Optimal Scheduling of the Maintenance and Improvement for Water Main System Using Markov Decision Process

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Abstract: Optimal scheduling of maintenance and improvement on the water main system is a substantial problem since it can greatly reduces the potential threat of failure and the expenses. The Markov decision process (MDP) based methodology to find the optimal schedule which minimizes the cost is proposed. Since it needs the information about the current state of pipe, cost, and deterioration model, the definition and the usage of auxiliary information are also presented. The objective function and detailed algorithm of dynamic programming (DP) are modified due to the difficulty of implementing the conventional DP approaches. The result is compared to two simple policies via Monte Carlo simulation. Validity of the solution and improvement in computational time are proved. Proposed decision framework provides an easy way to obtain the comprehensive and local-specialized policy.

Keywords: Markov decision processes, Dynamic Programming, Monte Carlo simulation, Periodic replacement, Weibull distribution

1. INTRODUCTION

Various kinds of policies for appropriate combination of maintenance and improvement on water main system are implemented, and heuristic policy or myopic policy is the most widely used policy among them. The former is to improve the pipe every 15-30 years, and the latter is to improve the pipe when failure occurs. It is trivial that both policies are very different from an effective policy. The objective of this research is to suggest a systematic decision-making framework for deriving an optimal policy that minimizes the total cost and reflects the overall circumstance of pipe.

A mathematical model explaining the deterioration of water pipe is necessary since the direct investigation of a water main system is time-consuming and costly. A statistical approach has been made to explain the failure risk of the pipe in Andreou et al. (1987a) and Andreou et al. (1987b), which is called survival data analysis. A proportional hazards model is used in the early stage of deterioration, while a Poisson model is used in the late stages. Le Gat and Eisenbeis (2000) applied survival analysis for the number of real data.

Markov model is one of the most widely used models to describe the deterioration model. It classifies the pipe into finite state with some criteria such as the number of failures (Li and Haimes, 1992) or current performance of pipe (Cesare et al., 1992). Once the state classification criteria has been declared, the state transition probability

can be obtained in two ways. One is to combine survival analysis with Markov model (Li and Haimes, 1992; Kleiner, 2001). It is an extended version of survival analysis. While survival analysis only considers two states (failure or not), Markov model makes a relationship between multiple states. The other method is to use a statistical method directly without using survival analysis (Madanat et al., 1995; Baik et al., 2006; Guignier and Madanat, 1999; Madanat and Ibrahim, 1995; Cesare et al., 1992; Wirahadikusumah et al., 2001; Micevski et al., 2002). Transition probabilities are calculated by regression methods.

The deterioration model explained above is then combined with the cost model to formulate the objective function. Objective function is expressed as minimizing the total expectation cost within the decision horizon. Several researchers added extra objective criterion such as pipe resilience or production rate by workers to make a multiobjective optimization problem (Dandy and Engelhardt, 2006; Nafi et al., 2008; Fwa et al., 2000). Heuristic optimization algorithms such as genetic algorithm (GA) are used to obtain the Pareto optimal set of these problems.

Various kinds of formulation have been suggested and a corresponding optimal poilcy has been withdrawn. Loganathan et al. (2002) used threshold break rate which concerns the failure history of a single pipe. Kleiner et al. (1998a) and Kleiner et al. (1998b) added the analysis of hydraulic pressure in order to expand the single pipe problem into the pipe network system. Both researches use the exponential failure rate which is the most simple form of deterioration models.

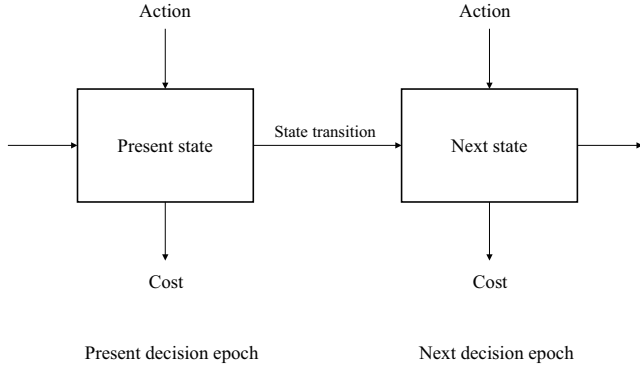[1] Assoc. Prof., Corresponding author, E-mail: jongmin@snu.ac.kr

Fig. 1. Illustration of sequential decision problem (Puterman, 2009)

More complex optimization strategies have also been studied. Linear programming based optimization strategies have been suggested in Li and Haimes (1992) and Guignier and Madanat (1999). A basic concept of implementing Markov decision process to the scheduling problem for infrastructure is suggested by Madanat and Ben-Akiva (1994). Monte Carlo simulation is also used to solve the optimization problem by Boston and Bettinger (1999) and Buxey (1979).

Those formulations are intuitive and convenient to get a solution, but the computational burden grows exponentially with the problem size. To overcome the problem, this work proposes a novel dynamic programming formulation to solve the Markov decision process. Monte Carlo simulations are used to show the validity of the result and reduction in computational time.

2. MARKOV DECISION PROCESS (MDP) FOR OPTIMAL SCHEDULING

2.1 Fundamentals of Markov decision process

Consider a single water pipeline in the water main system. Decision maker should decide whether to maintain the pipe or not at each decision epoch based on the information of system. Markov decision process is an appropriate decision-making method for a system whose system follows a Markov property. The fundamentals are treated in Puterman (2009) with considerable detail.

At each time step, the state of system is observed or calculated and the decision maker may choose any action which accompanies cost. At the next time step, the process moves into a new state and a corresponding cost occurs. The probability of the process moves into its new state is influenced by the chosen action. Given the state and action, they are conditionally independent of all previous states and actions; in other words, the state transitions of a Markov decision process possess the Markov property. The objective of Markov decision process is to find the sequence of action, which is called optimal policy, to minimize the total cost. Fig. 1 is the brief illustration of Markov decision process. The following data are given:

$T = \{1, \ldots, N\}$ is a set of decision epoch.
$S = \{1, \ldots, |S|\}$ is a finite set of states; state 1 denotes good, brand-new pipe and state n denotes failure.
$A = \{1, \ldots, |A|\}$ is a finite set of actions; 1: maintenance, 2: rehabilitation, 3: replacement.

$C_t(s_t, a_t)$ is a cost function at decision epoch t.
$p_t(s_{t+1}|s_t, a_t)$ is a transition probability at time t that gives the probability; when the state is in s_t and action a_t is taken, then the next state will be in s_{t+1} with probability of $p_t(s_{t+1}|s_t, a_t)$.
$X_t = [x_1, x_2, \ldots, x_{|S|}]$ is a state distribution at decision epoch t.
γ is a discount factor; future costs are discounted when converted into present value.

2.2 Deterioration and action matrix

It is convenient to express the transition probability as a matrix form. A Markov transition probability matrix $P_t(a_t)$ is a matrix whose element of ith row and jth column denotes the transition probability $p_t(s_{t+1} = j|s_t = i, a_t)$. It is assumed that the process can move from state i to state j only if $j \geqslant i$. And the pipeline can deteriorate only one state at a time (Kathuls and McKim, 1999). When $|S| = 5$, transition probability matrix can be expressed as follows.

$$P_t(a_t) =$$
$$\begin{bmatrix} p_t(1|1, a_t) & p_t(2|1, a_t) & 0 & 0 & 0 \\ 0 & p_t(2|2, a_t) & p_t(3|2, a_t) & 0 & 0 \\ 0 & 0 & p_t(3|3, a_t) & p_t(4|3, a_t) & 0 \\ 0 & 0 & 0 & p_t(4|4, a_t) & p_t(5|4, a_t) \\ 0 & 0 & 0 & 0 & p_t(5|5, a_t) \end{bmatrix}$$
$$(1)$$

Decision maker can employ three kinds of action, which means $|A| = 3$, at each time step; maintenance, rehabilitation, and replacement. Transition probability matrix $p_t(s_{t+1} = j|s_t = i, a_t)$ can be decomposed to a deterioration matrix D_t and an action matrix R_{a_t}. Action matrices have the same dimension with the transition matrix. Replacement and rehabilitation transfer the inferior states to state 1. When the improvement performance is r, corresponding action matrix is given by

$$M(r) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ r & 1-r & 0 & 0 & 0 \\ r & 0 & 1-r & 0 & 0 \\ r & 0 & 0 & 1-r & 0 \\ r & 0 & 0 & 0 & 1-r \end{bmatrix} \quad (2)$$

Maintenance does not alter the state distribution and its matrix is $R_1 = M(0)$. Assume that replacement recover the pipe perfectly and rehabilitation recover the pipe 70 percent, thus $R_2 = M(0.7)$, $R_3 = M(1)$.

State distribution at the next time step can be calculated by the left multiplication of action matrix and transition matrix to state distribution at current time step. Generally, state distribution at time τ, X_τ can be obtained by

$$X_\tau = X_1 \prod_{t=1}^{\tau-1} P_t(a_t) = X_1 \prod_{t=1}^{\tau-1} R_{a_t} D_t \quad (3)$$

3. DETERIORATION MODEL OF WATER PIPE

3.1 Deterioration matrix evaluation using waiting time

The pipe would deteriorate naturally if no improvement had been employed. Deterioration matrix D_t can be evaluated by the deterioration model of water pipe. The basic

idea of the deterioration model is to estimate a survival function or a hazard function for a water pipe. Estimating those functions is called survival analysis which has been widely studied. Models developed by Weibull are the most prominent, but they only considers the two state system ($|S| = 2$); failure or not. Kleiner (2001) generalizes the deterioration model to n state variables, and provides the methods to evaluate the deterioration matrix.

Let $\{T_1, T_2, \ldots, T_{|S|-1}\}$ be random variables representing the waiting time in states $\{1, 2, \ldots, |S| - 1\}$. For example, it takes T_i for the process to go from state i to $i + 1$. When we define the random variable $T_{i \to k}$ as the sum of waiting times in states $\{i, i+1, \ldots, k-1\}$, we can obtain the cumulative waiting time between states i and k. In general, summation of two or more random variables can be calculated analytically by convolution integral. Probability density function (PDF), survival function (SF) of $T_{i \to k}$ are denoted as $f_{i \to k}(T_{i \to k})$, $S_{i \to k}(T_{i \to k})$. Then the transition probability of state i to state $i + 1$ is the generalization of hazard function which can be expressed as follows.

$$
\begin{aligned}
\Pr[s_{t+1} = i + 1 | s_t = i] &= p_t(i + 1 | i, 1) \\
&= \frac{f_{1 \to i}(t)}{S_{1 \to i}(t) - S_{1 \to i-1}(t)} \\
&\text{for all} \quad i = \{1, 2, \ldots, |S| - 1\}
\end{aligned} \tag{4}
$$

Once the PDF and SF of waiting time $T_i(t)$ are established, every element of the deterioration matrix D_t can be calculated.

3.2 Weibull distribution of waiting time

The waiting time T_i of state i follows the Weibull probability distribution. Weibull model is the special case of the proportional hazards model whose physical interpretation is explained by Andreou et al. (1987b). Weibull model has two parameters and takes the following.

$$
\begin{aligned}
\text{SF}: S_i(t) &= \Pr[T_i \geq t] = \exp[-(\lambda_i t)^{\beta_i}] \\
\text{PDF}: f_i(t) &= \lambda_i \beta_i (\lambda_i t)^{\beta_i - 1} \exp[-(\lambda_i t)^{\beta_i}]
\end{aligned} \tag{5}
$$

Parameters λ_i and β_i can be calculated by regression using the survival history of water main system of target region (e.g. $x\%$ probability of being in state i more than t years). The criteria of classifying the state of water main pipe has been suggested by many researchers (Cesare et al., 1992; Madanat and Ibrahim, 1995) and municipal government. The data on which the time a pipe takes to shift from one state to other without any action employed would be recorded. Decision maker uses those historical data to find the parameters of Weibull model and evaluate the deterioration matrix $D_t, t \in T$.

4. DYNAMIC PROGRAMMING

4.1 Fundamentals of dynamic programming and Bellman equation

Dynamic programming algorithm is implemented to solve the Markov decision process. Dynamic programming is the method for solving the complex optimization problems by breaking down the big system into smaller subproblems. We need to get solutions of the subproblems, then take the solutions into account to reach an overall solution. This bottom-up approach method reduces the repeated calculation and complexity of a large-scale optimization problem.

The goal is to find an optimal policy (action) that minimizes the total cost over the whole period of the decision process. The optimal policy $\pi^* = (a_1, a_2, \ldots, a_N)$ is the sequence that minimizes the total cost and can be found by solving the following objective function.

$$
V_{\pi^*} = \min_{\pi} \mathrm{E}[\sum_{t=1}^{N} \gamma^t C_t(s_t, a_t)] \tag{6}
$$

Define the value function at the kth time-step as

$$
V_k(s_k) = \min_{a_k, a_{k+1}, \ldots, a_N} \mathrm{E}[\sum_{t=k}^{N} \gamma^{t-k} C_t(s_t, a_t)]] \tag{7}
$$

Then we can find the optimal policy by working backwards from N, which is called Bellman equation.

$$
V_k(s_k) = \tag{8}
$$
$$
\min_{a_k \in A} (C_k(s_k, a_k) + \gamma \sum_{s' \in S} P(s_{t+1} = s' | s_t, a_t) V_{k+1}(s'))
$$

Let v_t the column vector with ith element be $V_t(s_t = i)$, and $c_t(a_t)$ the column vector with ith element be $C_t(s_t, a_t)$. Then the standard Bellman equation can be expressed by a vector-matrix form.

$$
v_k = \min_{a_k \in A} (c_k(a_k) + \gamma v_{k+1} P_{k+1}(a_{k+1})) \tag{9}
$$

4.2 Modified DP algorithm

According to equation (4), deterioration matrices are time-varying. So proper matrix should be substituted to Bellman equation. Consider when replacement action takes place at decision epoch k. "Replacement" means replacing the deteriorated or aged pipe into the new pipe. Deterioration rate is then initialized at the next decision epoch $k+1$; $D_{k+1} = D_1$. In order to choose the deterioration matrix at decision epoch k, decision maker should know all information about policy employed before $(a_1, a_2, \ldots, a_{k-1})$, or just know about when the pipe was replaced recently. However, it is impossible to know that information since the optimal policy is evaluated from a_N to a_1, in a backward manner.

To avoid the algorithmic difficulty, replacement action should be separated from dynamic programming. Based on the idea that replacement initialize the time index of deterioration matrix, this problem also can be considered as a periodic problem. Let the replacement period τ, then the pipe would be replaced at decision epoch $\{\tau, 2\tau, \ldots, \lfloor \frac{N}{\tau} \rfloor \tau\}$. During the decision epoch within the first period $T_\tau = \{1, 2, \ldots, \tau - 1\}$, define the objective function V_η^τ.

$$
V_\eta^\tau = \min_{\eta} \mathrm{E}[\sum_{t=1}^{|T_\tau|} \gamma^{t-1} C_t(s_t, a_t)] + \gamma^{\tau-1} C_\tau(s_\tau, 3) \tag{10}
$$
$$
\eta = \{a_1, a_2, \ldots, a_{|T_\tau|}\} \quad a_i \in A \setminus \{3\}
$$

Fig. 2. Illustration of periodic subproblem
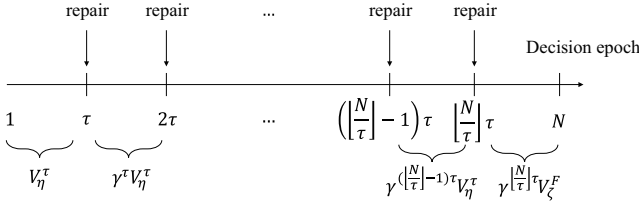

Fig. 3. Modified DP algorithm flow diagram

Fig. 4. Total cost of each subproblem

Optimal policy η^* minimizes the total cost within the first period, which can be found by using backward dynamic programming. The last period of the decision horizon is $T_F = \{\lfloor \frac{N}{\tau} \rfloor \tau + 1, \lfloor \frac{N}{\tau} \rfloor \tau + 2, \ldots, N\}$ and a corresponding objective function is defined as follows.

$$V_\zeta^F = \min_\zeta \mathrm{E}[\sum_{t=1}^{|T_F|} \gamma^{t-1} C_t(s_t, a_t)] \qquad (11)$$

$$\zeta = \{a_1, a_2, \ldots, a_{|T_F|}\}, a_i \in A \setminus \{3\}$$

Due to the periodic property of the problem, optimal policy π^* is equal to $\{\eta^*, 3, \eta^*, 3, \ldots, \eta^*, 3, \zeta^*\}$. The cost incurred during the kth period is equal to $\gamma^{k-1} V_\eta^\tau$ when discount factor γ is taken into account. The total cost of whole decision horizon is evaluated as follows.

$$V_\pi = \mathrm{E}[\gamma^{\lfloor \frac{N}{\tau} \rfloor \tau} V_\zeta^F + \sum_{k=1}^{\lfloor \frac{N}{\tau} \rfloor} \gamma^{k-1} V_\eta^\tau]$$

$$= \mathrm{E}[\gamma^{\lfloor \frac{N}{\tau} \rfloor \tau} V_\zeta^F + \frac{1 - \gamma^{\lfloor \frac{N}{\tau} \rfloor \tau}}{1 - \gamma^\tau} V_\eta^\tau] \qquad (12)$$

The object is to find the replacement period τ^* which minimizes the total cost V_π, where the minimum cost is V_{π^*}. The global optimality of proposed algorithm can be proofed as following.

$$V_{\pi^*} = \min_\pi \mathrm{E}[\sum_{t=1}^N \gamma^t C_t(s_t, a_t)]$$

$$= \min_{\tau, \eta, \zeta} \mathrm{E}[\gamma^{\lfloor \frac{N}{\tau} \rfloor \tau} V_\zeta^F + \frac{1 - \gamma^{\lfloor \frac{N}{\tau} \rfloor \tau}}{1 - \gamma^\tau} V_\eta^\tau]$$

$$= \min_\tau \mathrm{E}[\gamma^{\lfloor \frac{N}{\tau} \rfloor \tau} \min_\zeta V_\zeta^F + \frac{1 - \gamma^{\lfloor \frac{N}{\tau} \rfloor \tau}}{1 - \gamma^\tau} \min_\eta V_\eta^\tau]$$

$$= \min_\tau \mathrm{E}[\gamma^{\lfloor \frac{N}{\tau} \rfloor \tau} V_{\zeta^*}^F + \frac{1 - \gamma^{\lfloor \frac{N}{\tau} \rfloor \tau}}{1 - \gamma^\tau} V_{\eta^*}^\tau] \qquad (13)$$

The scheme explained above is illustrated in Fig. 2. And Fig. 3 is the flow diagram of the proposed algorithm of decision process. Denote the MDP problem with decision horizon N as $G(N)$. Then the optimal policy of $G(\tau)$ is solved by dynamic programming explained in Section 4.1. The total cost of $G(N)$ can be calculated by following the procedure explained in Section 4.2. The optimal replacement period τ^* is found exhaustively; solve the optimization problem by searching for all possible candidates.
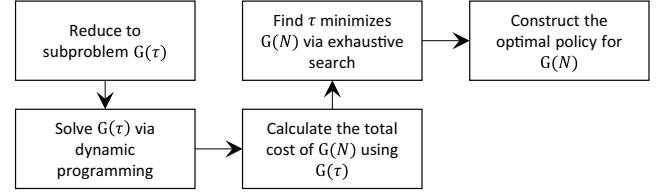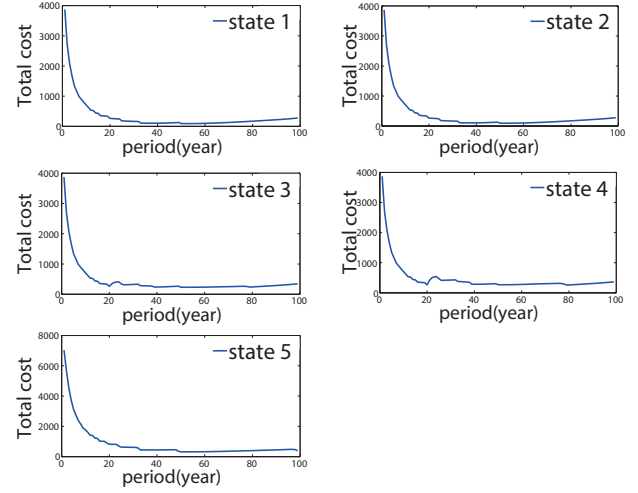
5. SIMULATION RESULTS

5.1 Example data

An illustrative example case is solved to validate the proposed algorithm to find the minimum cost. Given that there are 5 states ($|S| = 5$), and the decision horizon is 100 years ($t = 100$). Parameters of Weibull distribution model are arbitrarily chosen which makes the pipe deteriorate for the decision horizon. In this example, the parameter data of Kleiner (2001) are used. The rehabilitation performance is set to be 70 percent. The discount factor γ is 0.99. The opportunity cost of failure, rehabilitation and replacement are assumed to be $200, 80, 100$ (dimensionless).

5.2 Optimal policy from proposed algorithm

The proposed decision framework with the example data of Section 5.1 was implemented in MATLAB environment to verify the concept. Calculated optimal policy has the form of 5×100 matrix; ith row is the 100 years plan of the pipe whose initial state is estimated to be i. Fig. 4 shows the variation of the total cost of $G(N)$ as subproblem $G(\tau)$ changes when the initial state is given. All graphs have global minimum point and it is clear that those points are the optimal replacement period of a corresponding initial state.

Table 1. Optimal replacement period

Initial state	State1	State2	State3	State4	State5
τ^*(year)	53	53	54	80	53
V^*	94.04	97.40	234.88	256.06	476.52

Table. 1 shows the optimal replacement period and minimum total cost of an example case. Various constraints can

be added on this algorithm. For example, when the upper bound of the replacement period is set to be 50 years, then τ^*s are altered like Table. 2.

Table 2. Optimal replacement period with constraint

Initial state	State1	State2	State3	State4	State5
τ^*(year)	39	38	40	21	50
V^*	104.90	107.63	240.59	263.60	525.95

5.3 Comparison with other policies via Monte Carlo(MC) simulation

Monte Carlo simulation is widely used to solve the optimization problem of sequential stochastic process. This algorithm generates a set of random samples to obtain numerical results and to observe a stochastic dynamics. Total cost and state variation would be simulated at each policy. And the global optimal policy would be found by enumerating the whole possibility. However, the number of cases in this problem is $|A|^{N \times |S|}$, which is too huge to enumerate. Nevertheless, it is very useful to observe the state variation over the decision horizon and the cost when the action set is given.

To prove the validity and speed of the proposed algorithm, heuristic policies and optimal policies in Section 5.2. are compared by Monte Carlo simulation. The whole map of state transition probability $p_t(s_{t+1}|s_t, a_t)$ is revealed when the action set is given. Monte Carlo simulation creates the path regarding the revealed map from decision epoch 1 through 100 by using uniformly distributed random numbers. Each experiment is repeated 5000 times to reduce the effect of randomness.

Table 3. Policy comparison

Initial state	State1	State2	State3	State4	State5
Proposed	94.04	97.40	234.88	256.06	476.52
Heuristic15	369.54	369.54	369.54	369.54	469.54
Heuristic30	174.50	177.92	460.70	687.80	275.22
Myopic	642.40	648.70	946.30	1070.7	730.90

Monte Carlo simulation results of widely implemented policies are compared to the policy obtained by proposed algorithm and shown in Table. 3. Policies named Heuristics 15 and 30 replace the pipe every 15 and 30 years, respectively, regardless of its initial state. Myopic policy is to replace the pipe when failure occurs. Rehabilitation action is not considered in these reference policies. Simulation results show that the proposed algorithm leads to the smallest value on all initial states.

The strength of proposed algorithm is emphasized when the computational times are compared. The time complexity of proposed algorithm is $O(\mathrm{N}^2)$ while Monte Carlo simulation is $O(2^N)$. Thus, applying the dynamic programming method to solve the stochastic optimization reduces the computational time greatly.

Proof. Time complexity of subproblem $G(\tau)$ is $O(\tau)$, since dynamic programming has a linear time complexity. $G(N)$ is consisted of $\lfloor \frac{N}{\tau} \rfloor$ units of $G(\tau)$, so time complexity of $G(N)$ is $O(\mathrm{N})$. Total time complexity is then $O(\mathrm{N}^2)$ since $G(N)$ is solved N times to find optimal τ in the exhaustive manner.

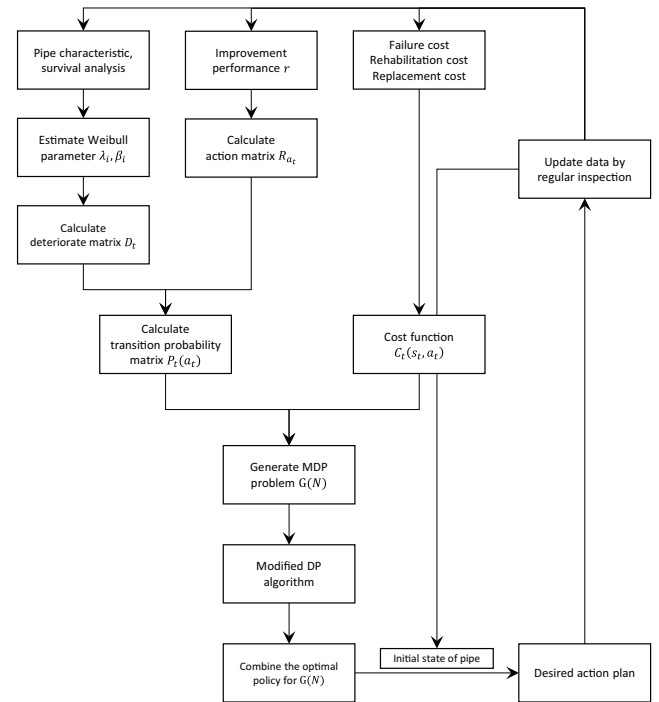In order to use Monte Carlo method to find the optimal

Fig. 5. Decision process flow diagram

policy, every possible cases ($|A|^{N \times |S|}$) should be explored. Which makes the time complexity exponential.

6. SUMMARY AND CONCLUSION

Fig. 5 summarizes the whole decision process. Decision maker needs the pipe characteristics such as geometry, pH, soil type, weather, population, etc. Using those raw data, MDP can be formulated by following the procedure in Section 2 and 3.

Due to the non-homogeneous property of transition probability matrix, conventional DP algorithm cannot be implemented. Hence, a modified method is proposed to get a global optimal policy; tear the problem into small-size subproblem and apply DP of each part, and reassemble them.

Calculated optimal policy is compared with two simple policies (heuristic policy, myopic policy) using Monte Carlo simulation in order to prove the validity and performance of the proposed algorithm. As a result, the proposed algorithm not only achieves the global optimal policy, but also reduces the computational time greatly.

Moreover, local-specialty would be one of the major difference compared to existing policies. It considers the effect of current state, property, cost, decision horizon comprehensively while heuristic and myopic policies do not. So the policy can be altered flexibly on frequently varying circumstance. By regular inspection, properties of pipe are updated and optimal policy can be varied regularly.

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