

Reliable H_∞ Control of Switched Linear Systems [★]

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Abstract

In this paper a reliable H_∞ controller for a class of uncertain switched linear systems is designed using the multiple Lyapunov function technique with the property that the solvability of reliable H_∞ control of the individual subsystems is not necessary. First, reliable H_∞ control is defined for this class of systems and the multiple Lyapunov function method is deployed to ensure that the magnitude of the controlled output is upper bounded by suppressing the external disturbance while guaranteeing internal stability. Last, the H_∞ control method above for the switched systems is directly applied to the standard reliable H_∞ control problem of a class of non-switched systems which a continuous control method cannot solve.

Key words: Guaranteed cost control, H_∞ control, reliable control, switched systems.

1 Introduction

Uncertainties and external disturbances exist in all practical systems. If they are not considered during the course of controller design, the resulting closed-loop practical system will usually exhibit degradation of performances, even leading to instability. Thus systems with uncertainties and external disturbances are paid much attention [22][2]. For such systems, we hope that not only can it be stable, but also certain least expected performances can be guaranteed or the impact of disturbances on the output variables can be controlled. H_∞ control is one of effective methods to deal with these issues [3] [12]. The advantage of the H_∞ control is that the norm of controlled output is upper bounded by suppressing the external disturbance while guaranteeing internal stability [12][11]. For H_∞ control, there have been many results, including latest ones [3] [12][5] [6] [7][1] [4].

Furthermore, actuator faults are another issue to impact on both stability and performance. Thus, the existence of uncertain faults and the possible occurrence of actu-

ator faults must be taken care of during the course of controller design to avoid heavy economic costs or life-threatening prices the faults may cause [20]. Thus reliable control also attracts much attention [8][15][10][18]. From a point of view of robustness, in a wide sense fault-tolerant control also can be considered one of robust control methods. Therefore, actuator faults also pose difficulties on our control objectives which require not only stability but certain expected performances.

On the other hand, switched systems, as a special hybrid system, have received a great amount of attention because of their importance from both theoretical and practical points of view (see, e.g., [9] [13][21][19] and the references therein). It is still interesting to extend some traditional topics such as guaranteed cost and H_∞ controls to switched systems and in the meanwhile consider some uncertain and/or random influences on the systems such as actuator faults [8]. On dealing with actuator faults, there are a few results [14][16][17]. References [18-20] designed reliable controllers for the nonlinear systems with actuator faults to guarantee reliable stability of the systems. Their common feature is that actuators are decomposed into two parts, one of which is fragile to faults, but the other part is robust to faults. But in order to implement the controller designs, the two-part decomposition must be known a priori. It is usually difficult to obtain because of uncertain and random features of faults.

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The difficulties above raise a question naturally: can we propose a control strategy to H_∞ control of switched systems without necessarily identifying faulty actuators from healthy ones and even without changing any structures and/or parameters of the controller to be designed to realize reliability of control (i.e., regardless of whether the faults occur or not)? This paper provides an affirmative answer. Specifically, this paper first describes a representation of fault models which facilitates the realization of reliable control, then presents a reliable H_∞ control method for the switched systems, and last develops a hybrid state feedback strategy, as a special application of the H_∞ control method, to solve the standard reliable H_∞ control problem of non-switched linear systems that a single continuous controller is assumed unable to solve.

The main features of this paper are summarized as follows:

- The reliable H_∞ control of switched systems is solved where the solvability of the H_∞ control of individual subsystems is not necessary.
- A co-design of controllers and a state-dependent switching rule, based on the multiple Lyapunov function technique, is proposed for reliable H_∞ control, which reduces the conservativeness that the common Lyapunov function method causes.
- The proposed methods achieve reliable control without necessarily identifying faulty actuators from healthy ones and even without needing to change any structures and/or parameters of the controller to be designed regardless of whether the faults occur or not.

2 Problem Statement

Consider the following switched system

$$\begin{aligned} \dot{x} &= (A_\sigma + \Delta A_\sigma)x + (B_\sigma + \Delta B_\sigma)u_\sigma + G_\sigma w, \\ z &= C_i x, \end{aligned} \quad (1)$$

where the function $\sigma(t) : [0, \infty) \rightarrow M = \{1, \dots, m\}$ is the switching signal which, for simplicity, is assumed to be a piecewise constant (from the right) function depending on time or state or both; m is the number of models (called subsystems) of the switched system. $x \in R^n$ and $u \in R^{q_i}$ represent state and control signal, respectively. $z \in R^{p_i}$ is controlled output. A_i and B_i are constant matrices with appropriate dimensions, ΔA_i and ΔB_i are uncertain real-valued matrix functions, w_i is the disturbance input of the function class $w \in L_2[0, \infty)$, and $G_i, C_i, i \in M$, are constant matrices with appropriate dimensions.

We need the following assumptions.

Assumption 1 Assume that (A_i, B_i) is controllable, and the state is available for feedback.

Assumption 2 Assume that ΔA_i and ΔB_i are norm-bounded, i.e., there exist constants δ and θ such that the following inequalities hold

$$\|\Delta A_i\| \leq \delta, \text{ and } \|\Delta B_i\| \leq \theta. \quad (2)$$

Assume that the controller is in the form of

$$u_i = K_i x, \quad (3)$$

where $K_i \in R^{q_i \times n}, i \in M = \{1, 2, \dots, m\}$ is a constant matrix.

Since fault possibly occurs on each actuator, a matrix L_s^i is used to represent the fault situations of the system

$$L_s^i = \text{diag}(l_1^i, l_2^i, \dots, l_{q_i}^i), \quad (4)$$

where $l_j^i (j \in 1, 2, \dots, q_i)$ is one or zero subject to $L_s^i \neq 0$. If $l_j^i = 1$ it indicates the j^{th} actuator works normally, otherwise if $l_j^i = 0$ the j^{th} actuator fails completely.

Thus the closed-loop switched system is

$$\dot{x} = [(A_i + \Delta A_i) + (B_i + \Delta B_i)L_s^i K_i]x + G_i w. \quad (5)$$

We will now define the reliable H_∞ control of switched systems (1).

Definition 1 For a given positive constant $\gamma > 0$, if there exist an individual control law $u_i = K_i x$ for subsystem i and a switching law $\sigma(t)$ such that the closed-loop switched system for all the admissible structural uncertainties and all the possible actuator faults described by (3) satisfies:

(i) when $w \equiv 0$, the resulting closed-loop system (5) is asymptotically stable,

(ii) when the initial state $x(t_0) = 0, \|z\|_2 < \gamma \|w\|_2$ holds,

then the $u_i = K_i x$ and the switching law $\sigma(t)$ achieves the reliable H_∞ control of switched systems (1).

Before designing the reliable guaranteed cost controller, we give the following notations.

The switching signal $\sigma(t)$ can be characterized by the following switching sequence:

$$\Sigma = \{(x_0); (i_0, t_0), (i_1, t_1), \dots, (i_n, t_n), \dots, \{i_k \in M, k \in N, \}, (6)$$

where t_0 is the initial time, x_0 is the initial state, and N is the set of nonnegative integers. When $t \in [t_k, t_{k+1})$,

$\sigma(t) = i_k$, that is, the i_k th subsystem is active. For any j , $1 \leq j \leq m$, we define

$$\Sigma_t(j) = \{[t_{j_1}, t_{j_1+1}), [t_{j_2}, t_{j_2+1}), \dots, [t_{j_n}, t_{j_n+1}), \dots, \sigma(t) = j, t_{j_k} \in [t_{j_k}, t_{j_k+1}), k \in N\}, (7)$$

which is the switching sequence of the subsystem k . In addition, we assume that the state of the switched system (5) does not jump at the switching instants, i.e., the trajectory $x(t)$ is everywhere continuous. We also assume that σ has finite number of switchings on any finite interval of time, which is a standard assumption in switched systems literature to rule out Zeno behavior [9].

3 Reliable H_∞ Control

In this section, we will design the H_∞ controller for a class of uncertain switched systems (1) using the multiple Lyapunov function technique.

Before presenting the main result, we need the following well-known lemma borrowed from literature (e.g., [8]).

Lemma 1 For any real numbers $x, y \in R^r$, a positive number ε and a symmetric positive definite matrix Π , we have that

$$x^T y + y^T x \leq \frac{x^T \Pi x}{\varepsilon} + \varepsilon y^T \Pi^{-1} y \leq \frac{x^T \Pi x}{\varepsilon} + \frac{\varepsilon y^T y}{\lambda(\Pi)}.$$

Now, we are ready to present the main result for the reliable H_∞ control for the switched system (1) using the multiple Lyapunov function technique.

Theorem 1 Consider the uncertain switched system (1) satisfying Assumption 1 and 2. If there exist positive constants β_{ij} ($i, j \in M$), ε, λ , and positive definite matrices P_i , such that the following matrix inequalities hold, for $\forall i \in M$,

$$\Lambda_i + C_i^T C_i + \gamma^{-2} P_i G_i G_i^T P_i + \sum_{j=1}^m \beta_{ij} (P_j - P_i) < 0, (8)$$

where $\Lambda_i = P_i A_i + A_i^T P_i + \frac{1}{\varepsilon} P_i H_i P_i + \frac{\varepsilon}{\lambda_{\min}(H)} \delta^2 I_r + \varepsilon P_i B_i B_i^T P_i + \varepsilon \theta^2 P_i P_i + \frac{2}{\varepsilon} P_i B_i (\lambda^{-1})^2 B_i^T P_i$, then, for the switched system with all the possible actuator faults described by (3), there exist a switching law $i = i(x)$ and state feedback controllers $u_i = K_i x$ with $K_i = -R^{-1} B_i^T P_i$ such that the reliable H_∞ control of switched systems is solvable.

Proof: For all $i \in M$, we define

$$\Omega_i = \{x \in R^n | x^T (P_j - P_i) x \geq 0, x \neq 0, \forall j \in M\}, (9)$$

then, $\bigcup_{i=1}^m \Omega_i = R^n \setminus \{0\}$.

It is obvious to see that for $\forall x \in \Omega_i$, there is an $i \in M$ such that $x^T (P_j - P_i) x \geq 0, \forall j \in M$. So, from (8) we can obtain that:

$$x^T (\Lambda_i + \gamma^{-2} P_i G_i G_i^T P_i + C_i^T C_i) x < 0. (10)$$

We construct that

$$\bar{\Omega}_1 = \Omega_1, \dots, \bar{\Omega}_i = \Omega_i - \bigcup_{j=1}^{i-1} \Omega_j, \dots, \bar{\Omega}_m = \Omega_m - \bigcup_{j=1}^{m-1} \Omega_j. (11)$$

It is easy to obtain that

$$\bigcup_{i=1}^m \bar{\Omega}_i = R^n \setminus \{0\}, \bar{\Omega}_j \cap \bar{\Omega}_i = \emptyset, i \neq j. (12)$$

We define the Lyapunov functional candidate

$$V_i(x) = x^T P_i x, (13)$$

where $P_i > 0$ satisfies (8).

Next, we design the following switching law:

$$i(t) = i, \text{ when } x(t) \in \bar{\Omega}_i, i \in M. (14)$$

When $w \equiv 0, x(t) \in \bar{\Omega}_i$, the derivative of $V_i(x)$ along the trajectory of the switched system (5) is given by

$$\begin{aligned} \dot{V}_i(x) &= x^T (A_i^T P_i + P_i A_i) x + x^T (\Delta A_i^T P_i + P_i \Delta A_i) x \\ &\quad - 2x^T P_i B_i L_s^i R_i^{-1} B_i^T P_i x - x^T P_i B_i L_s^i R_i^{-1} \Delta B_i^T P_i x \\ &\quad - x^T P_i \Delta B_i L_s^i R_i^{-1} B_i^T P_i x. \end{aligned} (15)$$

Using Lemma 1 and the fact that $(-L_s^i) (-L_s^i) = 1$, we can easily have

$$\begin{aligned} &-2x^T P_i B_i L_s^i R_i^{-1} B_i^T P_i x \\ &\leq x^T \left[\frac{1}{\varepsilon} P_i B_i R_i^{-1} (-L_s^i) (-L_s^i) R_i^{-1} B_i^T P_i \right. \\ &\quad \left. + \varepsilon P_i B_i B_i^T P_i \right] x \\ &\leq x^T \left[\frac{1}{\varepsilon} P_i B_i R_i^{-1} \|(-L_s^i) (-L_s^i)\| R_i^{-1} B_i^T P_i \right. \\ &\quad \left. + \varepsilon P_i B_i B_i^T P_i \right] x \\ &= x^T \left[\frac{1}{\varepsilon} P_i B_i (R_i^{-1})^2 B_i^T P_i + \varepsilon P_i B_i B_i^T P_i \right] x, \end{aligned} (16)$$

$$x^T (-P_i B_i L_s^i R_i^{-1} \Delta B_i^T P_i - P_i \Delta B_i L_s^i R_i^{-1} B_i^T P_i) x$$

$$\begin{aligned}
 &= x^T [P_i B_i (-L_s^i) R_i^{-1} \Delta B_i^T P_i - P_i \Delta B_i (-L_s^i) \\
 &\quad \times R_i^{-1} B_i^T P_i] x \\
 &\leq x^T \left[\varepsilon P_i \Delta B_i \Delta B_i^T P_i + \frac{1}{\varepsilon} P_i B_i R_i^{-1} (-L_s^i) (-L_s^i) \right. \\
 &\quad \left. R_i^{-1} B_i^T P_i \right] x \\
 &\leq x^T \left[\varepsilon \theta^2 P_i P_i + \frac{1}{\varepsilon} \|L_s^i L_s^i\| P_i B_i (R_i^{-1})^2 B_i^T P_i \right] x \\
 &\leq x^T \left[\varepsilon \theta^2 P_i P_i + \frac{1}{\varepsilon} P_i B_i (R_i^{-1})^2 B_i^T P_i \right] x. \tag{17}
 \end{aligned}$$

Therefore, we obtain the following inequality:

$$\begin{aligned}
 \dot{V}_i(x) &\leq x^T \left[A_i^T P_i + P_i A_i + \frac{1}{\varepsilon} P_i H_i P + \frac{\varepsilon \delta^2}{\lambda_{\min}(H_i)} I_r \right. \\
 &\quad \left. + \varepsilon P_i B_i B_i^T P_i + \varepsilon \theta^2 P_i P_i + \frac{2}{\varepsilon} P_i B_i (R_i^{-1})^2 B_i^T P_i \right] x \tag{18}
 \end{aligned}$$

Therefore, from (10)-(18), we obtain the following inequality:

$$\dot{V}_i < -x^T (\gamma^{-2} P_i G_i G_i^T P_i + C_i^T C_i) x < 0. \tag{19}$$

By using the multiple Lyapunov function theory, the closed-loop switched system (5) is asymptotically stable. Now, we will prove that the closed-loop switched system satisfies the following performance index J :

$$J = \int_0^{+\infty} [z^T(t)z(t) - \gamma^2 w^T w] dt. \tag{20}$$

Without loss of generality, we assume that $x_0^T P_{i_0} x_0 = \min_{i_k \in M} \{x_0^T P_{i_k} x_0\}$. According to the switching sequence (6), for any $w \in L_2[0, +\infty)$, one has

$$\begin{aligned}
 J &= \sum_{i=1}^m \sum_{j=1}^{\infty} \int_{t_{i_j}}^{t_{i_{j+1}}} \left[\|z(t)\|^2 - \gamma^2 \|w_i\|^2 + \dot{V}_i(x(t)) \right] dt \\
 &\quad - \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \dot{V}_{i_k}(x(t)) dt \\
 &\leq \sum_{i=1}^m \sum_{j=1}^{\infty} \int_{t_{i_j}}^{t_{i_{j+1}}} \left[\|C_i x(t)\|^2 - \gamma^2 \|w_i\|^2 + 2x^T P_i G_i w \right. \\
 &\quad \left. + x^T \Lambda_i x \right] dt - \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \dot{V}_{i_k}(x(t)) dt
 \end{aligned}$$

$$\begin{aligned}
 &\leq \sum_{i=1}^m \sum_{j=1}^{\infty} \int_{t_{i_j}}^{t_{i_{j+1}}} \|C_i x(t)\|^2 + x^T (\gamma^{-2} (P_i G_i)^2 + \Lambda_i) x dt \\
 &\quad - \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \dot{V}_{i_k}(x(t)) dt, \tag{21}
 \end{aligned}$$

where the last inequality is obtained from the inequality

$$2x^T P_i G_i w \leq \gamma^{-2} x^T P_i G_i G_i^T P_i x + \gamma^2 w^T w. \tag{22}$$

Then, by using (10) and (14), we obtain that

$$\begin{aligned}
 J &\leq - \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \dot{V}_{i_k}(x(t)) dt \\
 &= - [V_{i_0}(x(t_1)) - V_{i_0}(x(t_0)) + V_{i_1}(x(t_2)) \\
 &\quad - V_{i_1}(x(t_1)) + \dots] \\
 &= V_{i_0}(x(t_0)) = x_0^T P_{i_0} x_0 = 0, \tag{23}
 \end{aligned}$$

that is, $\|z\| < \gamma \|w\|, \forall w \in L_2[0, +\infty)$. \square

Remark 1 *without loss of generality, the initial state $x(t_0)$ is assumed zero in the second condition of Definition 1 and thus $\|z\|_2 < \gamma \|w\|_2$ is guaranteed. From the above proof, it can be seen that if the initial state is not zero the disturbances can still be suppressed by tuning γ .*

Remark 2 *By pre- and post multiplying both sides of inequality (8) by $X_i = P_i^{-1}$ and by using the Schur Complement Lemma, inequality (8) is equivalent to the following linear matrix inequality:*

$$\begin{bmatrix} \bar{\Gamma}_i & \bar{\Psi}_i^T \\ \bar{\Psi}_i & \bar{\Upsilon}_i \end{bmatrix} < 0, \tag{24}$$

where

$$\begin{aligned}
 \bar{\Gamma}_i &= X_i A_i + A_i^T X_i + \varepsilon B_i B_i^T + \frac{1}{\varepsilon} H_i + B_i R_i^{-1} B_i^T \\
 &\quad + \varepsilon \theta^2 I + \frac{2}{\varepsilon} B_i \lambda^{-2} B_i^T + \lambda^{-2} G G^T - \sum_{j=1}^m \beta_{ij} X_j, \\
 \bar{\Psi}_i^T &= [X_i, C X_i, X_i, X_i, \dots, X_i], \\
 \bar{\Upsilon}_i &= \text{diag} \left\{ -\frac{\lambda_{\min}(H_i)}{\varepsilon \delta^2} I_r, -\beta_{i1}^{-1} X_1, \right. \\
 &\quad \left. \dots, -\beta_{i,i-1}^{-1} X_{i-1}, -\beta_{i,i+1}^{-1} X_{i+1}, \dots, -\beta_{im}^{-1} X_m \right\}.
 \end{aligned}$$

4 Hybrid reliable H_∞ control application

For certain class of systems in practical engineering if only *one* continuous controller is designed to achieve certain control objective such as stabilization, it can lead to

complicated structure of the controller or poor performances, sometimes such a controller does not exist [9]. Thus for this class of systems, switching among finite candidate controllers can give a desired solution. This section will apply the result of Section 3 to a class of non-switched systems and propose the hybrid control strategy accordingly.

We consider the following switched system:

$$\begin{aligned} \dot{x} &= (A + \Delta A)x + (B + \Delta B)u_i + Gw, \\ z &= Cx. \end{aligned} \quad (25)$$

Assume that there exist a set of candidate controllers:

$$u_i = K_i x, i = 1, 2, \dots, m. \quad (26)$$

To show the usage of the switching mechanism, we also assume that the uncertain systems with faulty actuators cannot be stabilized by each single controller if the systems did not have disturbance.

In the following, we use the switching technique to properly choose which individual controller is applied to the studied system:

$$u = K_{\sigma(t)} x, \sigma(t) : [0, +\infty) \rightarrow M = 1, 2, \dots, m. \quad (27)$$

Theorem 2 Consider the uncertain linear system (25) satisfying Assumption 2. If there exist positive constants $\beta_{ij}, \varepsilon, \lambda$, and positive definite matrices P_i , such that the following matrix inequalities hold, for $\forall i \in M$,

$$\Theta_i + C^T C + \gamma^{-2} P_i G G^T P_i + \sum_{j=1}^m \beta_{ij} (P_j - P_i) < 0, \quad (28)$$

where $\Theta_i = P_i A + A^T P_i + \frac{1}{\varepsilon} P_i H_i P_i + \frac{\varepsilon}{\lambda_{\min}(H)} \delta^2 I_r + \varepsilon P_i B B^T P_i + \varepsilon \theta^2 P_i P_i + \frac{2}{\varepsilon} P_i B (\lambda^{-1})^2 B^T P_i$, then, for the system (25) with all the possible actuator faults described by (6), there exist state feedback controllers $u_\sigma = K_\sigma x$ and a switching law $\sigma = \sigma(x)$ such that the reliable H_∞ control of the system (25) is solvable, where $K_i = -R^{-1} B_i^T P_i$.

Proof: Given the proof of Theorem 2, this proof is straightforward and thus omitted.

5 A Numerical Example

In this section, an example is provided to verify the developed results. Consider the switched linear systems

consisting of two subsystems described by

$$\begin{aligned} A_1 &= \begin{bmatrix} -5 & 2 \\ 0 & 2 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 2 & 0 \\ 1 & -5 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ G_1 = G_2 &= \begin{bmatrix} 0.01 & 0.02 \\ 0.02 & 0 \end{bmatrix}. \end{aligned}$$

We choose the initial state $x(0) = (-5, 3)^T$ and the disturbance $w = \frac{\sin t}{t}$. When the actuator is faulty, we set

$$L_s^1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, L_s^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \quad (29)$$

The state responses of each subsystem are shown in Figs. 1(a) and 1(b), which can be seen that each subsystem is unstable and thus the H_∞ problems of the individual subsystems are not solvable. On the other hand, by using the proposed method, we can construct a reliable state-feedback controller to achieve the asymptotic stabilization when $w = 0$. We choose $\varepsilon = 1.2, \gamma = 1.3, \theta = 5, \delta = 5$ in a trial-and-error way but satisfying the assumptions, and from (24) then obtain that

$$P_1 = \begin{bmatrix} 0.0012 & 0.0009 \\ 0.0009 & 0.0045 \end{bmatrix}, P_2 = \begin{bmatrix} 0.0031 & 0.0007 \\ 0.0007 & 0.0015 \end{bmatrix}. \quad (30)$$

Let $\Omega_1 = \{x \in R^n | x^T (P_1 - P_2) x \geq 0, x \neq 0\}$, and $\Omega_2 = \{x \in R^n | x^T (P_2 - P_1) x \geq 0, x \neq 0\}$. Then, one has $\Omega_1 \cup \Omega_2 = R^n \setminus \{0\}$. We then design the switching signal as follows:

$$\sigma(t) = \begin{cases} 1, & x(t) \in \Omega_1, \\ 2, & x(t) \in \Omega_2 \setminus \Omega_1. \end{cases} \quad (31)$$

The individual controller for each subsystem is

$$u_i = -\lambda_i^{-1} B_i^T P_i x, i = 1, 2. \quad (32)$$

Fig. 1(c) describes the state responses of the closed-loop switched system. Fig. 2(a) shows the switching signal. In order to make the switching process clearer, the scope of such a switching signal is depicted in Fig. 2(b). Therefore, it can be seen that the proposed control strategy can achieve the reliable H_∞ control of the closed-loop switched system containing fault actuators.

6 Conclusions

This paper first presents the reliable H_∞ control method for a class of uncertain switched linear systems using the

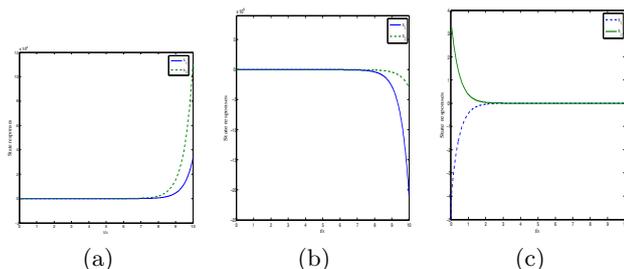


Fig. 1. (a) The trajectories of subsystem 1; (b) The trajectories of subsystem 2; (c) The trajectories of the switched system.

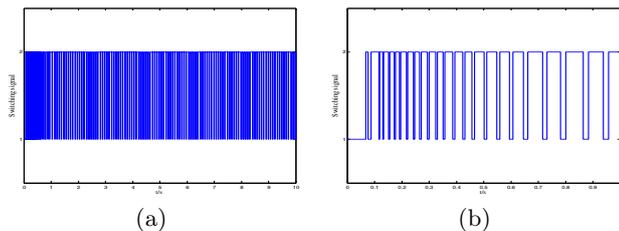


Fig. 2. (a) The switching signal; (b) The enlarged switching signal.

multiple Lyapunov function technique to guarantee that the norm of the controlled output is upper bounded by suppressing the external disturbance. Then, the H_∞ control method is directly deployed to the standard reliable H_∞ control problem of a class of non-switched systems which a single controller cannot internally stabilize. The on-going work is on the reliable guaranteed cost control methods for the same class of uncertain switched linear systems using the multiple Lyapunov function technique to ensure the cost performance index is always guaranteed to be smaller than a fixed upper bound regardless the degradation of the system performances.

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