Lyapunov based nonlinear control of tubular chemical reactors \star

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Abstract: This paper is concerned with the stabilization of tubular reactors in which convection, dispersion, conduction phenomena as well as chemical reaction take place. The stabilization is performed by using a Lyapunov function derived from the second law of thermodynamics called availability function. This function is used to design a stabilizing distributed control law around a stationary profile of a tubular reactor driven far from the thermodynamic equilibrium. A numerical example illustrates the proposed control strategy.

Keywords: Infinite-dimensional systems, Tubular reactors, Irreversible Thermodynamics, Lyapunov function, Distributed control

1. INTRODUCTION

The stabilization of tubular chemical reactor systems described by non linear Partial Differential Equations (PDEs) remains an open problem. In this paper we show that it is possible to use the intrinsic properties of the physical system to derive suitable control laws. For that purpose the model of the tubular reactor is derived by using local mass and energy balance equations in the context of irreversible thermodynamics. The stabilization problem can be formulated for a possibly open loop unstable stationary profile (Hlavek and Hofmann (1970)). Even if the reactor is fully actuated its control is challenging due to its distributed parameter and non linear aspects. Many studies have been dedicated to the control of tubular reactors described by partial differential equations models (Georgakis (1977); Christofides (2001); Orlov and Dochain (2001); Boskovic and Krstic (2002); Aksikas et al. (2009)).

In Ruszkowski et al. (2005), the stability analysis of the open loop tubular reactor system is derived using a Lyapunov function issued from irreversible thermodynamic considerations: thermodynamic availability (de Groot and Mazur (1962)). Its positivity and convexity are by definition directly related to the properties of the entropy function issued from the second law of thermodynamics (Callen (1985)).

In this paper we propose a generalization of the work presented in Hoang *et al.* (2012) for the stabilization of a continuous stirred tank reactor using the availability function. We consider the one dimensional model of a non isothermal tubular chemical reactor in which occur convection, diffusion, conduction and chemical reaction phenomena. These phenomena are represented by non linear constitutive equations. The distributed jacket temperature is used as the control variable. The model is established in the framework of irreversible thermodynamics by using the so called local equilibrium assumption (de Groot and Mazur (1962)).

The control presented in this paper is different from the one previously presented in Zhou et al. (2013). In Zhou et al. (2013), we have considered a multivariable approach for the control. The inlet temperature as well as the distributed jacket temperature were used as the control variables.

It is important to notice that in this work we do not study the existence of solutions associated to the control problem (such a study can be found in Laabissi et al. (2001) in the open loop case) but we focus on their properties.

The paper is organized as follows: in section 2 we present the model of the tubular reactor issued from mass and enthalpy balances. In section 3 we recall the main properties of irreversible thermodynamic systems. In section 4 the availability function is defined for infinite dimensional systems. In section 5, the distributed control using the jacket temperature as the manipulated variable is derived. We give some simulation results in section 6 and we end with some conclusions and perspectives.

2. THE MODEL OF THE REACTOR

The model of the tubular reactor is given by considering the following assumptions :

- Symmetries are assumed in the reactor so only longitudinal axis is under consideration. The spatial coordinate is $x \in [0, L]$.
- We consider the chemical reaction $\nu_A A \longrightarrow \nu_B B$ with $\nu_A = 1$, ν_B the stoichiometric coefficients.

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The reaction rate r is modeled with the Arrhenius law. $r = kc_A e^{-\frac{\alpha}{T}}$ is first order with respect to the concentration c_A of species A and with positive constant k and α .

- The convective flow has a constant velocity v.
- The total mass concentration ρ is constant.
- The axial dispersion density flux of species i is given by $F_{ax}^i = -D\rho \frac{\partial \theta_i}{\partial x}$ where $\theta_i = \frac{\rho_i}{\rho}$ is the mass fraction of species i for i = A, B and ρ_i the mass concentration of species i, and D the dispersion constant.
- The conduction flux is chosen as $F_{cond} = -\lambda \frac{\partial T}{\partial x}$ where λ is the conduction constant and T(x) is the temperature inside the reactor at x.
- The distributed heat exchange q(x) with the jacket is given by $q(x) = C(T(x) T_j(x))$ where $T_j(x)$ is the jacket temperature.
- The pressure P is constant.
- The reacting mixture is ideal and incompressible.
- The specific heat capacities c_{p_A} and c_{p_B} are constant.

Let us note the state vector as $z^T = (h \ \rho_A \ \rho_B)$ where ρ_A, ρ_B are the mass concentration of species A and species B respectively ad h is the concentration of enthalpy.

We obtain the following parabolic model of the fixed bed reactor (Bird *et al.* (2006)):

$$\begin{split} &\frac{\partial}{\partial t} \begin{pmatrix} h\\ \rho_A\\ \rho_B \end{pmatrix} = -\frac{\partial}{\partial x} \left[vI_3 - D \ W \frac{\partial}{\partial x} \right] \begin{pmatrix} h\\ \rho_A\\ \rho_B \end{pmatrix} \quad (1) \\ &-\frac{\partial}{\partial x} \begin{pmatrix} -\lambda \frac{\partial T}{\partial x}\\ 0\\ 0 \end{pmatrix} + \begin{pmatrix} 0\\ -\nu_A r\\ +\nu_B r \end{pmatrix} \\ &+g \ C(T_j - T) \end{split}$$

where I_3 is the 3×3 identity matrix, $W(z) = \begin{pmatrix} 0 & h_A & h_B \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

 $g^T = (1 \ 0 \ 0)$. The enthalpy concentration is given by $h = \rho_A h_A + \rho_B h_B$ with h_A , h_B the partial mass enthalpy of species A and B respectively. We have $h_i = c_{p_i}(T - T_{ref}) + h_{iref}$ for i = A, B.

The temperature T(z) is a nonlinear function of the state z:

$$T = \frac{h - \rho_A h_{Aref} - \rho_B h_{Bref}}{c_p} + T_{ref}$$

with $c_p = \rho_A c_{p_A} + \rho_B c_{p_B}$ is the total heat capacity.

Let us define

$$F_{con}^T = (vh \ v\rho_A \ v\rho_B)$$

the convective flux,

$$F_{dis}^T = \left(F_{ax}^h + F_{cond} \ F_{ax}^A \ F_{ax}^B \right)$$

the dispersive flux with $F_{ax}^h = h_A F_{ax}^A + h_B F_{ax}^B$ and $R^T = (0 - \nu_A r \ \nu_B r)$

the reaction rate vector.

With these notations, the system (1) can be written in the compact form:

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial x} (\overbrace{F_{con}(t,x) + F_{dis}(t,x)}^{F}) + R(t,x) + gq(t,x) \quad (2)$$

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Based on the assumption that there is no dispersion outside the reactor, the boundary conditions are the following at the inlet of the reactor:

$$F^{h}_{con}|_{in} = F^{h}|_{0} = (F^{h}_{con} + F^{h}_{ax} + F_{cond})|_{0}$$
(3)

$$F_{acm}^{A}|_{in} = F^{A}|_{0} = (F_{acm}^{A} + F_{am}^{A})|_{0}$$
(4)

$$0 = F_{con}^B|_{in} = F^B|_0 = (F_{con}^B + F_{ax}^B)|_0$$
(5)

and at the outlet of the reactor:

$$F_{ax}^{A}|_{L} = 0 \Longrightarrow \frac{\partial \rho_{A}}{\partial x}|_{L} = 0 \tag{6}$$

$$F_{ax}^{B}|_{L} = 0 \Longrightarrow \frac{\partial \rho_{B}}{\partial x}|_{L} = 0 \tag{7}$$

$$F_{cond}|_L = 0 \Longrightarrow \frac{\partial T}{\partial x}|_L = 0$$
 (8)

Finally we consider the initial conditions:

$$\rho_A(0,x) = \rho_{A_0}(x) \tag{9}$$

$$\rho_B(0,x) = \rho - \rho_{A_0}(x) \tag{10}$$

$$h(0,x) = h_0(x) \tag{11}$$

3. GIBBS EQUATION FOR INFINITE DIMENSIONAL THERMODYNAMIC SYSTEMS

In this section we briefly recall the fundamentals of irreversible thermodynamics. We leave the reader refer to Callen (1985) and de Groot and Mazur (1962) for more details.

In the case of infinite dimensional thermodynamic systems the fundamental Gibbs equation can be stated by using concentrations as:

$$ds = w^T dz \tag{12}$$

with $w^T = \left(\frac{1}{T} \frac{P}{T} \dots - \frac{\mu_i}{T} \dots\right)$ and $z = \left(u \ 1 \dots \rho_i \dots\right)$ where T, P, μ_i represent the temperature, the pressure and the chemical potential of species *i* respectively and u, ρ_i represent the energy and mass concentrations. An alternative representation at constant pressure is given by:

$$ds = w^T dz \tag{13}$$

with $w^T = \left(\frac{1}{T}\dots - \frac{\mu_i}{T}\dots\right)$ and $z = \left(h\dots\rho_i\dots\right)$ where *h* represents the enthalpy concentration (h = u + R) used as a state variable for the resetor model simpled

P) used as a state variable for the reactor model given in section 2. As in finite dimensional case, we can write $s = w^T z$, $w = \frac{\partial s}{\partial z}$ and $w(\gamma z) = w(z)$ with γ a positive constant.

The generalization of equilibrium thermodynamics to nonequilibrium thermodynamics systems is done by assuming the so called local equilibrium assumption. The main points of this hypothesis are

- the local state z and the specific properties are equivalent (see de Groot and Mazur (1962)).
- we have the following relation between the different local balances:

$$\frac{Ds(t,x)}{Dt} = w^T(t,x)\frac{Dz(t,x)}{Dt}$$
(14)

where $\frac{D}{Dt}$ stands for the material derivative: $\frac{D}{Dt} = \frac{\partial}{\partial t} + v \frac{\partial}{\partial x}$ where v is the mean velocity of the matter.

We recall also the Gibbs-Duhem relation:

$$\frac{\partial w^T}{\partial x}z = 0. \tag{15}$$

v

From the Gibbs equation we obtain the local entropy time derivative equation. Rewriting this latter equation as a balance, we obtain :

$$\frac{\partial s}{\partial t} = -\frac{\partial}{\partial x} (F^s_{cond} + F^s_{ax} + F^s_{con}) + \frac{q}{\rho T_j} + \frac{\sigma_s}{\rho} \qquad (16)$$

where T_j is the distributed temperature of the jacket, $F_{ax}^s = \sum_{i=A,B} F_{ax}^i s_i$ is the entropy diffusion flux due to species diffusion where s_i is the partial mass entropy of species i. $F_{cond}^s = \frac{1}{T}F_{cond}$ is the entropy flux due to heat conduction. The irreversible entropy production σ_s is given by:

$$\sigma_{s} = \overbrace{q(\frac{1}{T_{j}} - \frac{1}{T})}^{\sigma_{ext}} + \overbrace{\frac{r}{T}(\mu_{A}M_{A}\nu_{A} - \mu_{B}M_{B}\nu_{B})}^{\sigma_{r}}$$

$$\overbrace{-\frac{1}{T}\sum F_{ax}^{i}\frac{\partial\mu_{i}}{\partial x} - (F_{ax}^{s} + F_{cond}^{s})\frac{\partial T}{\partial x}}^{\sigma_{d}^{therm}}}$$
(17)

where the M_i are the molar mass of species. Each term of (17) is assumed to be positive in the context of irreversible thermodynamics Callen (1985) with local equilibrium assumption. The first term σ_{ext} corresponds to the exchange with the jacket of the reactor. The second term σ_r is due to the chemical reaction. The third and the fourth terms σ_d^{mat} and σ_d^{therm} are due to diffusion in the material domain and heat conduction in the thermal domain.

4. DISTRIBUTED THERMODYNAMIC AVAILABILITY FUNCTION

In this section we show the availability function can be used as a Lyapunov function for control purpose. Let us first recall the definition of a Lyapunov function for finite dimensional systems:

Definition 1. (Khalil (2002)). Let Z = 0 be a stable equilibrium state and $D \subset R^n$ be a domain containing Z = 0. Let $V : D \to R$ be a continuously differentiable function such that :

$$V(0) = 0 \quad and \quad V(Z) > 0 \quad in \quad D - \{0\}$$
 (18)

$$\dot{V}(Z) \le 0 \quad in \quad D \tag{19}$$

The function V(Z) satisfying (18) and (19) is called a Lyapunov function .

Let us denote T the temperature of the system, P the pressure, μ_i the chemical potential of the species i, S the entropy, V the volume of the matter, M_i the mass of the species i and U the internal energy of the system.

The availability function for finite dimensional systems A(Z) is defined in Ruszkowski et al. (2005) as:

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$$A(Z) = -S(Z) + (W_d)^T Z = -\left(W - W_d\right)^T Z \qquad (20)$$

with $W^T = \left(\frac{1}{T} \frac{P}{T} - \frac{\mu_i}{T}\right)$ and $Z^T = \left(U \ V \ M_i\right).$

This function is defined with respect to some reference state (W_d, Z_d) . This reference state may be a desired stationary state Z_d or a time varying reference state.

Using the concavity of S(Z) (provided by the second law of thermodynamics, see Callen (1985)) we deduce that A(Z) is a non negative convex function such that:

$$A(Z) \ge 0 \quad \forall Z \neq \gamma Z_d , \quad A(\gamma Z_d) = 0 \tag{21}$$

It can be shown (see Ruszkowski et al. (2005); Hoang *et al.* (2012)) that as soon the strict concavity of S is ensured we ensure the strict convexity of A:

$$A(Z) > 0 \quad \forall Z \neq Z_d , \quad A(Z_d) = 0 \tag{22}$$

Indeed strict concavity of S can be achieved by fixing an additional linear constraint on the extensive variables Z in the sense of an inventory (Jillson and Ydstie (2007)). This linear constraint reduces the singular line where $w(\gamma Z) = w(Z_d)$ on the desired state Z_d . Figure 1 illustrates the availability function in the isothermal case for a reacting ideal mixture of two species with mole numbers $Z = [N_1 N_2]^T$ with Z_d such that $N_{1d} = N_{2d}$. So in the illustrated case the availability function vanishes on the line $N_1 = N_2$.



Fig. 1. The availability function in the isothermal case

Since the availability function satisfies the condition (18), it represents a Lyapunov function candidates for the stabilization problem of the chemical reactor at a desired equilibrium state Z_d (see Ruszkowski et al. (2005) or Hoang *et al.* (2012)). Hence the stabilizing control design consists to compute the feedback law for the considered inputs such that the availability function satisfies condition (19):

$$\dot{A}(Z) \le 0 \tag{23}$$

As in the finite dimensional case, we define the availability function for the distributed parameter system as:

$$a(z(x,t)) = -(w(x,t) - w_d(x))^T z(x,t)$$
(24)

where z_d and w_d refer to the desired steady profile and its associated effort respectively. The function a(z(x,t))can be qualified as a *local availability* function since it involves a distributed variable z(x,t). In our case, the fixed extensive (inventory) function corresponds to the total mass inside the reactor induced by a constant total mass density ρ on a fixed total volume. With this assumption we have:

$$a(z) > 0 \quad \forall z \neq z_d , \quad a(z_d) = 0 \tag{25}$$

Thanks to equation (14) and (15), we have $\frac{\partial a}{\partial z} = \frac{\pi}{2} \frac{\partial z}{\partial z}$

$$\frac{\partial a}{\partial t} = -\tilde{w}^T \frac{\partial z}{\partial t} \tag{26}$$

where $\tilde{w} = w - w_d$. Let us consider the global availability function A defined by:

$$A(z) = \int_{0}^{L} a dx = \int_{0}^{L} \left(-\tilde{w}(x,t)^{T} z(x,t)\right) dx \qquad (27)$$

From (26) the variation of A along the system trajectories has the following expression by considering (2):

$$\frac{dA(z)}{dt} = \int_0^L -\left(\tilde{w}^T \frac{\partial F}{\partial x}\right) dx - \int_0^L \tilde{w}^T (R+gq) dx \quad (28)$$

In the next section we use the global availability function A to design the stabilizing control for the chemical reactor.

5. THE CONTROL PROBLEM

The objective of the control is to stabilize the system around an admissible stationary profile considering the distributed jacket temperature as the control variable. For the control design we use the availability A defined by (27) as a Lyapunov function.

Proposition 2. The dynamic system (2) closed with the nonlinear state feedback:

$$T_j(x) = \frac{\left\lfloor \tilde{w}^T \frac{\partial F}{\partial x} - \tilde{w}^T R + K(z, x, t) a(z) \right\rfloor}{C \tilde{w_h}} + T(x) \quad (29)$$

with K(z, x, t) a positive function and $\tilde{w}_h = (\frac{1}{T} - \frac{1}{T_d})$ is globally asymptotically stable at z_d .

Proof Using the expression of the state feedback (29) for the jacket temperature in (28) we obtain:

$$\frac{dA(z)}{dt} = -\int_{0}^{L} K(z, x, t)a(z)dx \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{dA(z)}{dt} < 0 , \forall z \neq z_{d} \\ \frac{dA(z)}{dt} = 0 , z = z_{d} \end{cases}$$
(30)

Thus A(z) is a Lyapunov function for the system in closed loop. Using the positivity of a(z) we can write:

$$\lim_{t \to \infty} A(z) = 0 \Rightarrow \lim_{t \to \infty} a(z) = 0 \quad \forall x \in [0, L]$$
$$\Rightarrow \lim_{t \to \infty} z = z_d. \tag{31}$$

From this Lyapunov type arguments, the closed loop system with state feedback control (29) is globally asymptotically stable.

Remark 3. It can be shown as in Hoang *et al.* (2012) for finite dimensional control that T_j remains well posed when $T \to T_d$, meaning that the numerator of the control converges to zero faster than \tilde{w}_h .

6. SIMULATIONS

6.1 Open-loop simulation

This section illustrates an open-loop simulation scenario. The model is spatially discretized with the use of a cen-

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tered finite difference scheme with 120 spatial discretization elements. For this simulation, the initial conditions correspond to the steady state profile when $T_j(x) = 350K$ along the reactor and with the temperature at the inlet of the reactor $T_{in} = 330K$ and the inlet mass fraction $\theta_{A_{in}} = 1$ and $\theta_{B_{in}} = 0$. The numerical values of the parameters are given in Table 1. In Figures 2 and 3 is given

Symb.	Numerical value	Symb.	Numerical value
c_{pA}	$150.48J/(K \cdot g)$	c_{pB}	$120J/(K \cdot g)$
C	$1.25 \cdot 10^5 W/(m \cdot K)$	E	72.335 KJ/mol
h_{Aref}	0J/g	h_{Bref}	-9150 J/g
k_0	$0.12 \ 10^{10} 1/s$	L	1m
M_A	0.5g/mol	M_B	0.5g/mol
R	$8.314J/(K \cdot mol)$	s_{Aref}	$210.4J/(K \cdot g)$
s_{Bref}	$180.2J/(K \cdot g)$	T_{ref}	300 K
v	$0.0005m^3/mol$	V	$0.001m^{3}$
λ	$1.25 \cdot 10^8 J/(K \cdot m \cdot s)$		
ν_A	1	ν_B	1
Table 1 Numerical values of nemerications			

Table 1. Numerical values of parameters.

the time response of the difference between the reactor temperature T and the initial profile $T_{initial}$ with mass fraction θ_B and its initial profile $\theta_{Binitial}$ to a uniform step change of the jacket temperature from $T_j = 350K$ to $T_j = 370K$ while keeping the same boundary conditions. In Figure 4 we present the final steady state profile for the temperature and mass fraction θ_B which will be taken as the desired profile target for the stabilizing control.



Fig. 2. Open loop time response of T(x, t)



Fig. 3. Open loop time response of $\theta_B(x,t)$



Fig. 4. Open loop steady state profile

6.2 Stabilisation results

First of all we propose to tune the feedback gain K(x,t)in order to avoid excessive control values. Indeed, choosing spatially uniform and constant gain K in the simulations, leads to control inputs T_j with very important (non admissible) transient magnitudes. At the same time, this uniform constant gain does not give an interesting time response for the closed loop system. The feedback gain K(x,t) is chosen as follows:

$$K(x,t) = \min\left(\delta exp^{-\frac{x}{2}} + \delta vt(1 - exp^{-\frac{x}{2}}), \delta\right)$$
(32)

Remark 4. This particular choice is interesting because it allows to obtain the available amplitudes and variations of control input T_j . It is based on the spatial evolution of K(x,t) from an initial exponential profile with a saturation value δ in the input side which decrease in the space. The gain profile increases linearly but it has a saturation value equal to δ with δ a positive constant. It follows the direction of flow with the same speed as the convection flow.

The time evolution of the gain K(x, t) is presented in figure 5 with $\delta = 0.08$.



Fig. 5. Dynamical distributed gain K(x, t)

The time evolutions of the error between the closed loop temperature T and its desired equilibrium profile T_d as well as the closed loop mass fraction θ_B and its equilibrium profile θ_{Bd} of the closed loop system are presented in figures 6 and 7 respectively. These closed loop simulations

are given with T_j satisfying (29) and with K defined as in (32).



Fig. 6. Temperature of the closed loop system



Fig. 7. θ_B of the closed loop system

The objective of the stabilisation is well achieved at settling time: approximatively 110s with respect to the open loop case approximatively 220s. Figure 8 shows the time evolution of the control variable $T_j(x)$. It can be noticed that this control is admissible.



Fig. 8. Time evolution of the jacket temperature

Figure 9 shows the time evolution of the local availability function a. As expected the local availability tends to zero. At this moment the performances of the control could be improved by setting the gain function K(x) more precisely.

6.3 Stabilisation results in presence of perturbation

In presence of perturbation the closed loop control plays its role although the performances of the control could be improved. Figure 10 shows the final error of temperature with respect to the desired steady profile with a perturbation



Fig. 9. Time evolution of the local availability function

of 3 degrees of the external temperature at the boundaries in the open loop and closed loop case.



Fig. 10. Error with respect to the desired steady state profile at t=1200s open loop and closed loop case

7. CONCLUSION

In this paper we propose an introductory study of the distributed stabilization of tubular reactors using the thermodynamic global availability function as a Lyapunov function. The control is achieved considering the distributed actuation of the jacket temperature. The presented simulations show the effectiveness of the design control especially in presence of perturbations.

Two perspectives can be formulated for this control problem.

The first one is to generalize the modification of the availability function as proposed in Hoang *et al.* (2012). In this work, the author presented a reduced availability function which is mainly based on the classical availability in which the chemical potentials are modified in order to remove the mixing term. This new function is again strictly convex and positive. It has been shown in Hoang *et al.* (2012) that it reduces considerably the amplitude variation of the temperature of the jacket.

The second perspective is to combine the distributed and boundary control inputs to stabilize the reactor. As we can see from the closed loop simulation results, this alternative control strategy should upgrade the settling time of the stabilization at the boundary of the reactor.

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