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Data filtering based parameter estimation algorithms for multivariable Box-Jenkins-like systems ⋆

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Abstract: This paper proposes an auxiliary model based hierarchical least squares algorithm for multivariable Box-Jenkins-like systems using the hierarchical identification principle. To improve the computational efficiency, a multivariable system is decomposed into two subsystems by using the data filtering technique. Furthermore, this paper presents a data filtering based auxiliary model hierarchical least squares algorithm for multivariable Box-Jenkins-like systems. The simulation example shows that the proposed identification algorithms are effective.

Keywords: Least squares, Data filtering, Parameter estimation, Hierarchical identification, Multivariable system

1. INTRODUCTION

In the areas of the system identification, system control and analysis, the study of optimization (Castillo & Melin, 2012), prediction (Markopoulos, Karystinos, & Pados, 2014; Zhang, 2011) and parameter estimation (Ding & Lin, 2014; Vörös, 2010) has become a hotspot path for the past decades. Multivariable systems widely exist in industrial process control (Karacan, Hapoğlu, & Alpbaz, 2007), system control (Liu, Wang, & Wu, 2011; Owens, Freeman, & Bing, 2013; Zhang, Shi, & Saadat, 2011), complex networks and so on. The identification of multivariable systems has received many control scientists' attention and many identification methods have been reported. Martin (2013) presented a optimal level-crossing prediction for the jump linear multi–input and multi–output dynamical systems; Ding (2013) studied the coupled-least-squares identification algorithm for multivariable systems.

The data filtering technique as a new method has been commonly involved in the fields of signal processing (Qaisar, Fesquet, & Renaudin, 2014; Yu, Shi, & Huang, 2008;), communication (Hu & Dong, 2012), and system identification (Gaspar & Oliveira, 2014; Korbel, et al., 2014). Hung and Ba (2014) studied the sensor management for multi-target tracking via multi-Bernoulli filtering. Wang, Ding, and Zhu (2013) presented a hierarchical least squares algorithm by means of the filtering technique for multivariable CARAR-like systems which revealed faster convergence and higher estimation capability compared with the implementation of hierarchical least squares algorithm; Xie, Yang, and Ding (2011) proposed the data filtering based RLS parameter estimation method for non-uniformly sampled systems to achieve better computational efficiency.

The hierarchical identification principle is based on the decomposition technique and can deal with parameter estimation problems for linear or nonlinear multivariable systems (Ding & Chen, 2005a,b). Recently, a hierarchical least squares algorithm based on the data filtering technique was developed for multivariable equation error systems using the hierarchical identification principle (Wang, Ding, & Zhu, 2013). Based on the work in Wang, Ding, and Zhu (2013), this paper presents an auxiliary model based hierarchical least squares (AM-HLS) algorithm and a data filtering based auxiliary model hierarchical least squares (F-AM-HLS) algorithm for the multivariable Box-Jenkins-like systems using the hierarchical identification principle.

The rest of this paper is organized as follows. Section 2 simply presents an AM-HLS algorithm for multivariable Box-Jenkins-like systems. Section 3 employs the data filtering technique and presents the F-AM-HLS identification algorithm. Section 4 gives an example for the AM-HLS and the F-AM-HLS algorithms. Finally, the concluding remarks are given in Section 5.

2. THE AUXILIARY MODEL BASED HIERARCHICAL LEAST SQUARES ALGORITHM

Consider a multivariable Box-Jenkins-like system,

$$\boldsymbol{y}(t)=\boldsymbol{x}(t)+\boldsymbol{w}(t), \tag{1}$$

$$\boldsymbol{x}(t)=-\sum_{i=1}^{n}\alpha_i\boldsymbol{x}(t-i)+\sum_{i=1}^{n}\boldsymbol{Q}_i\boldsymbol{u}(t-i), \tag{2}$$

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$$\boldsymbol{w}(t)=-\sum_{i=1}^{n_c}c_i\boldsymbol{w}(t-i)+\sum_{i=1}^{n_d}d_i\boldsymbol{v}(t-i)+\boldsymbol{v}(t). \quad (3)$$

Define the parameter vectors $\boldsymbol{\vartheta}_{\rm s}$, $\boldsymbol{\vartheta}_{\rm n}$ and $\boldsymbol{\vartheta}$, the parameter matrix $\boldsymbol{\theta}$, and the information vectors $\boldsymbol{\psi}_{\rm s}(t)$, $\boldsymbol{\psi}_{\rm n}(t)$, $\boldsymbol{\psi}(t)$ and $\boldsymbol{\varphi}(t)$ as

$$\begin{aligned}
\boldsymbol{\vartheta}_{\rm s}&:=[\alpha_1,\alpha_2,\cdots,\alpha_n]^{\rm T}\in\mathbb{R}^n,\\
\boldsymbol{\vartheta}_{\rm n}&:=[c_1,c_2,\cdots,c_{n_c},d_1,d_2,\cdots,d_{n_d}]^{\rm T}\in\mathbb{R}^{n_c+n_d},\\
\boldsymbol{\vartheta}&:=\begin{bmatrix}\boldsymbol{\vartheta}_{\rm s}\\ \boldsymbol{\vartheta}_{\rm n}\end{bmatrix}\in\mathbb{R}^{n+n_c+n_d},\\
\boldsymbol{\theta}^{\rm T}&:=[\boldsymbol{Q}_1,\boldsymbol{Q}_2,\cdots,\boldsymbol{Q}_n]\in\mathbb{R}^{m\times(nr)},\\
\boldsymbol{\psi}_{\rm s}(t)&:=[\boldsymbol{x}(t-1),\boldsymbol{x}(t-2),\cdots,\boldsymbol{x}(t-n)]\in\mathbb{R}^{m\times n},\\
\boldsymbol{\psi}_{\rm n}(t)&:=[\boldsymbol{w}(t-1),\boldsymbol{w}(t-2),\cdots,\boldsymbol{w}(t-n_c),-\boldsymbol{v}(t-1),\\
&\quad -\boldsymbol{v}(t-2),\cdots,-\boldsymbol{v}(t-n_d)]\in\mathbb{R}^{m\times(n_c+n_d)},\\
\boldsymbol{\psi}(t)&:=[\boldsymbol{\psi}_{\rm s}(t),\boldsymbol{\psi}_{\rm n}(t)]\in\mathbb{R}^{m\times(n+n_c+n_d)},\\
\boldsymbol{\varphi}(t)&:=[\boldsymbol{u}^{\rm T}(t-1),\boldsymbol{u}^{\rm T}(t-2),\cdots,\boldsymbol{u}^{\rm T}(t-n)]^{\rm T}\in\mathbb{R}^{nr}.
\end{aligned}$$

From the above definitions, Equation (1) can be rewritten as

$$\begin{aligned}
\boldsymbol{y}(t)&=-\boldsymbol{\psi}_{\rm s}(t)\boldsymbol{\vartheta}_{\rm s}+\boldsymbol{\theta}^{\rm T}\boldsymbol{\varphi}(t)-\boldsymbol{\psi}_{\rm n}(t)\boldsymbol{\vartheta}_{\rm n}+\boldsymbol{v}(t)\\
&=-\boldsymbol{\psi}(t)\boldsymbol{\vartheta}+\boldsymbol{\theta}^{\rm T}\boldsymbol{\varphi}(t)+\boldsymbol{v}(t). \qquad (4)
\end{aligned}$$

Equation (4) is the identification model of the multivariable Box-Jenkins-like system, which contains both the unknown parameter vector $\boldsymbol{\vartheta}$ and the unknown parameter matrix $\boldsymbol{\theta}$. Define the quadratic cost function:

$$J(\boldsymbol{\vartheta},\boldsymbol{\theta}):=\sum_{j=1}^{t}\|\boldsymbol{y}(j)+\boldsymbol{\psi}(j)\boldsymbol{\vartheta}-\boldsymbol{\theta}^{\rm T}\boldsymbol{\varphi}(j)\|^2.$$

Let $\hat{\boldsymbol{\vartheta}}(t):=\begin{bmatrix}\hat{\boldsymbol{\vartheta}}_{\rm s}(t)\\ \hat{\boldsymbol{\vartheta}}_{\rm n}(t)\end{bmatrix}$ be the estimate of $\boldsymbol{\vartheta}$, and $\hat{\boldsymbol{\theta}}(t)$ be the estimate of $\boldsymbol{\theta}$. Referring to the method in Ding and Chen (2005a), minimizing the cost function $J(\boldsymbol{\vartheta},\boldsymbol{\theta})$, and replacing the unknown variables $\boldsymbol{\psi}(t)$, $\boldsymbol{x}(t)$, $\boldsymbol{w}(t)$ and $\boldsymbol{v}(t)$ with their estimates, we can obain the following auxiliary model based hierarchical least squares (AM-HLS) algorithm:

$$\begin{aligned}
\hat{\boldsymbol{\vartheta}}(t)&=\hat{\boldsymbol{\vartheta}}(t-1)+\boldsymbol{L}_1(t)[\boldsymbol{y}(t)\\
&\quad+\hat{\boldsymbol{\psi}}(t)\hat{\boldsymbol{\vartheta}}(t-1)-\hat{\boldsymbol{\theta}}^{\rm T}(t-1)\boldsymbol{\varphi}(t)], \qquad (5)\\
\boldsymbol{L}_1(t)&=-\boldsymbol{P}_1(t-1)\hat{\boldsymbol{\psi}}^{\rm T}(t)\\
&\quad[\boldsymbol{I}+\hat{\boldsymbol{\psi}}(t)\boldsymbol{P}_1(t-1)\hat{\boldsymbol{\psi}}^{\rm T}(t)]^{-1}, \qquad (6)\\
\boldsymbol{P}_1(t)&=[\boldsymbol{I}+\boldsymbol{L}_1(t)\hat{\boldsymbol{\psi}}(t)]\boldsymbol{P}_1(t-1), \qquad (7)\\
\hat{\boldsymbol{\theta}}(t)&=\hat{\boldsymbol{\theta}}(t-1)+\boldsymbol{L}_2(t)[\boldsymbol{y}(t)\\
&\quad+\hat{\boldsymbol{\psi}}(t)\hat{\boldsymbol{\vartheta}}(t-1)-\hat{\boldsymbol{\theta}}^{\rm T}(t-1)\boldsymbol{\varphi}(t)]^{\rm T}, \qquad (8)\\
\boldsymbol{L}_2(t)&=\boldsymbol{P}_2(t-1)\boldsymbol{\varphi}(t)[1+\boldsymbol{\varphi}^{\rm T}(t)\boldsymbol{P}_2(t-1)\boldsymbol{\varphi}(t)]^{-1}, \qquad (9)\\
\boldsymbol{P}_2(t)&=[\boldsymbol{I}-\boldsymbol{L}_2(t)\boldsymbol{\varphi}^{\rm T}(t)]\boldsymbol{P}_2(t-1), \qquad (10)\\
\boldsymbol{\varphi}(t)&=[\boldsymbol{u}^{\rm T}(t-1),\boldsymbol{u}^{\rm T}(t-2),\cdots,\boldsymbol{u}^{\rm T}(t-n)]^{\rm T}, \qquad (11)\\
\hat{\boldsymbol{\psi}}(t)&=[\hat{\boldsymbol{\psi}}_{\rm s}(t),\hat{\boldsymbol{\psi}}_{\rm n}(t)], \qquad (12)\\
\hat{\boldsymbol{\psi}}_{\rm s}(t)&=[\hat{\boldsymbol{x}}(t-1),\hat{\boldsymbol{x}}(t-2),\cdots,\hat{\boldsymbol{x}}(t-n)], \qquad (13)\\
\hat{\boldsymbol{\psi}}_{\rm n}(t)&=[\hat{\boldsymbol{w}}(t-1),\hat{\boldsymbol{w}}(t-2),\cdots,\hat{\boldsymbol{w}}(t-n_c),\\
&\quad-\hat{\boldsymbol{v}}(t-1),-\hat{\boldsymbol{v}}(t-2),\cdots,-\hat{\boldsymbol{v}}(t-n_d)], \qquad (14)\\
\hat{\boldsymbol{x}}(t)&=-\hat{\boldsymbol{\psi}}_{\rm s}(t)\hat{\boldsymbol{\vartheta}}_{\rm s}(t)+\hat{\boldsymbol{\theta}}^{\rm T}(t)\boldsymbol{\varphi}(t), \qquad (15)\\
\hat{\boldsymbol{w}}(t)&=\boldsymbol{y}(t)-\hat{\boldsymbol{x}}(t), \qquad (16)\\
\hat{\boldsymbol{v}}(t)&=\boldsymbol{y}(t)+\hat{\boldsymbol{\psi}}(t)\hat{\boldsymbol{\vartheta}}(t)-\hat{\boldsymbol{\theta}}^{\rm T}(t)\boldsymbol{\varphi}(t). \qquad (17)
\end{aligned}$$

To initialize the AM-HLS algorithm, we let $\hat{\boldsymbol{\vartheta}}(0)=\mathbf{1}_{n+n_c+n_d}/p_0$, $\hat{\boldsymbol{\theta}}(0)=\mathbf{1}_n/p_0$, $\boldsymbol{P}_1(0)=p_0\boldsymbol{I}$, $\boldsymbol{P}_2(0)=p_0\boldsymbol{I}$, $\hat{\boldsymbol{x}}(i)=\mathbf{1}_m/p_0$, $\hat{\boldsymbol{w}}(i)=\mathbf{1}_m/p_0$, $\hat{\boldsymbol{v}}(i)=\mathbf{1}_m/p_0$, $\boldsymbol{u}(i)=\mathbf{0}$, $\boldsymbol{y}(i)=\mathbf{0}$, $i\leqslant 0$, $p_0=10^6$.

3. THE FILTERING BASED HIERARCHIAL LEAST SQUARES ALGORITHM

Define the filtered input vector $\boldsymbol{u}_{\rm f}(t)$ and the filtered output vector $\boldsymbol{y}_{\rm f}(t)$ as

$$\begin{aligned}
\boldsymbol{u}_{\rm f}(t)&=-\sum_{i=1}^{n_d}d_i\boldsymbol{u}_{\rm f}(t-i)+\boldsymbol{u}(t)+\sum_{i=1}^{n_c}c_i\boldsymbol{u}(t-i),\\
\boldsymbol{y}_{\rm f}(t)&=-\sum_{i=1}^{n_d}d_i\boldsymbol{y}_{\rm f}(t-i)+\boldsymbol{y}(t)+\sum_{i=1}^{n_c}c_i\boldsymbol{y}(t-i).
\end{aligned}$$

Then, we obtain the filtered system,

$$\boldsymbol{y}_{\rm f}(t)=\boldsymbol{x}_{\rm f}(t)+\boldsymbol{v}(t), \qquad (18)$$

where

$$\boldsymbol{x}_{\rm f}(t)=-\sum_{i=1}^{n}\alpha_i\boldsymbol{x}_{\rm f}(t-i)+\sum_{i=1}^{n}\boldsymbol{Q}_i\boldsymbol{u}_{\rm f}(t-i).$$

Define the filtered information vectors:

$$\begin{aligned}
\boldsymbol{\psi}_{\rm f}(t)&:=[\boldsymbol{x}_{\rm f}(t-1),\boldsymbol{x}_{\rm f}(t-2),\cdots,\boldsymbol{x}_{\rm f}(t-n)]\in\mathbb{R}^{m\times n},\\
\boldsymbol{\varphi}_{\rm f}(t)&:=[\boldsymbol{u}_{\rm f}^{\rm T}(t-1),\boldsymbol{u}_{\rm f}^{\rm T}(t-2),\cdots,\boldsymbol{u}_{\rm f}^{\rm T}(t-n)]^{\rm T}\in\mathbb{R}^{nr}.
\end{aligned}$$

Equation (18) can be described as

$$\boldsymbol{y}_{\rm f}(t)+\boldsymbol{\psi}_{\rm f}(t)\boldsymbol{\vartheta}_{\rm s}=\boldsymbol{\theta}^{\rm T}\boldsymbol{\varphi}_{\rm f}(t)+\boldsymbol{v}(t), \qquad (19)$$

Equation (3) can be written as

$$\boldsymbol{w}(t)=-\boldsymbol{\psi}_{\rm n}(t)\boldsymbol{\vartheta}_{\rm n}+\boldsymbol{v}(t). \qquad (20)$$

According to Equations (19) and (20), define two quadratic criterion functions:

$$\begin{aligned}
J_1(\boldsymbol{\vartheta}_{\rm s},\boldsymbol{\theta})&:=\sum_{j=1}^{t}\|\boldsymbol{y}_{\rm f}(j)+\boldsymbol{\psi}_{\rm f}(j)\boldsymbol{\vartheta}_{\rm s}-\boldsymbol{\theta}^{\rm T}\boldsymbol{\varphi}_{\rm f}(j)\|^2,\\
J_2(\boldsymbol{\vartheta}_{\rm n})&:=\sum_{j=1}^{t}\|\boldsymbol{w}(j)+\boldsymbol{\psi}_{\rm n}(j)\boldsymbol{\vartheta}_{\rm n}\|^2.
\end{aligned}$$

Referring to the auxiliary model identification idea, replacing the unknown variables with their estimates, and minimizing the cost functions $J_1(\boldsymbol{\vartheta}_{\rm s},\boldsymbol{\theta})$ and $J_2(\boldsymbol{\vartheta}_{\rm n})$ with respect to $\boldsymbol{\vartheta}_{\rm s}$, $\boldsymbol{\theta}$ and $\boldsymbol{\vartheta}_{\rm n}$ yield the following data filtering based auxiliary model hierarchical least squares (F-AM-HLS) algorithm:

$$\begin{aligned}
\hat{\boldsymbol{\vartheta}}_{\rm s}(t)&=\hat{\boldsymbol{\vartheta}}_{\rm s}(t-1)+\boldsymbol{L}_3(t)[\hat{\boldsymbol{y}}_{\rm f}(t)\\
&\quad+\hat{\boldsymbol{\psi}}_{\rm f}(t)\hat{\boldsymbol{\vartheta}}_{\rm s}(t-1)-\hat{\boldsymbol{\theta}}^{\rm T}(t-1)\hat{\boldsymbol{\varphi}}_{\rm f}(t)], \qquad (21)
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{L}_3(t) &= -\boldsymbol{P}_3(t-1)\hat{\boldsymbol{\psi}}_{\rm f}^{\rm T}(t)\\
&\qquad [\boldsymbol{I}+\hat{\boldsymbol{\psi}}_{\rm f}(t)\boldsymbol{P}_3(t-1)\hat{\boldsymbol{\psi}}_{\rm f}^{\rm T}(t)]^{-1}, \qquad (22)\\
\boldsymbol{P}_3(t) &= [\boldsymbol{I}+\boldsymbol{L}_3(t)\hat{\boldsymbol{\psi}}_{\rm f}(t)]\boldsymbol{P}_3(t-1), \qquad (23)\\
\hat{\boldsymbol{\theta}}(t) &= \hat{\boldsymbol{\theta}}(t-1)+\boldsymbol{L}_4(t)[\hat{\boldsymbol{y}}_{\rm f}(t)\\
&\qquad +\hat{\boldsymbol{\psi}}_{\rm f}(t)\hat{\boldsymbol{\vartheta}}_{\rm s}(t-1)-\hat{\boldsymbol{\theta}}^{\rm T}(t-1)\hat{\boldsymbol{\varphi}}_{\rm f}(t)]^{\rm T}, \qquad (24)\\
\boldsymbol{L}_4(t) &= \boldsymbol{P}_4(t-1)\hat{\boldsymbol{\varphi}}_{\rm f}(t)[1+\hat{\boldsymbol{\varphi}}_{\rm f}^{\rm T}(t)\boldsymbol{P}_4(t-1)\hat{\boldsymbol{\varphi}}_{\rm f}(t)]^{-1}, \qquad (25)\\
\boldsymbol{P}_4(t) &= [\boldsymbol{I}-\boldsymbol{L}_4(t)\hat{\boldsymbol{\varphi}}_{\rm f}^{\rm T}(t)]\boldsymbol{P}_4(t-1), \qquad (26)\\
\hat{\boldsymbol{\varphi}}_{\rm f}(t) &= [\hat{\boldsymbol{u}}_{\rm f}^{\rm T}(t-1),\hat{\boldsymbol{u}}_{\rm f}^{\rm T}(t-2),\cdots,\hat{\boldsymbol{u}}_{\rm f}^{\rm T}(t-n)]^{\rm T}, \qquad (27)\\
\hat{\boldsymbol{\psi}}_{\rm f}(t) &= [\hat{\boldsymbol{x}}_{\rm f}(t-1),\hat{\boldsymbol{x}}_{\rm f}(t-2),\cdots,\hat{\boldsymbol{x}}_{\rm f}(t-n)], \qquad (28)\\
\hat{\boldsymbol{u}}_{\rm f}(t) &= -\hat{d}_1(t)\hat{\boldsymbol{u}}_{\rm f}(t-1)-\hat{d}_2(t)\hat{\boldsymbol{u}}_{\rm f}(t-2)-\cdots\\
&\qquad -\hat{d}_{n_d}(t)\hat{\boldsymbol{u}}_{\rm f}(t-n_d)+\boldsymbol{u}(t)+\hat{c}_1(t)\boldsymbol{u}(t-1)\\
&\qquad +\hat{c}_2(t)\boldsymbol{u}(t-2)+\cdots+\hat{c}_{n_c}(t)\boldsymbol{u}(t-n_c), \qquad (29)\\
\hat{\boldsymbol{y}}_{\rm f}(t) &= -\hat{d}_1(t)\hat{\boldsymbol{y}}_{\rm f}(t-1)-\hat{d}_2(t)\hat{\boldsymbol{y}}_{\rm f}(t-2)-\cdots\\
&\qquad -\hat{d}_{n_d}(t)\hat{\boldsymbol{y}}_{\rm f}(t-n_d)+\boldsymbol{y}(t)+\hat{c}_1(t)\boldsymbol{y}(t-1)\\
&\qquad +\hat{c}_2(t)\boldsymbol{y}(t-2)+\cdots+\hat{c}_{n_c}(t)\boldsymbol{y}(t-n_c), \qquad (30)\\
\hat{\boldsymbol{x}}_{\rm f}(t) &= -\hat{\boldsymbol{\psi}}_{\rm f}(t)\hat{\boldsymbol{\vartheta}}_{\rm s}(t)+\hat{\boldsymbol{\theta}}^{\rm T}(t)\hat{\boldsymbol{\varphi}}_{\rm f}(t), \qquad (31)\\
\hat{\boldsymbol{\vartheta}}_{\rm n}(t) &= \hat{\boldsymbol{\vartheta}}_{\rm n}(t-1)+\boldsymbol{L}_5(t)[\hat{\boldsymbol{w}}(t)+\hat{\boldsymbol{\psi}}_{\rm n}(t)\hat{\boldsymbol{\vartheta}}_{\rm n}(t-1)], \qquad (32)\\
\boldsymbol{L}_5(t) &= -\boldsymbol{P}_5(t-1)\hat{\boldsymbol{\psi}}_{\rm n}^{\rm T}(t)\\
&\qquad [\boldsymbol{I}+\hat{\boldsymbol{\psi}}_{\rm n}(t)\boldsymbol{P}_5(t-1)\hat{\boldsymbol{\psi}}_{\rm n}^{\rm T}(t)]^{-1}, \qquad (33)\\
\boldsymbol{P}_5(t) &= [\boldsymbol{I}+\boldsymbol{L}_5(t)\hat{\boldsymbol{\psi}}_{\rm n}(t)]\boldsymbol{P}_5(t-1), \qquad (34)\\
\boldsymbol{\varphi}(t) &= [\boldsymbol{u}^{\rm T}(t-1),\boldsymbol{u}^{\rm T}(t-2),\cdots,\boldsymbol{u}^{\rm T}(t-n)]^{\rm T}, \qquad (35)\\
\hat{\boldsymbol{\psi}}(t) &= [\hat{\boldsymbol{\psi}}_{\rm s}(t),\hat{\boldsymbol{\psi}}_{\rm n}(t)], \qquad (36)\\
\hat{\boldsymbol{\psi}}_{\rm s}(t) &= [\hat{\boldsymbol{x}}(t-1),\hat{\boldsymbol{x}}(t-2),\cdots,\hat{\boldsymbol{x}}(t-n)], \qquad (37)\\
\hat{\boldsymbol{\psi}}_{\rm n}(t) &= [\hat{\boldsymbol{w}}(t-1),\hat{\boldsymbol{w}}(t-2),\cdots,\hat{\boldsymbol{w}}(t-n_c),\\
&\qquad -\hat{\boldsymbol{v}}(t-1),-\hat{\boldsymbol{v}}(t-2),\cdots,-\hat{\boldsymbol{v}}(t-n_d)], \qquad (38)\\
\hat{\boldsymbol{x}}(t) &= -\hat{\boldsymbol{\psi}}_{\rm s}(t)\hat{\boldsymbol{\vartheta}}_{\rm s}(t-1)+\hat{\boldsymbol{\theta}}^{\rm T}(t-1)\boldsymbol{\varphi}(t), \qquad (39)\\
\hat{\boldsymbol{w}}(t) &= \boldsymbol{y}(t)-\hat{\boldsymbol{x}}(t), \qquad (40)\\
\hat{\boldsymbol{v}}(t) &= \boldsymbol{y}(t)+\hat{\boldsymbol{\psi}}(t)\hat{\boldsymbol{\vartheta}}(t)-\hat{\boldsymbol{\theta}}^{\rm T}(t)\boldsymbol{\varphi}(t). \qquad (41)
\end{aligned}$$

To initialize the F-AM-HLS algorithm, we let $\hat{\boldsymbol{\vartheta}}_{\rm s}(0) = \boldsymbol{1}_n/p_0$, $\hat{\boldsymbol{\vartheta}}_{\rm n}(0) = \boldsymbol{1}_{n_c+n_d}/p_0$, $\hat{\boldsymbol{\theta}}(0) = \boldsymbol{1}_{nr\times m}/p_0$, $\boldsymbol{P}_3(0) = p_0\boldsymbol{I}$, $\boldsymbol{P}_4(0) = p_0\boldsymbol{I}$, $\boldsymbol{P}_5(0) = p_0\boldsymbol{I}$, $\hat{\boldsymbol{u}}_{\rm f}(i) = \boldsymbol{1}_m/p_0$, $\hat{\boldsymbol{y}}_{\rm f}(i) = \boldsymbol{1}_m/p_0$, $\hat{\boldsymbol{x}}_{\rm f}(i) = \boldsymbol{1}_m/p_0$, $\hat{\boldsymbol{x}}(i) = \boldsymbol{1}_m/p_0$, $\hat{\boldsymbol{w}}(i) = \boldsymbol{1}_m/p_0$, $\hat{\boldsymbol{v}}(i) = \boldsymbol{1}_m/p_0$, $\boldsymbol{u}(i) = \boldsymbol{0}$, $\boldsymbol{y}(i) = \boldsymbol{0}$, $i \leqslant 0$, $p_0 = 10^6$.

4. EXAMPLE

Consider a two-input two-output Box-Jenkins-like system,

$$\begin{aligned}
\boldsymbol{y}(t) &= \boldsymbol{x}(t)+\boldsymbol{w}(t),\\
\boldsymbol{x}(t) &= -\sum_{i=1}^{n}\alpha_i\boldsymbol{x}(t-i)+\sum_{i=1}^{n}\boldsymbol{Q}_i\boldsymbol{u}(t-i),\\
\boldsymbol{w}(t) &= -\sum_{i=1}^{n_c}c_i\boldsymbol{w}(t-i)+\sum_{i=1}^{n_d}d_i\boldsymbol{v}(t-i)+\boldsymbol{v}(t),\\
\boldsymbol{\vartheta} &= [\alpha_1,c_1,d_1]^{\rm T} = [0.13, 0.16, -0.46]^{\rm T},\\
\boldsymbol{\theta}^{\rm T} &= \boldsymbol{Q}_1 = \begin{bmatrix} 1.24 & 1.11\\ 1.03 & 1.20\end{bmatrix}.
\end{aligned}$$

In simulation, the inputs $\{u_1(t)\}$ and $\{u_2(t)\}$ are taken as two persistent excitation signal sequences with zero mean and unit variances, $\{v_1(t)\}$ and $\{v_2(t)\}$ are taken as two white noise sequences with zero mean and variances σ_1^2 for $v_1(t)$ and σ_2^2 for $v_2(t)$. Applying the proposed algorithms to estimate parameters of this example system, the parameter estimates and the estimation errors are shown in Tables 1–3 and Figs. 1–2, where δ is defined the same as in Wang, Ding, and Zhu (2013).

Table 1. The AM-HLS estimates and errors ($\sigma_1^2 = \sigma_2^2 = 0.50^2$)

t	50	100	200	500	1000	True values
α_1	0.14415	0.12975	0.13642	0.14420	0.13856	0.13000
$Q_1(1,1)$	1.52012	1.34256	1.30165	1.24772	1.24702	1.24000
$Q_1(1,2)$	0.95922	1.02381	1.11341	1.10536	1.10487	1.11000
$Q_1(2,1)$	1.65917	1.29630	1.13303	1.07360	1.05612	1.03000
$Q_1(2,2)$	0.31280	0.70464	0.99308	1.11683	1.16623	1.20000
c_1	0.08208	0.06832	0.07515	0.08883	0.08659	0.16000
d_1	-0.26969	-0.27952	-0.28486	-0.32701	-0.38436	-0.46000
δ (%)	49.00329	26.06074	13.12435	7.59284	4.86609	

Table 2. The F-AM-HLS estimates and errors ($\sigma_1^2 = \sigma_2^2 = 0.50^2$)

t	50	100	200	500	1000	True values
α_1	0.12471	0.12782	0.13212	0.13501	0.13219	0.13000
$Q_1(1,1)$	1.38847	1.31352	1.29232	1.24654	1.25475	1.24000
$Q_1(1,2)$	1.25798	1.17663	1.18217	1.13205	1.11401	1.11000
$Q_1(2,1)$	0.98072	0.97199	0.97965	1.02318	1.02849	1.03000
$Q_1(2,2)$	1.26351	1.23753	1.24112	1.21108	1.21584	1.20000
c_1	0.18100	0.11150	0.07716	0.15214	0.14272	0.16000
d_1	-0.34950	-0.40997	-0.43876	-0.38981	-0.43873	-0.46000
δ (%)	10.68548	5.93759	5.93937	3.21533	1.49963	

Table 3. The F-AM-HLS estimates and errors ($\sigma_1^2 = \sigma_2^2 = 1.00^2$)

t	50	100	200	500	1000	True values
α_1	0.10016	0.11746	0.13578	0.14126	0.13651	0.13000
$Q_1(1,1)$	1.66711	1.51070	1.45511	1.31634	1.30533	1.24000
$Q_1(1,2)$	1.25307	1.09390	1.15041	1.10655	1.09389	1.11000
$Q_1(2,1)$	1.19204	1.12750	1.07653	1.08002	1.06515	1.03000
$Q_1(2,2)$	1.12427	1.10170	1.17287	1.18600	1.21235	1.20000
c_1	0.04348	-0.01819	-0.00431	0.12435	0.12441	0.16000
d_1	-0.50391	-0.55402	-0.51438	-0.41598	-0.45900	-0.46000
δ (%)	21.32663	15.54227	12.09293	4.63555	3.61654	

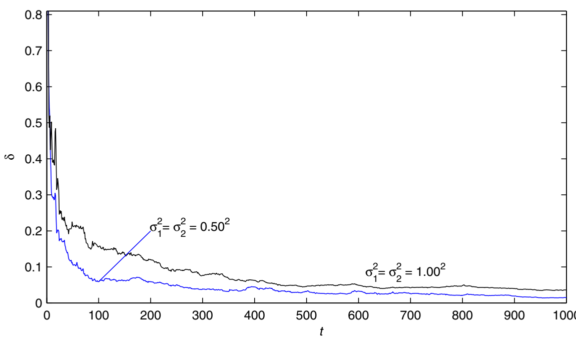

Fig. 1. The F-AM-HLS estimation errors δ versus t.

From Tables 1–3 and Figs. 1–2, we can draw the following conclusions.

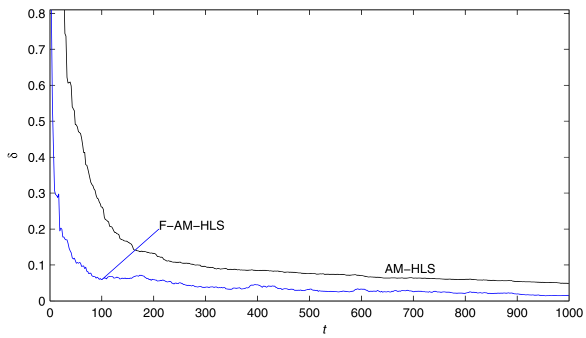

Fig. 2. The estimation errors δ versus t ($\sigma_1^2 = \sigma_2^2 = 0.50^2$).

- The parameter estimation errors become smaller (in general) with the increasing of t and the parameter estimation accuracy of the F-AM-HLS algorithm is higher than that of the AM-HLS algorithm.
- The parameter estimates given by the F-AM-HLS algorithm faster converge to their true values compared with the AM-HLS algorithm.
- For lower noise levels, the parameter estimation errors given by the F-AM-HLS algorithm become small.

5. CONCLUSIONS

In this paper, we employ the hierarchical identification principle and the data filtering technique to present an AM-HLS algorithm and an F-AM-HLS algorithm for Box-Jenkins-like systems. The F-AM-HLS algorithm can improve the computational efficiency due to filtering the input-output data of the system. But the convergence of the proposed algorithm needs further researching, and the idea can be extended to the other multivariable systems or other linear or nonlinear systems with colored noise (Li, 2013; Wang & Tang, 2014; Hu, et al., 2014).

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