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Stability Margin Interpretation of the SIMC Tuning Rule for PI Controllers and its Applications

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Abstract: Model reduction and tuning rules given in the SIMC (Simple Internal Model Control) method are very effective in tuning PI controllers. For some processes with large lead elements, control performances by the SIMC method are somewhat oscillatory or sluggish. To mitigate such drawbacks, additional tuning rules based on the second order plus time delay model with lead term are proposed. Improvements for certain types of models are critical. For such processes, besides the SIMC tuning rule, no PI controller tuning rules that are analytic and given in terms of process parameters are not available. Since the proposed tuning rules are very simple, they can be used in the field, effectively complementing the SIMC method.

Keywords: PI controller tuning, SIMC method, SOPTD model, Model reduction, Stability margin.

1. INTRODUCTION

There are many analytic tuning rules for PI and PID controllers (Seborg et al., 2010; Astrom and Hagglund, 1995; O'Dwyer, 2009). The internal model control (IMC) method (Rivera et al., 1986) provides simple algebraic tuning rules. Applying the IMC method to a first order plus time delay (FOPTD) model with a Pade approximation of the time delay term, PI controllers can be obtained analytically. The simple IMC (SIMC) method (Skogestad, 2003) modifies the IMC-PI tuning rules slightly and utilizes model reduction rules to obtain FOPTD models from higher order models. The SIMC method is popular due to its simplicity and excellent performance for a wide range of processes (Grimholt and Skogestad, 2012) with a few exceptions. Some SIMC model reduction rules were modified by Lee et al. (2014) for more consistent performances.

The SIMC tuning rules are based on the FOPTD model. The model reduction to FOPTD model can be very poor for processes with large lead elements due to their structural limitations, which appear often in process models (Ogunnaike and Ray, 1979; Luyben, 1986). For such processes, PI controllers designed by the SIMC method can yield somewhat oscillatory or sluggish responses. For PI controllers, besides the SIMC tuning rule, no PI controller tuning rules that are analytic and given in terms of process parameters are not available (O'Dwyer, 2009). Numerical methods that use the time-domain and frequency-domain optimization can be used to tune PI controllers. However, they can suffer from convergence and local optimization problems. To mitigate drawbacks of the SIMC method for processes with large lead elements, tuning rules based on second order plus time delay models with lead terms are proposed. Analytic tuning rules are obtained by applying the stability margin characteristics of the SIMC tuning rules. The procedure applying the SIMC model reduction rules is slightly modified for better approximate models and consistent control performance.

The proposed method designs the proportional and integral gains of PI controller separately. It is very similar to the sequential tuning method (Lee et al, 1998). The process information needed are frequency responses at two frequencies whose phase angles are -90° and -180°, respectively. They can be obtained from the process model or from two relay feedback tests. Hence the method can be applied for on-line autotuning (Lee et al., 2007).

2. MOTIVATION

Consider a first order plus time delay (FOPTD) process

$$G(s) = \frac{k\ \exp(-\theta s)}{\tau s + 1} \tag{1}$$

For this process, one of the simplest tuning rules for PI controllers, $C(s) = k_C + k_I / s$, is

$$kk_C = \tau/(\lambda + \theta),\ kk_I = 1/(\lambda + \theta) \tag{2}$$

which can be derived through the direct synthesis method (Seborg et al., 2010), the internal model control (IMC) method (Rivera et al., 1986) and the SIMC method (Skogestad, 2003) .

For real processes, the process transfer functions are first approximated by FOPTD models;

$$G(s) \approx \frac{k_{eff}\ \exp(-\theta_{eff} s)}{\tau_{eff} s + 1} \tag{3}$$

The SIMC method also provides simple model reduction rules to obtain approximate FOPTD models. However, for some processes with large lead terms that can appear in process models (Ogunnaike and Ray column (Ogunnaike and Ray, 1979), Tyreus Stabilizer (Luyben, 1986)), reduced order models can be very poor due to its structural limitation and the above advantages of SIMC method are not guaranteed.

For example, consider the 1x1 element of the Tyreus Stabilizer process;

$$G(s) = \frac{(10s+1)\exp(-0.1s)}{(4s+1)^3} \tag{4}$$

The SIMC method uses the approximation

$$G(s) = \frac{(10s+1)\exp(-0.1s)}{(4s+1)^3} \approx 2.5\frac{\exp(-2.1s)}{6s+1} \tag{5}$$

Then the PI controller is designed as k_C=6/2.5/(2.1+2.1) and τ_I=6 (λ=θ=2.1). Figure 1(a) shows step set point responses when the PI controller is applied to the process of Eq. (4) and the FOPTD model of Eq. (5). For the FOPTD model of Eq. (5), the PI controller is satisfactory and the closed-loop response is very similar to the desired closed-loop response of $G_{cl}(s)=exp(-2.1s)/(2.1s+1)$. However, the closed-loop response for the process of Eq. (4) is rather sluggish and much different from the desired closed-loop response.

Secondly, consider the process

$$G(s) = \frac{(2s+1)\exp(-s)}{(5s+1)(0.1s+1)} \tag{6}$$

The SIMC method reduces *G(s)* to the FOPTD model of

$$G(s) = \frac{(2s+1)\exp(-s)}{(5s+1)(0.1s+1)} \approx \frac{\exp(-1.05s)}{3.05s+1} \tag{7}$$

Then the PI controller is obtained as k_C=3.05/(1.05+1.05) and τ_I=3.05 (λ=θ=1.05). Figure 1(b) shows step set point responses when the PI controller is applied to the process of Eq. (6) and the FOPTD model of Eq. (7).

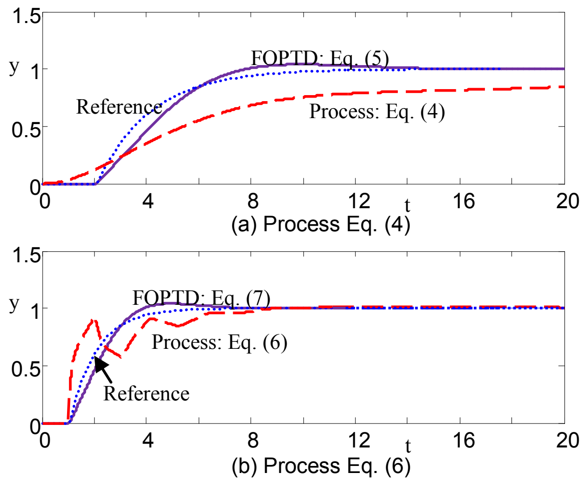

Fig. 1. Step set point responses of PI control systems.

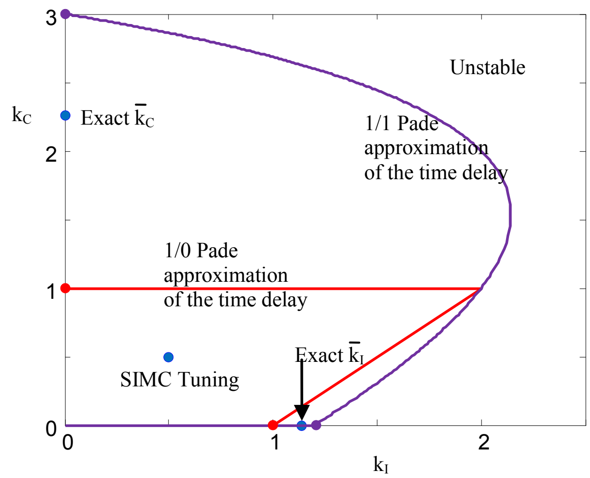

Fig. 2. Stability region for *G(s)=exp(-s)/(s+1)*.

For the FOPTD model of Eq. (7), the PI controller is satisfactory and the closed-loop response is very similar to the desired closed-loop response. However, the closed-loop response for the process of Eq. (6) is rather oscillatory and is much different from the first order response. The tuning parameter λ should be increased for responses that are less oscillatory. In this process, the effective time constant estimated is too large.

3. STABILITY MARGIN INTERPRETATION OF THE SIMC PI CONTROLLER TUNING RULE

With approximating the time delay term in FOPTD process of Eq. (1) by the 1/0 Pade method, the characteristic equation is

$$1 + \left(k_C + \frac{k_I}{s}\right)\frac{k(-\theta s+1)}{\tau s+1} = 0 \tag{8}$$

The stable region for which Eq. (8) has roots with negative real parts is shown in Fig. 2. The maximum stable k_C for k_I=0 is

$$k\bar{k}_C = \tau/\theta \tag{9}$$

and the maximum stable k_I for k_C=0 is

$$k\bar{k}_I = 1/\theta \tag{10}$$

The SIMC method can be interpreted as $k_C = \bar{k}_C/\nu$ and $k_I = \bar{k}_I/\nu$, with ν=λ/θ+1. The design parameter ν can be considered as a gain margin, representing the speed of the control system. When the 1/1 Pade approximation for the time delay term is used, the stable region is enlarged (Fig. 2) and the margin parameter ν should be increased for control responses similar to those of SIMC method.

4. PROPOSED METHOD

For a general process *G(s)*, we tune the PI controller as

$$k_C = \frac{1}{\nu}\bar{k}_C, \quad k_I = \frac{1}{\nu}\bar{k}_I \tag{11}$$

where $\bar{k}_C$ and $\bar{k}_I$ are the maximum stable controller gains of *G(s)* and *G(s)/s*, respectively.

For some processes, k_I can be very large. In SIMC tuning, $1/kk_I=\lambda+\theta$ (Eq. (4)). Based on this, we limit k_I as

$$kk_I < 1/\theta_{eff} \tag{12}$$

Similarly, k_C can be very large and should be limited as the SIMC method limits the controller integral time for lag dominant processes. The controller zero is k_C/k_I and its large value can cause sluggish load responses. We limit this to be less than 5 times the effective closed loop time constant of $1/kk_I=\lambda+\theta$. That is, $k_C/k_I<5(\lambda+\theta)=5/(kk_I)$. Equivalently,

$$kk_C < 5 \tag{13}$$

Equation (11) determines the controller proportional and integral gains independently. Instead of the independent design, they can be designed sequentially (Lee et al., 1998). First design k_I based on Eq. (11) and then design k_C by obtaining the maximum stable gain from the characteristic equation,

$$1 + (k_C + k_I/s)G(s) = 0 \tag{14}$$

for a given k_I. For the FOPTD process, this sequential design method provides the same results as the SIMC method when the 1/0 Pade approximation of time delay is used. As shown in Fig. 2, this sequential design procedure will guarantee the given stability margins.

[Relay Feedback Autotuning]

The proposed tuning method requires two pieces of process information of the maximum stable controller gains of *G(s)* and *G(s)/s*, equivalently frequency responses at two frequencies whose process phase angles are -90^o and -180^o. They can be obtained from two relay feedback tests (Lee et al., 2007), one for the process itself and one for the process with integral action. The proposed method can be applied for on-line autotuning.

5. ANALYTIC TUNING RULES: STABILITY MARGIN (SM) METHOD

Analytic tuning rules can be obtained for low order processes. Consider a second order process with large lead term,

$$G(s) = \frac{k(\alpha s+1)\exp(-\theta s)}{(\tau_1 s+1)(\tau_2 s+1)} \tag{15}$$

The characteristic equation is, with the 1/0 Pade approximation for the time delay term,

$$1 + \left(k_C + \frac{k_I}{s}\right)\frac{k(\alpha s+1)(-\theta s+1)}{(\tau_1 s+1)(\tau_2 s+1)} = 0 \tag{16}$$

Equivalently,

$$s(\tau_1 s+1)(\tau_2 s+1) + k(k_C s + k_I)(\alpha s+1)(-\theta s+1) = 0 \tag{17}$$

There are several analytic methods to find the largest stable gains of k_C and k_I such as the Routh method and the direct substitution method (Seborg et al., 2010; Lee et al., 2005).

The maximum stable gain for k_I with k_C=0 is (Lee et al., 2005)

$$\begin{aligned} k\bar{k}_I &= \frac{1}{\nu}\sqrt{\frac{w^2(1+\tau_1^2 w^2)(1+\tau_2^2 w^2)}{(1+\alpha^2 w^2)(1+\theta^2 w^2)}} \\ w^2 &= \frac{2}{P+\sqrt{P^2+4\alpha\tau_1\tau_2\theta}} \\ P &= \tau_1\tau_2 + \theta\tau_1 + \theta\tau_2 - \alpha(\tau_1+\tau_2+\theta) \end{aligned} \tag{18}$$

Applying the Routh stability theorem (Seborg et al., 2010), we have the maximum stable gain for k_C with k_I=0 as

$$k\bar{k}_C = \begin{cases} \dfrac{\tau_1\tau_2}{\alpha\theta}, & \alpha > \dfrac{\tau_1\tau_2\theta}{\tau_1\tau_2+\tau_1\theta+\tau_2\theta} \\ (\tau_1+\tau_2)/(\theta-\alpha), & otherwise \end{cases} \tag{19}$$

The proposed tuning rules of Eqs. (18) and (19) may fail to provide proper controller parameters for some extreme cases of very small α, τ_2, and θ.

[Small τ_2 Case]

Consider a first order process with large lead term (τ_2=0 in Eq. (15)),

$$G(s) = \frac{k(\alpha s+1)\exp(-\theta s)}{\tau_1 s+1} \tag{20}$$

The characteristic equation for the controller integral gain k_I is, with the 1/0 Pade approximation for the time delay term, $s(\tau_1 s+1) + kk_I(\alpha s+1)(-\theta s+1) = 0$. Applying the Routh stability theorem, we can obtain the maximum stable gain for k_I with k_C=0 as

$$\bar{k}_I = \begin{cases} \dfrac{\tau_1}{\alpha\theta}, & \alpha > \dfrac{\tau_1\theta}{\tau_1+\theta} \\ 1/(\theta-\alpha), & otherwise \end{cases} \tag{21}$$

The 1/0 Pade approximation for the time delay term cannot be used for k_C because there is no stable k_C. So we use the 1/1 Pade approximation and the characteristic equation for the controller proportional gain k_C is

$$1 + kk_C\frac{(\alpha s+1)(-\theta s+2)}{(\tau_1 s+1)(\theta s+2)} = 0, \tag{22}$$

The maximum stable gain for k_C with k_I=0 is

$$\bar{k}_C = \begin{cases} \dfrac{\tau_1}{\alpha}, & \alpha > \dfrac{\tau_1\theta}{4\tau_1+\theta} \\ \dfrac{2\tau_1+\theta}{\theta-2\alpha}, & otherwise \end{cases} \quad (23)$$

Since there are different Pade approximations for the time delay terms, we use different gain margins for ν. Here we use

$$k_C = \frac{\bar{k}_C}{2\nu}, \quad k_I = \frac{\bar{k}_I}{\nu} \quad (24)$$

This rule is used for $\alpha > \tau_1\theta/(\tau_1+\theta)$ .When $\alpha > \tau_1$, the controller proportional term shows bad performance and we set k_C=0. When $\alpha \le \tau_1\theta/(\tau_1+\theta)$, we apply the SIMC model reduction rules.

When τ_2 is small in the above proposed tuning rules, k_C of Eq. (19) becomes too small and results in sluggish responses. To avoid this, the half-rule for the SIMC method is first applied as

$$G(s) = \frac{k(\alpha s+1)\exp(-\theta s)}{(\tau_1 s+1)(\tau_2 s+1)} \approx \frac{k(\alpha s+1)\exp(-(\theta+\tau_2/2)s)}{(\tau_1+\tau_2/2)s+1} \quad (25)$$

when $\tau_2 > \dfrac{2\tau_1\theta}{4\tau_1+\theta}$. Then the tuning rule of Eq. (24) is applied. The τ_2 condition is such that tuning rules of Eqs. (19) and (24) provide the same k_C.

[Small α Case]

Consider a SOPTD process (α=0 in Eq. (17)),

$$G(s) = \frac{k\exp(-\theta s)}{(\tau_1 s+1)(\tau_2 s+1)} \quad (26)$$

With 0/1 Pade approximation of the time delay term, the characteristic equation is $1+\left(k_C+\dfrac{k_I}{s}\right)\dfrac{k(-\theta s+1)}{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1}=0$ and, applying the Routh stability theorem, we have

$$k\bar{k}_C = \frac{\tau_1+\tau_2}{\theta}, \quad k\bar{k}_I = 1/\left(\theta+\frac{\tau_1\tau_2}{\tau_1+\tau_2}\right) \quad (27)$$

Because the controller zero is $\bar{k}_C/\bar{k}_I \ge (\tau_1+\tau_2/2)$, we limit this to be equal to $\tau_1+\tau_2/2$ (τ_1 is the dominant time constant)

$$k_I = \frac{1}{k\nu\left(\theta+\dfrac{\tau_1\tau_2}{\tau_1+\tau_2}\right)}, \quad k_C = (\tau_1+\tau_2/2)k_I \quad (28)$$

When $\alpha \le \dfrac{\tau_1\tau_2\theta}{\tau_1+\tau_2+\theta}$, this tuning rule is used for the process of Eq. (15) with the SIMC model reduction as

$$G(s) = \frac{k(\alpha s+1)\exp(-\theta s)}{(\tau_1 s+1)(\tau_2 s+1)} \approx \frac{k\exp(-\theta s)}{(\tau_1 s+1)((\tau_2-\alpha)s+1)} \quad (29)$$

[Small θ Case]

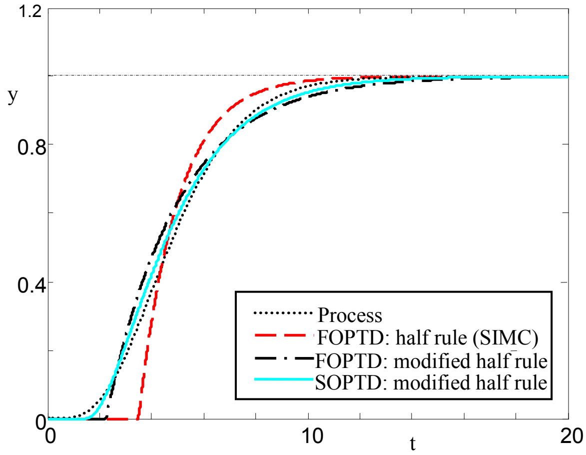

Fig. 3. Open-loop step responses for $G(s)=1/(s+1)^5$ and its reduced order models.

When SIMC model reduction yields an integrating process, the SIMC method is used. This case can occur for lag dominant processes with very small θ.

[Modification of the Half -Rule]

The above tuning rule can be applied to reduced order models for higher order processes. For this, the SIMC model reduction rules and those in Lee et al. (2014) can be used. Here, the half-rule of SIMC is slightly modified. We apply the half-rule sequentially from the smallest time constant. For example, for the 5th order process, the original half-rule is

$$G(s) = \frac{1}{(s+1)^5} \approx \frac{\exp(-3.5s)}{1.5s+1} \quad (30)$$

On the other hand, applying the half-order rule sequentially, we obtain

$$\begin{aligned} G(s) &= \frac{1}{(s+1)^5} \approx \frac{\exp(-0.5s)}{(s+1)^3(1.5s+1)} \\ &\approx \frac{\exp(-s)}{(1.5s+1)^2(s+1)} \approx \frac{\exp(-1.5s)}{(1.5s+1)(2s+1)} \end{aligned} \quad (31)$$

The approximate FOPTD model becomes

$$G(s) \approx \frac{\exp(-1.5s)}{(1.5s+1)(2s+1)} \approx \frac{\exp(-2.25s)}{2.75s+1} \quad (32)$$

Figure 3 shows the step responses of models for $G(s)=1/(s+1)^5$. Integral of absolute errors (IAE) values are 0.7023, 0.4934 and 0.2499 for models of the half-rule (Eq. (34)), the modified half-rule (Eq. (36)) and the second order one (Eq. (35)), respectively. The IAE of the modified half-rule is 0.7 times the half rule value. We can see that the half rule of SIMC method provides a somewhat large time delay and can result in sluggish tuning.

6. SIMPLIFIED SM (SM^S) METHOD

Oscillatory responses occur when

$$\tau_1 > \alpha > \tau_2 > \frac{2\tau_1\theta}{4\tau_1+\theta},\ \tau_1\tau_2 < \alpha^2 \tag{33}$$

or

$$\tau_1 > \alpha > \tau_2 > \frac{2\tau_1\theta}{4\tau_1+\theta},\ \alpha > 1.6\tau_2 \tag{34}$$

The proposed SM method is effective in this case. However, the tuning rule of Eq. (18) for k_I is quite complicated. To avoid this, k_I can be replaced by the SIMC method while k_C is computed by Eq. (19). With slight degradation of performances, this simplified method can avoid complicated equations for k_I. Tables 1 and 2 summarize the proposed SM and SM^S methods.

7. EXAMPLES

Example 1: Consider the process with inverse response

$$G(s) = \frac{(2s+1)(-0.5s+1)}{(5s+1)(0.2s+1)^3} \tag{35}$$

The SIMC method uses the reduced order model

$$G(s) = \frac{(2s+1)(-0.5s+1)}{(5s+1)(0.2s+1)^3} \approx \frac{\exp(-s)}{3.1s+1} \tag{36}$$

The proposed method performs the model reduction as

$$G(s) = \frac{(2s+1)(-0.5s+1)}{(5s+1)(0.2s+1)^3} \approx \frac{(2s+1)\exp(-0.7s)}{(5s+1)(0.4s+1)} \tag{37}$$

Figure 4 shows closed-loop responses. The SIMC method shows oscillatory closed-loop responses. Such troublesome responses are removed in the proposed SM and SM^S methods.

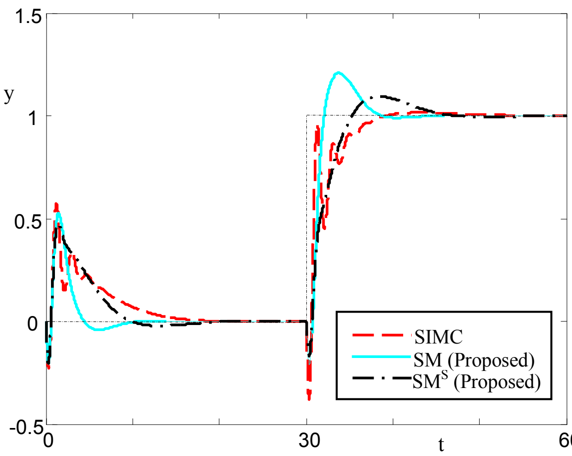

Fig. 4. Closed-loop responses for the Example 1 process.

Example 2: Consider the process

$$G(s) = \frac{0.5s+1}{(s+1)^2(0.1s+1)} \tag{38}$$

Assuming that the effective dead time is less than 0.1, the SIMC method develops the reduced order model

$$G(s) = \frac{0.5s+1}{(s+1)^2(0.1s+1)} = \frac{0.5s+1}{s+1}\frac{1}{(s+1)(0.1s+1)} \approx 0.5\frac{\exp(-0.05s)}{1.05s+1} \tag{39}$$

If one assumes that the effective dead time is greater than 0.2, the SIMC method uses the reduced order model

$$G(s) = \frac{0.5s+1}{(s+1)^2(0.1s+1)} = \frac{0.5s+1}{s+1}\frac{1}{(s+1)(0.1s+1)} \approx \frac{1}{(0.5s+1)(s+1)(0.1s+1)} \approx \frac{\exp(-0.35s)}{1.25s+1} \tag{40}$$

The proposed SM method uses

$$G(s) = \frac{0.5s+1}{(s+1)^2(0.1s+1)} \approx \frac{(0.5s+1)\exp(-0.05s)}{(s+1)(1.05s+1)} \tag{41}$$

The SM^S method obtains by applying the SIMC model reduction rules to Eq. (41),

$$G(s) \approx \frac{(0.5s+1)\exp(-0.05s)}{(1.05s+1)(s+1)} \approx \frac{\exp(-0.05s)}{(1.05s+1)(0.5s+1)} \tag{42}$$

Figure 5 shows closed-loop responses. The SIMC method based on Eq. (39) provides PI control parameters showing responses with large overshoot and poor robustness (the peak amplitude ratio of sensitivity function (Ms) is 3.2). The SIMC method based on Eq. (40) shows slower closed-loop responses. The proposed SM method increases the control speed and the proposed SM^S method shows closed-loop responses similar to the slow SIMC method.

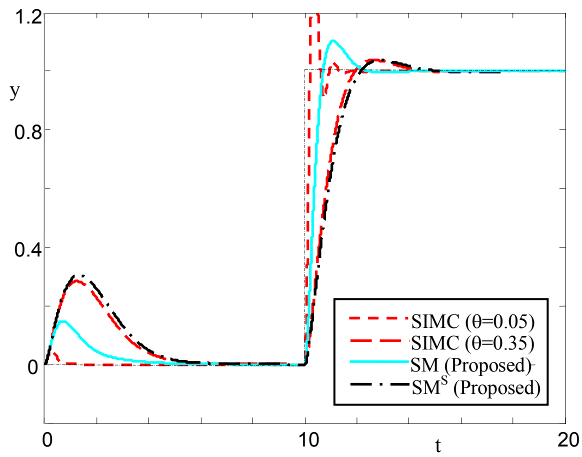

Fig. 5. Closed-loop responses for the Example 2 process.

Example 3 (Tyreus Stabilizer): Consider the 1x1 element of Tyreus Stabilizer process of Eq. (4). The SIMC method reduces the process to FOPTD model as in Eq. (5). The LCE method (Lee et al., 2014) uses

$$G(s)=\frac{(10s+1)\exp(-0.1s)}{(4s+1)^3}\approx\frac{2.2618\ \exp(-2.1s)}{6s+1} \tag{43}$$

The proposed methods use the following reduced models; for the SM method,

$$G(s)=\frac{(10s+1)\exp(-0.1s)}{(4s+1)^3}\approx\frac{(10s+1)\exp(-2.1s)}{(4s+1)(6s+1)} \tag{44}$$

and, for the SM^S method,

$$G(s)\approx\frac{(10s+1)\exp(-2.1s)}{(6s+1)(4s+1)}\approx\frac{10}{6}\frac{\exp(-2.1s)}{4s+1} \tag{45}$$

Figure 6 shows closed-loop responses. The SIMC and LCE methods show slower closed-loop responses and their control speed is increased by the proposed SM and SM^S methods.

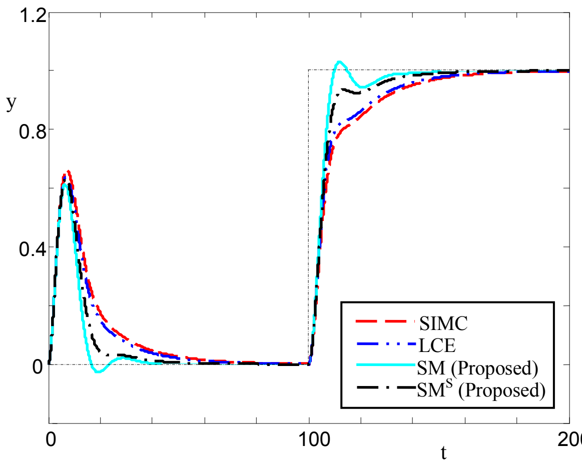

Fig. 6. Closed-loop responses for the Example 3 process.

8. CONCLUSIONS

PI controller design method based on the stability margin property of SIMC method is proposed. The proportional and integral gains are obtained from the ultimate gains of the process G(s) and G(s)/s, respectively. Because such ultimate gains can be obtained by the relay feedback oscillations, the method can be applied to auto-tune PI controllers with relay feedback experiments. Applying the method to second order plus time delay models, $G(s)=\frac{k(\alpha s+1)\exp(-\theta s)}{(\tau_1 s+1)(\tau_2 s+1)}$, analytic tuning rules have been obtained. To obtain the SOPTD model, the half-rule of SIMC method is applied sequentially. This slight modification of the half-rule shows better approximations for some processes. Compared to the FOPTD model, our SOPTD model can provide better approximations, resulting PI controllers with consistent closed-loop performances. The proposed method can remove some flaws in the SIMC method, enhancing its usefulness.

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