] \cdot \Delta t \quad (12)$$

where  $u_{m',m}^{i,1}, u_{m',m}^{i,2}, \dots, u_{m',m}^{i,n}$  are sampled values of the manipulated variables at each sampling step  $i$ .

### 3.4 Additional transition cost

Within the transition period, a certain amount of off-spec product is produced and sent to waste. Therefore, the cost of wasted raw material ( $C_{\text{raw}}^h$ ) should be incorporated as an additional transition cost when the process system triggers a mode switching, which is given as follows:

$$C_{\text{raw}}^h = \mathcal{G}_r \cdot \sum_{m',m \in M} (z_{m',m}^h \cdot t_{m',m} \cdot y_m^h \cdot p_m) \quad (13)$$

where  $\mathcal{G}_r$  is the unit price of raw material.

### 3.5 Objective Function

To achieve economically optimal operation of the process, the objective is to minimize the total cost, which can be calculated as follows:

$$J = \min \{ \Phi_1 + \Phi_2 + \Phi_3 \} \quad (14)$$

with

$$\Phi_1 = \sum_{h=1}^H (\Delta E^h \cdot \mathcal{G}_{el}^h) \quad (15)$$

$$\Phi_2 = \sum_{h=1}^H \sum_{i=1}^{N_{m/m}} (\Delta e_i^h \cdot \mathcal{G}_{el}^h) + \sum_{h=1}^H C_{raw}^h \quad (16)$$

$$\Phi_3 = \sum_{h=1}^H s^h \cdot \mathcal{G}_s \quad (17)$$

where  $\Phi_1$  is the cost of purchased electricity during the production period,  $\Phi_2$  is the transition cost that consists of wasted energy and raw material, and  $\Phi_3$  is the inventory cost. Note that the electricity price  $\mathcal{G}_{el}^h$  is time-varying on an hourly basis and the sold electricity price is proportional to the current electricity price with ratio  $\eta$ , and  $\mathcal{G}_s$  is the unit inventory cost.

### 3.6. Solution Algorithm

The problem of demand energy management of process systems is cast as a MINLP problem, which can be solved efficiently by the solver BONMIN available in the GAMS software. Although the global optimum solution may be difficult to obtain for MINLP, useful solutions can still be obtained as will be seen in the next section.

It should be noted that, although the transition times and transition profiles are not considered as decision variables, the transition times and profiles obtained from the differential equations or historical data will be close to the actual values. The only reason to prefer the strategy as presented in this work, compared to the case in which the differential equations are discretized and incorporated into the optimization model, is because the proposed optimization formulation is simpler to deal with. With the pre-calculated transition profile, the non-convexity of the underlying optimization formulation is reduced, and the resulting optimization problem is easier to solve.

## 4. ILLUSTRATIVE CASE STUDY

To motivate and illustrate the proposed scheduling and control framework for energy demand response, we consider a single CSTR unit as shown in Figure 3, which conceptually represents a simplified version of the plant unit in Figure 1. It should be noted that even though the CSTR is not a power-intensive process, it is still possible to illustrate the proposed formulation if we only consider energy demand management, where the required energy is assumed to be roughly proportional to the material flow.

Considering a single product that is to be produced by the reaction  $A \rightarrow B$ , the following model equations can be obtained from standard material and energy balances:

$$\frac{dC}{dt} = \frac{F}{V} (C_i - C) - k_0 \cdot e^{-E/RT} \cdot C \quad (18)$$

$$\frac{dT}{dt} = \frac{-\Delta H \cdot k_0 \cdot e^{-E/RT} \cdot C}{\rho C_p} + \frac{F}{V} (T_i - T) + \frac{UA_c \cdot (T_c - T)}{\rho C_p \cdot V} \quad (19)$$

$$\frac{dT_c}{dt} = \frac{F_c}{V_c} (T_{ci} - T_c) + \frac{UA_c}{\rho_c C_{pc} \cdot V_c} (T - T_c) \quad (20)$$

The inlet concentration is  $C_i = 0.5 \text{ mol/ft}^3$  ( $17.657 \text{ mol/m}^3$ ) and the concentration of the on-spec product is  $0.0591 \text{ mol/ft}^3$  ( $2.087 \text{ mol/m}^3$ ). The details of the other model parameters can be found in Feital *et al.* [16]. The CSTR is equipped with proportional-integral (PI) controllers with fixed parameters. The scheduling and control strategy provides optimal set-points for these controllers and manipulates the raw material inlet flow rate  $F$ .

Furthermore, the feasible operating modes for the CSTR are generated and tabulated in Table I. All operating modes have the same steady-state product concentration to ensure that mode switching does not sacrifice product quality. The energy required for the different modes is proportional to the sum of material flow rates (i.e.,  $F$  and  $F_c$ ). Operating mode 3 is the nominal operating mode used to meet the hourly demand,  $35 \text{ ft}^3/\text{h}$  (or  $0.991 \text{ m}^3/\text{h}$ ) before the demand response strategy is carried out.

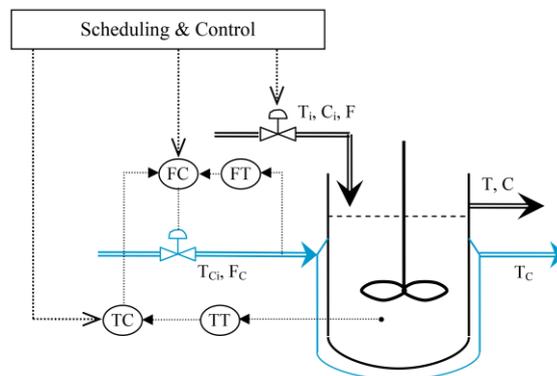


Figure 3. Conceptual case study involving a CSTR process equipped with feedback control.

Plant schedules are established for one day with electricity prices varying at three different levels: 70 \$/MWh, 94.8 \$/MWh, and 163 \$/MWh, as can be seen from Figure 4(a). Figures 4(b) and 4(c) present a comparison of the solutions obtained for two cases: (i) when transition costs are ignored in the optimization formulation (Figure 4(b)) and (ii) when the proposed optimization model with transitions explicitly considered is implemented (Figure 4(c)). The initial state of operating mode is set to the nominal one and the storage level is initialized at  $30 \text{ ft}^3$ , as shown by the dashed line in Figure 4.

Table I. Feasible operating modes for the CSTR process

Operating mode	Production rates $F$ ( $\text{ft}^3/\text{h}$ )	Jacket flow $F_c$ ( $\text{ft}^3/\text{h}$ )
6	50 (1.416 $\text{m}^3/\text{h}$ )	58.227 (1.648 $\text{m}^3/\text{h}$ )
5	45 (1.274 $\text{m}^3/\text{h}$ )	54.040 (1.530 $\text{m}^3/\text{h}$ )
4	40 (1.133 $\text{m}^3/\text{h}$ )	49.747 (1.409 $\text{m}^3/\text{h}$ )
<b>3</b>	<b>35 (0.991 <math>\text{m}^3/\text{h}</math>)</b>	<b>45.327 (1.284 <math>\text{m}^3/\text{h}</math>)</b>
2	30 (0.850 $\text{m}^3/\text{h}$ )	40.752 (1.154 $\text{m}^3/\text{h}$ )
1	25 (0.708 $\text{m}^3/\text{h}$ )	35.985 (1.019 $\text{m}^3/\text{h}$ )

As expected, the process switches to a lower production level when the electricity price rises and vice versa. Compared to the result with case (ii) given in Figure 4(c), less frequent mode switching occurs when no transition costs are considered (see Figure 4(b)). Given that instantaneous mode switching rarely exists in practice, to compensate for the transition waste, the process should increase the time spent at the higher production level and reduce the hours at the lower production level. Moreover, it should be noticed that the process tends to trigger mode switching when the electricity price is relatively low in order to reduce the electricity cost during the transition period, as shown in Figure 4(c). The corresponding storage profile for case (ii) is depicted in Figure 4(d), together with lower and upper bounds of storage capacity. We can clearly observe how the product demand is met from the storage unit when the process operates at the low production level.

To assess the economic impact of the developed optimization model, comparisons between the total costs for three different cases over the considered time horizon ( $H = 24$ ) are provided in Figure 5. Cases (i) and (ii) correspond to the demand responsive cases without and with transition costs included, respectively, while case (iii) corresponds to the case when no demand response strategy is enacted and the process operates only at the nominal operating mode for all times.

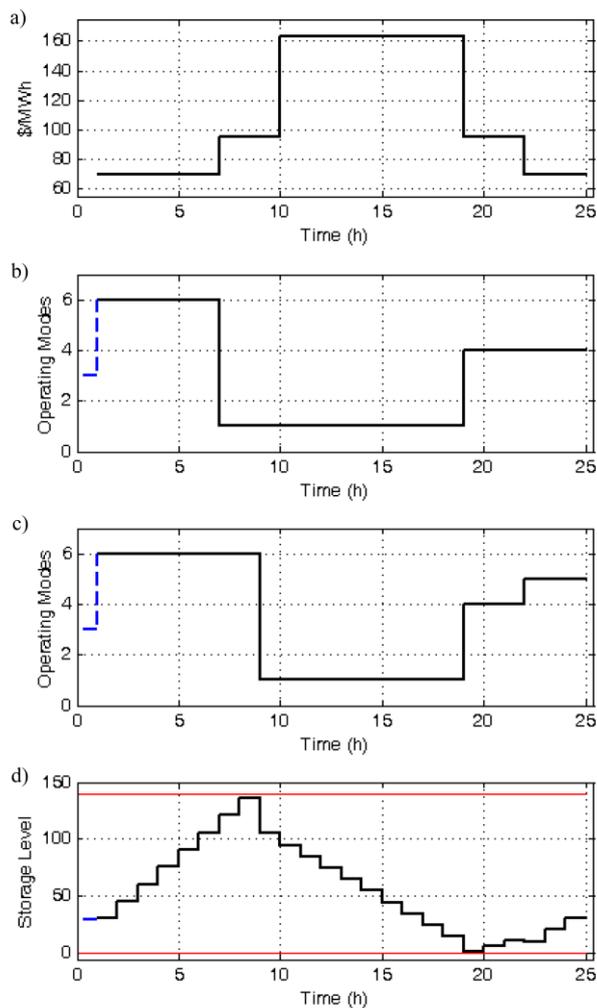


Figure 4. Production schedule: (a) time-sensitive electricity prices, (b) demand response schedule for case (i) when no transition costs are accounted for, and (c) demand response schedule for case (ii) when transition costs are included, (d) storage profile for case (ii).

Not surprisingly, case (i) achieves the lowest cost due to the absence of transition costs. With the proposed energy demand management, however, the total cost is reduced compared with the case when no demand management is considered (case (iii)). It should be noted here that while the simulation results are dependent on the choice of model parameters, the parameter values chosen are considered typical for the system under study and, therefore, the results presented above are representative.

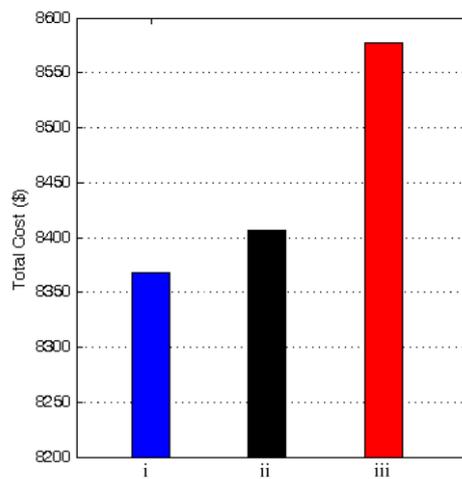


Figure 5. Total cost for (i) the conventional case ignoring the transition cost, (ii) the proposed demand-responsive strategy with transitions considered, and (iii) the case when only the nominal operating mode is active.

## 5. CONCLUDING REMARKS

In this work, we presented an optimization-based approach for the management of energy demand in dynamic process systems. The approach considered both transition behavior as well as time-sensitive electricity prices. The formulated optimization model was shown to be capable of realizing demand responsiveness in terms of operating mode switches and transitional behavior. Based on the assumption that the transition behavior between the same pair-wise operating modes should follow similar trajectories, the proposed scheduling and control formulation was then cast into a MINLP framework. It should be noted that in the presence of highly nonlinear behavior, the non-convexity in the optimization formulation makes convergence towards the globally optimal solution difficult to achieve, thus leading to possibly suboptimal solutions.

Additionally, the demand response problem was addressed in a closed-loop manner (i.e., in the presence of controllers), and this is a key feature of the proposed formulation given that the optimal transition is dependent on the type of controller and

the way in which it is tuned. While the current work has focused on a single processing unit with a fixed controller structure and parameters (which are computed off-line), future work will focus on the integration of the scheduling and control problems whereby the selection of the control structure and parameters are decision variables to be considered within the optimization formulation in order to optimize the transition behavior as well. Other possible extensions include the integration of renewable energy resources to supply part of the required energy demand, and how to handle the uncertainties introduced by the intermittent generation in this case.

## 5. ACKNOWLEDGMENTS

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