Preprints of the
9th International Symposium on Advanced Control of Chemical Processes
The International Federation of Automatic Control
June 7-10, 2015, Whistler, British Columbia, Canada

A Model Predictive Controller for Inverse Response Control Systems

Ky M. Vu

AuLac Technologies Inc, Ottawa, Ontario Canada
(e-mail: kymvu@aulactechnologies.com)

Abstract: A discrete model predictive controller for inverse response control systems is presented. The controller has two degrees of freedom (DOF): The feedback path contains current and past errors, and the feedforward path contains future setpoint values for tracking control. The controller can eliminate the inverse response completely due to its characteristic of having an infinite number of predictive values of the setpoint disturbance.

Keywords: Degree of freedom, inverse response, model predictive control, nonminimum phase.

1. INTRODUCTION

Nonminimum phase control systems are difficult to control. A nonminimum phase control system gives an inverse response due to an unstable zero of the transmission zero polynomial. Chemical process control engineers are familiar with these control systems. Some controllers such as the Dahlin controller cannot be used for this system. Other controllers like the Vogel-Edgar and PID can control this system, but they cannot suppress its inverse response, Vu (2008a). In Camacho, E.F. and Bordons, C. (2003), chapter 7, the authors praise model predictive control and have a picture showing model predictive control can suppress part of the inverse response of a nonminimum phase system, a feat that no other controllers or control algorithms can do so far.

Predictive control is a characteristic of discrete feedback control systems. The control system loses one control interval due to feedback, so it has to predict for a future value of the controlled variable to control it. If prediction is made at the dead time value of the control system, the designed controller has a dead time compensator. If, however, predictions are made beyond the dead time value of the control system, the predictions have future values of the control variable. This is an important and controversial topic of predictive control, but it is only serious for regulating control. For tracking control, the issue of future values of the control variable is less severe; see Vu (2008b).

In this paper, a model predictive tracking controller with an infinite control and prediction horizon is presented. It makes use of the fact that prediction of the setpoint disturbance is exact, and there can be an infinite number of predictions. The controller has been discussed in the author's book, Vu (2008a). Since the book is still little known and the controller was not presented as a predictive controller, this paper was written for this purpose. The paper gives an improvement of the controller discussed in Vu (2008a), and it is organized as follows. Section I is the introduction section. In section II, we discuss a little bit about the similarity and difference of model predictive tracking and regulating controls. In section III, we present our infinite-prediction-horizon predictive controller. In section IV, we give an example, and section V is the conclusion of the paper.

2. MODEL PREDICTIVE CONTROL

In this section, we discuss the application of predictive control in both tracking and regulating controls. Prediction of the setpoint is exact, and we can have an infinite number of predictions. Prediction of the load disturbance is more difficult because it requires the feedback signal.

2.1 An Account of Historical Development

In a year of the mid 90's, the author of this paper had a task, and that was to evaluate an algorithm of model predictive control for the employer the author worked for. The paper that presents the control algorithm is a conference paper by Cutler, C.R. and Radamaker, B.C. (1980). The algorithm is named DMC (Dynamic Matrix Control). The author was awed by the most striking characteristic of the algorithm: It is the calculation of the future values of the input or control variable. This led to the paper Vu (2008b) on the author company's website www.aulactechnologies.com. In Ding, B.C. (2010), its author said that the future values of the control variable are discarded. But different algorithms might have different ways to deal with the future values of the control variable. While the characteristic seemed to be a commercial hype, it has created a number of algorithms, in the ensuing years, that have been known in the industry. In Camacho, E.F. and Bordons, C. (1995), the authors listed these algorithms as: MUSMAR (Multistep Multivariable Adaptive Control), MURHAC (Multipredictor Receding Horizon Adaptive Control), PFC (Predictive Functional Control), UPC (Unified Predictive Control). One algorithm that has an academic origin is the GPC (Generalized Predictive Control) algorithm. Because of its academic origin, it has been described in a technical journal Clarke, D.W., Mohtadi, C. and Tuffs, P.S. (1987) and textbooks Camacho, E.F. and Bordons, C. (1995).

2.2 Regulating and Tracking Model Predictive Controls

Fig. 1 below is the block diagram of a feedback discrete control system.

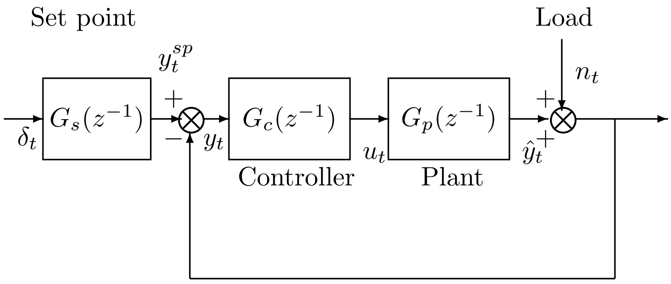

Figure 1. A Feedback Control System.

From this figure, we can write the following equation

$$\begin{aligned} y_t &= y_t^{sp} - \hat{y}_t - n_t, \\ &= y_t^{sp} - G_p(z^{-1})u_t - n_t. \end{aligned}$$

There are two variables that change the current state of the control system: One comes from the setpoint y_t^{sp}, and the other comes from the load n_t. Both these variables are called a disturbance. As long as the design of the controller $G_c(z^{-1})$ is concerned, the feedback controller is the same if the models of the setpoint and the load are the same. This fact constitutes a duality of tracking, with a setpoint change, and regulating control, with a load change. The setpoint can be known exactly. The load disturbance is usually not known; its existence is usually inferred from the feedback signal. Because of the unknown or uncertainty of the load disturbance, control theorists use a stochastic model for the case of a load change. The following model

$$\begin{aligned} y_t &= G_p(z^{-1})u_t + n_t, \\ &= \frac{\omega(z^{-1})}{\delta(z^{-1})}u_{t-f-1} + \frac{\theta(z^{-1})}{\phi(z^{-1})}a_t \end{aligned}$$

is called the Box-Jenkins stochastic control model. In this model, the variable a_t is a white noise. Reference Box, G.E.P. and Jenkins, G.M. (1976) gives a fundamental discussion of this model, and reference Vu (2008a) gives an extended discussion of the controllers for this model. This model can be obtained from the block diagram of Fig. 1 by setting the setpoint variable y_t^{sp} to zero and not negating the feedback signal.

While a stochastic model might have a strong argument for a load disturbance change, industrial experiences have shown that the load change can very well be deterministic in the sense that its successive values can be predicted exactly from others. What is not known is the prediction formula for prediction or the model of the disturbance. From the time series literature, see Vu (2007), we can say the white noise a_t in the Box-Jenkins model can be replaced by the discrete Dirac delta function.

If we take this modified model and compare it with the model given by Fig. 2 in the next section, we can say that there is a similarity between tracking and regulating controls. However, since the load n_t is not known like the setpoint y_t^{sp}, its value must be inferred from the feedback signal and the control variable. A long-range prediction creates the problem of the future control variable values. We will see this problem clearer in the next section.

3. THE INFINITE-PREDICTION-HORIZON MODEL PREDICTIVE CONTROLLER

A feedback tracking control system has its configuration described by the block diagram shown in Fig. 2 below.

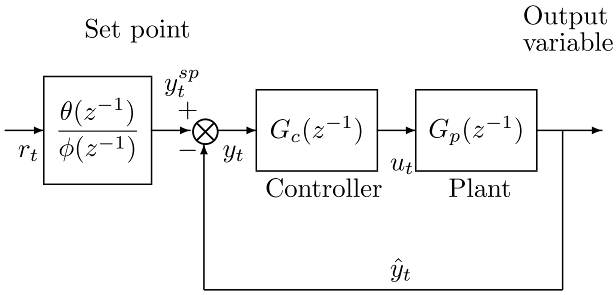

Figure 2. A Feedback Tracking Control System.

From the block diagram of Fig. 2, we can write

$$\begin{aligned} y_t &= -G_p(z^{-1})u_t + y_t^{sp}, \\ &= -\frac{\omega(z^{-1})}{\delta(z^{-1})}u_{t-f-1} + \frac{\theta(z^{-1})}{\phi(z^{-1})}r_t, \\ &= -\frac{\omega(z^{-1})}{\delta(z^{-1})}u_{t-f-1} + \frac{\theta(z^{-1})}{\phi^*(z^{-1})(1-z^{-1})^d}r_t. \end{aligned}$$

The variable r_t is actually a multiple value of the discrete Dirac delta function. We define it here so that the polynomial $\theta(z^{-1})$ can be monic and the model of the system has a similarity with the Box-Jenkins stochastic control model, except for the minus sign in front of the transfer function of the dynamic part. The parameter f is the dead time of the system, and the parameter d usually has value one because of a change in the mean level of the setpoint. Also from this figure, we can see that the source of disturbance of the system is the variable r_t. This means that we can write the controller in the following form

$$(1 - z^{-1})^d u_t = l(z^{-1}, z)r_t.$$

In this form, the controller will calculate its control action from all the past, current and future disturbance values. To proceed further, we define the following functions:

$$u(z^{-1}) = \sum_{t=0}^{\infty} u_t z^{-t}, \quad y(z^{-1}) = \sum_{t=0}^{\infty} y_t z^{-t},$$

$$r(z^{-1}) = \sum_{t=0}^{\infty} r_t z^{-t} = \text{const.},$$

$$y^{sp}(z^{-1}) = \frac{\theta(z^{-1})}{\phi^*(z^{-1})(1-z^{-1})^d} r(z^{-1}).$$

Having defined these quantities, we now use them to construct our performance index for the controller. We can define the performance index for our controller as

$$\hat{s}^2 = Min\ s^2,$$

with

$$s^2 = \underset{z=0}{Residue}\,[\frac{y(z)y(z^{-1})}{r(z)r(z^{-1})}+\lambda\frac{(1-z)^d u(z)(1-z^{-1})^d u(z^{-1})}{r(z)r(z^{-1})}]\frac{1}{z}.$$

The performance index value is a sum of two normalized sums of squares of infinite values of the error variable y_t and the control or input variable $(1-z^{-1})^d u_t$ with a weight or constraint on the input variable. The positive weight λ is called a penalty constant.

From the equation of the controller, we can obtain

$$\frac{(1-z^{-1})^d u(z^{-1})}{r(z^{-1})} = l(z^{-1}, z).$$

And by defining the following Diophantine equation

$$\frac{\theta(z^{-1})}{\phi(z^{-1})} = \psi(z^{-1}) + \frac{\gamma(z^{-1})}{\phi(z^{-1})} z^{-f-1},$$

we can obtain the equation for the error variable as below

$$\frac{y(z^{-1})}{r(z^{-1})} = \psi(z^{-1}) - \frac{\omega(z^{-1})\phi^*(z^{-1})l(z^{-1},z) - \delta(z^{-1})\gamma(z^{-1})}{\delta(z^{-1})\phi(z^{-1})} z^{-f-1}.$$

By replacing the input variable in the performance index equation, we can write it as

$$s^2 = \underset{z=0}{Residue}\ [\frac{y(z)y(z^{-1})}{r(z)r(z^{-1})} + \lambda l(z,z^{-1})l(z^{-1},z)]\frac{1}{z},$$

and by replacing the error variable in the performance index equation, we can write

$$s^2 = \underset{z=0}{Residue}\ [\psi(z) - \frac{\omega(z)\phi^*(z)l(z,z^{-1}) - \delta(z)\gamma(z)}{\delta(z)\phi(z)} z^{f+1}] [\psi(z^{-1}) - \frac{\omega(z^{-1})\phi^*(z^{-1})l(z^{-1},z) - \delta(z^{-1})\gamma(z^{-1})}{\delta(z^{-1})\phi(z^{-1})} z^{-f-1}]\frac{1}{z} + \lambda\ \underset{z=0}{Residue}\ l(z,z^{-1})l(z^{-1},z)\frac{1}{z}.$$

The cross-product terms of the first term in the last equation vanish with residue calculus, so we can write the performance index as

$$s^2 = \underset{z=0}{Residue}\ \Big[\psi(z)\psi(z^{-1})\frac{1}{z} + \frac{\omega(z)\phi^*(z)l(z,z^{-1}) - \delta(z)\gamma(z)}{z\delta(z)\phi(z)} \times \frac{\omega(z^{-1})\phi^*(z^{-1})l(z^{-1},z) - \delta(z^{-1})\gamma(z^{-1})}{\delta(z^{-1})\phi(z^{-1})} + \lambda\frac{l(z,z^{-1})\delta(z)\phi^*(z)(1-z)^d}{z\delta(z)\phi(z)} \times \frac{\delta(z^{-1})\phi^*(z^{-1})(1-z^{-1})^d l(z^{-1},z)}{\delta(z^{-1})\phi(z^{-1})}\Big].$$

By adding the last two terms in the last equation and defining the spectral factorization equation

$$\alpha(z)\alpha(z^{-1}) = \omega(z)\omega(z^{-1}) + \lambda\delta(z)(1-z)^d\delta(z^{-1})(1-z^{-1})^d, \quad (1)$$

we can arrive at the following equation

$$s^2 = \underset{z=0}{Residue}\ \Big[\psi(z)\psi(z^{-1})\frac{1}{z} + \lambda\frac{\delta(z)\gamma(z)\delta(z^{-1})\gamma(z^{-1})}{z\alpha(z)\phi^*(z)\alpha(z^{-1})\phi^*(z^{-1})} + [\frac{\alpha(z)l(z,z^{-1})}{\delta(z)(1-z)^d} - \frac{\gamma(z)\omega(z^{-1})}{\phi(z)\alpha(z^{-1})}] \times [\frac{\alpha(z^{-1})l(z^{-1},z)}{\delta(z^{-1})(1-z^{-1})^d} - \frac{\gamma(z^{-1})\omega(z)}{\phi(z^{-1})\alpha(z)}]\frac{1}{z}\Big].$$

The spectral factorization always gives a stable polynomial $\alpha(z^{-1})$ even though the right hand side of equation (1) contains unstable zeros of $\omega(z^{-1})$ and zeros of unit value of $(1-z^{-1})^d$. Therefore, the performance index value has a finite positive number and we can minimize it by setting the third term on the right hand side of the last equation to zero. This means that we must have

$$\frac{\alpha(z^{-1})}{\delta(z^{-1})(1-z^{-1})^d} l(z^{-1},z) = \frac{\gamma(z^{-1})\omega(z)}{\phi(z^{-1})\alpha(z)}.$$

The last equation gives us the controller in one form. To obtain the controller in an implementable form, we write

$$\frac{\alpha(z^{-1})}{\delta(z^{-1})(1-z^{-1})^d}\frac{(1-z^{-1})^d u(z^{-1})}{r(z^{-1})} = \frac{\gamma(z^{-1})\omega(z)}{\phi(z^{-1})\alpha(z)},$$

$$\frac{\alpha(z^{-1})u(z^{-1})}{\delta(z^{-1})r(z^{-1})} = \frac{\beta(z^{-1})}{\phi(z^{-1})} + \frac{\zeta(z)}{\alpha(z)}z,$$

$$\frac{\alpha(z^{-1})\theta(z^{-1})u(z^{-1})}{\delta(z^{-1})\phi(z^{-1})y^{sp}(z^{-1})} = \frac{\beta(z^{-1})}{\phi(z^{-1})} + \frac{\zeta(z)}{\alpha(z)}z$$

or

$$\frac{\alpha(z^{-1})\theta(z^{-1})}{\delta(z^{-1})\phi(z^{-1})}u(z^{-1}) = [\frac{\beta(z^{-1})}{\phi(z^{-1})} + \frac{\zeta(z)}{\alpha(z)}z]y^{sp}(z^{-1}).$$

In terms of the variables in the time domain, we can write

$$\frac{\alpha(z^{-1})}{\delta(z^{-1})}\frac{\theta(z^{-1})}{\phi(z^{-1})}u_t = \frac{\beta(z^{-1})}{\phi(z^{-1})}y_t^{sp} + \frac{\zeta(z)}{\alpha(z)}zy_t^{sp}, = \frac{\beta(z^{-1})}{\phi(z^{-1})}y_t^{sp} + v_t.$$

The variable v_t is a converging sum of the weighted future setpoint values. From the above equation, we can derive the equation for the controller as

$$\frac{\alpha(z^{-1})}{\delta(z^{-1})}\frac{\theta(z^{-1})}{\phi(z^{-1})}u_t = \frac{\beta(z^{-1})}{\phi(z^{-1})}y_t^{sp} + v_t, = \frac{\beta(z^{-1})}{\phi(z^{-1})}[\hat{y}_t + y_t] + v_t, = \frac{\beta(z^{-1})}{\phi(z^{-1})}[\frac{\omega(z^{-1})}{\delta(z^{-1})}z^{-f-1}u_t + y_t] + v_t.$$

By moving the term with the input variable from the right hand side of the above equation to its left hand side, we can write

$$[\frac{\alpha(z^{-1})}{\delta(z^{-1})}\frac{\theta(z^{-1})}{\phi(z^{-1})} - \frac{\beta(z^{-1})}{\phi(z^{-1})}\frac{\omega(z^{-1})}{\delta(z^{-1})}z^{-f-1}]u_t =$$
$$\underbrace{\frac{\beta(z^{-1})}{\phi(z^{-1})}y_t}_{\text{Current and past errors}} + \underbrace{v_t.}_{\text{Infinite future setpoint values}}$$

In model predictive tracking control, the variable v_t can be calculated because we have all the values of the setpoint y_t^{sp}. This is not so in model predictive regulating control: We do not know the future values of the load variable n_t. The future values of the disturbance n_t are inferred from the future values of the control and controlled variables u_t and y_t. This is a characteristic of model predictive control.

The controller for our case is

$$u_t = \frac{\delta(z^{-1})\beta(z^{-1})}{\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}}y_t + \frac{\delta(z^{-1})\phi(z^{-1})}{\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}}v_t. \quad (2)$$

This is the controller given in Vu (2008a). The polynomial in the denominators of the terms on the right hand side of the last equation usually has a zero of unit value, so calculation u_t by long division is impractical. Also the variable v_t has to be calculated separately. Therefore, for practical purposes, we can improve the calculation by introducing another equation as follows. We write

$$u_t = \frac{\delta(z^{-1})\beta(z^{-1})}{\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}}y_t + \frac{\delta(z^{-1})\phi(z^{-1})}{\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}}\frac{\zeta(z)}{\alpha(z)}zy_t^{sp}.$$

By replacing the model of the setpoint change, we obtain

$$u_t = \frac{\delta(z^{-1})\beta(z^{-1})}{\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}}y_t + \frac{\delta(z^{-1})\theta(z^{-1})}{\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}}\frac{\zeta(z)}{\alpha(z)}zr_t.$$

From this equation, we can calculate the control variable from the following equation

$$[\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}]u_t = \delta(z^{-1})\beta(z^{-1})y_t + \frac{z\zeta(z)\delta(z^{-1})\theta(z^{-1})}{\alpha(z)}r_t. \quad (3)$$

The controller gives the following optimal performance index value

$$\hat{s}^2 = \underset{z=0}{Residue}\,[\psi(z)\psi(z^{-1}) + \lambda\frac{\delta(z)\gamma(z)\delta(z^{-1})\gamma(z^{-1})}{\alpha(z)\phi^*(z)\alpha(z^{-1})\phi^*(z^{-1})}]\frac{1}{z}.$$

To judge and compare the performance of the controller, we need to calculate the sums of squares of the error and input variables. The normalized sum of squares of the input variable $(1-z^{-1})^d u_t$ values for the controller can be calculated as

$$s_u^2 = \underset{z=0}{Residue}\,\frac{\delta(z)\gamma(z)\omega(z)\delta(z^{-1})\gamma(z^{-1})\omega(z^{-1})}{z\phi^*(z)\alpha(z)\alpha(z)\phi^*(z^{-1})\alpha(z^{-1})\alpha(z^{-1})}.$$

To find the normalized sum of squares of the error variable, we need to obtain the expression for the output variable $\hat{y}_t$ first. This can be obtained as follows. First, we find an expression for the output variable

$$\begin{aligned}\frac{\hat{y}(z^{-1})}{r(z^{-1})} &= \frac{\omega(z^{-1})}{\delta(z^{-1})}\frac{u(z^{-1})}{r(z^{-1})}z^{-f-1},\\ &= \frac{\omega(z^{-1})}{\delta(z^{-1})}[\frac{\delta(z^{-1})\gamma(z^{-1})\omega(z)}{\alpha(z^{-1})\phi(z^{-1})\alpha(z)}]z^{-f-1},\\ &= \frac{\gamma(z^{-1})\omega(z^{-1})\omega(z)}{\phi(z^{-1})\alpha(z^{-1})\alpha(z)}z^{-f-1}.\end{aligned}$$

The existence of the polynomial $\omega(z)$ beside of the polynomial $\omega(z^{-1})$ is the proof that the controller can suppress the inverse response of a nonminimum phase system. This is because by the spectral factorization, we can always find a stable polynomial $\omega^*(z^{-1})$ that will satisfy the equation $\omega^*(z^{-1})\omega^*(z) = \omega(z^{-1})\omega(z)$. It is as if we have the response of a minimum phase transfer function, which implies no inverse response for a step change in the setpoint, with the transmission zero polynomial $\omega^*(z^{-1})$.

Then, from the above equation, we write the error variable as

$$\begin{aligned}\frac{y(z^{-1})}{r(z^{-1})} &= \frac{\theta(z^{-1})}{\phi(z^{-1})} - \frac{\gamma(z^{-1})\omega(z^{-1})\omega(z)}{\phi(z^{-1})\alpha(z^{-1})\alpha(z)}z^{-f-1},\\ &= \psi(z^{-1}) + \frac{\gamma(z^{-1})}{\phi(z^{-1})}z^{-f-1} - \frac{\gamma(z^{-1})\omega(z^{-1})\omega(z)}{\phi(z^{-1})\alpha(z^{-1})\alpha(z)}z^{-f-1},\\ &= \psi(z^{-1}) + \frac{\gamma(z^{-1})[\alpha(z^{-1})\alpha(z) - \omega(z^{-1})\omega(z)]}{\phi(z^{-1})\alpha(z^{-1})\alpha(z)}z^{-f-1},\\ &= \psi(z^{-1}) + \lambda\frac{\gamma(z^{-1})\delta(z^{-1})(1-z)^d\delta(z)}{\phi^*(z^{-1})\alpha(z^{-1})\alpha(z)}z^{-f-1}.\end{aligned}$$

From this equation, we can calculate the normalized sum of squares of the error variable values for the controller from the following equation

$$s_y^2 = \underset{z=0}{Residue}\,[\psi(z)\psi(z^{-1})\frac{1}{z} + \lambda^2\frac{\gamma(z)\delta(z)\delta(z)(1-z)^d\gamma(z^{-1})\delta(z^{-1})\delta(z^{-1})(1-z^{-1})^d}{z\phi^*(z)\alpha(z)\alpha(z)\phi^*(z^{-1})\alpha(z^{-1})\alpha(z^{-1})}].$$

4. AN EXAMPLE

For a numerical example, let us consider the following nonminimum phase feedback control system with the transfer function

$$\begin{aligned}\hat{y}_t &= \frac{\omega(z^{-1})}{\delta(z^{-1})}u_{t-1},\\ &= \frac{-0.4322 + 0.7806z^{-1} + 0.4655z^{-2} - 0.1942z^{-3}}{1+0.0835z^{-1}-1.2126z^{-2}-0.0635z^{-3}+0.3475z^{-4}}u_{t-1}.\end{aligned}$$

The system is required to follow an exponential change to a new set point with the equation

$$\frac{\theta(z^{-1})}{\phi(z^{-1})} = \frac{1}{(1-0.2z^{-1})(1-z^{-1})}.$$

From the last equation and with the value of the dead time $f = 0$, we can obtain the following polynomials:

$$\psi(z^{-1}) = 1,$$
$$\gamma(z^{-1}) = 1.2 - 0.2z^{-1}.$$

The system is nonminimum phase; therefore, a penalty constant is imperative for the system. Assuming that the penalty constant has a value of $\lambda = 0.05$, we can obtain the necessary polynomials as follows. From the spectral factorization equation

$$\alpha(z)\alpha(z^{-1}) = \omega(z)\omega(z^{-1}) + \lambda\delta(z)(1-z)^d\delta(z^{-1})(1-z^{-1})^d,$$

we can obtain the polynomial

$$\alpha(z^{-1}) = 1.0272 - 0.0873z^{-1} - 0.4736z^{-2} + 0.1363z^{-3} + 0.0341z^{-4} - 0.0169z^{-5}.$$

From the spectral separation equation

$$\frac{\gamma(z^{-1})\omega(z)}{\phi(z^{-1})\alpha(z)} = \frac{\beta(z^{-1})}{\phi(z^{-1})} + \frac{\zeta(z)}{\alpha(z)}z,$$

we can obtain the following polynomials:

$$\beta(z^{-1}) = 1.2631 - 0.2631z^{-1},$$
$$\zeta(z^{-1}) = 1.7833 + 0.7238z^{-1} - 0.4267z^{-2} - 0.0218z^{-3} + 0.0214z^{-4}.$$

With all the necessary polynomials procured, we can get the controller, given by (2), as

$$u_t = \frac{\begin{array}{r}1.2297 - 0.1534z^{-1} - 1.5125z^{-2} + 0.2325z^{-3}\\ +0.4436z^{-4} - 0.0890z^{-5}\end{array}}{\begin{array}{r}1 + 0.4465z^{-1} - 1.5317z^{-2} - 0.2398z^{-3}\\ +0.3912z^{-4} - 0.0662z^{-5}\end{array}} y_t + \frac{\begin{array}{r}0.9736 - 1.087z^{-1} - 1.0834z^{-2} + 1.3711z^{-3}\\ +0.1764z^{-4} - 0.4183z^{-5} + 0.0677z^{-6}\end{array}}{\begin{array}{r}1 + 0.4465z^{-1} - 1.5317z^{-2} - 0.2398z^{-3}\\ +0.3912z^{-4} - 0.0662z^{-5}\end{array}} v_t.$$

The controller has integral action in the feedback loop, because the denominator polynomial in the above equation has a zero with the value $z^{-1} = 1$. However, the feedforward path does not have integral action, because this zero of integration is canceled out by a zero of the same value.

Many setpoint-change or tracking controllers have only one control path, called degree of freedom (DOF), symbolized by the signal y_t. So the controller with only the variable y_t on the right hand side of the last equation is called an 1-DOF controller. Then some researchers, see Mosca (1995) page 69, broke the signal y_t into two signals $\hat{y}_t$ and y_t^{sp} and called their controllers a 2-DOF controller. But this idea is a *faux pas* in control theory because the controller is nonperforming. This was mentioned in Grimble (1994). The purpose of feedback control is to force the signal $\hat{y}_t$ to be the same as the signal y_t^{sp}. The best way to do it is to subtract them together and force the difference to be zero. Breaking the error variable signal y_t into two signals might give a control engineer some degrees of freedom, but the act defeats the purpose of feedback control.

The controller given by last equation can also be called a 2-DOF controller due to the two signals y_t and and v_t. But these two signals can be broken into 3 control paths: the available output variable $\hat{y}_t$, the current and past setpoint values and the future setpoint values. The setpoint variable y_t^{sp}, however, cannot be legitimately named as two distinct variables. Reference Grimble (1994), therefore, uses the name 2.5-DOF for the controller given by the last equation, which is what this paper also uses.

The responses of the variables by the two controllers, 1-DOF and 2.5-DOF, are shown in Fig. 3. From the top graph of this figure, we can see that the 1-DOF controller cannot overcome an inverse response by a change of the set point to a new level, but the 2.5-DOF controller can. The output variable given by this controller follows the setpoint closely without dipping first and rising later like the one given by the 1-DOF controller.

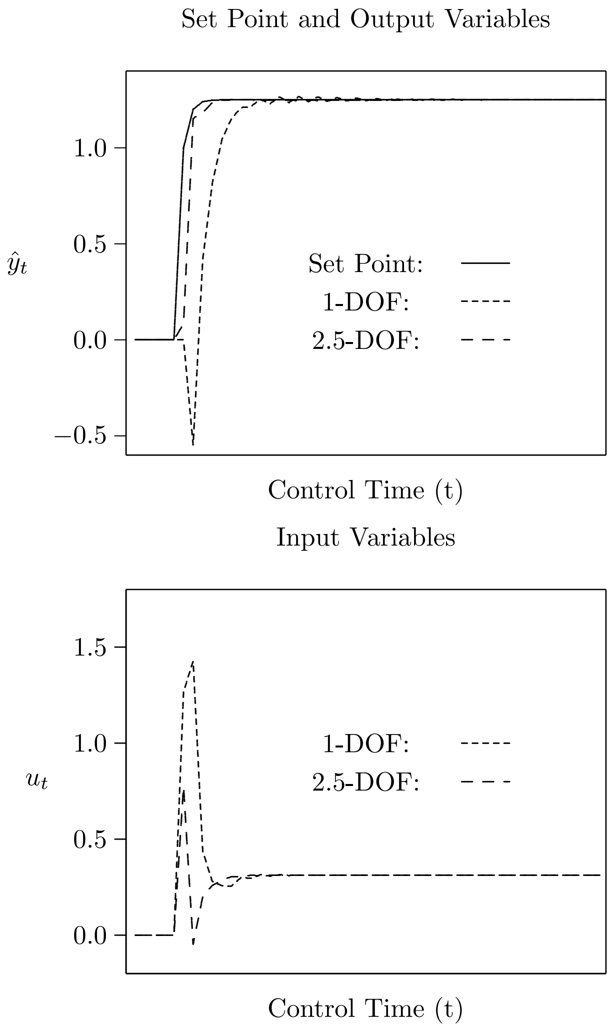

Figure 3: Responses of Controllers.

We now check the value of the performance index for the controller. We can write

$$\hat{s}^2 = \underset{z=0}{Residue}\,[\frac{\psi(z)\psi(z^{-1})}{z} + \lambda\frac{\delta(z)\gamma(z)\delta(z^{-1})\gamma(z^{-1})}{z\alpha(z)\phi^*(z)\alpha(z^{-1})\phi^*(z^{-1})}],$$
$$= 1 + 0.05 \times 2.2030,$$
$$= 1.1101,$$
$$= 1.0201 + 0.05 \times 1.8001,$$
$$= s_y^2 + \lambda s_u^2.$$

We can see that the controller obeys its performance index. The control algorithm has an infinite control horizon, but the sums of squares s_y^2 and s_u^2 and the performance index value $\hat{s}^2$ have finite values: This means that the controller gives stable feedback control values. For a nonminimum phase system, the 2.5-DOF controller usually gives a significant improved performance over the 1-DOF controller. For a minimum phase control system, the improvement in performance of the 2.5-DOF controller over that of the 1-DOF controller is usually negligible.

The formulae for the performance index and the sums of squares of the 1-DOF controller are given in Vu (2008a) as follows.

$$\hat{s}^2_{1-dof} = \underset{z=0}{Residue}[\psi(z)\psi(z^{-1})\frac{1}{z} + \lambda\frac{\delta(z)\gamma(z)\delta(z^{-1})\gamma(z^{-1})}{z\alpha(z)\phi^*(z)\alpha(z^{-1})\phi^*(z^{-1})} + \frac{\zeta(z)\zeta(z^{-1})}{z\alpha(z)\alpha(z^{-1})}],$$

$$s^2_{u,1-dof} = \underset{z=0}{Residue}\frac{\delta(z)\beta(z)\delta(z^{-1})\beta(z^{-1})}{z\alpha(z)\phi^*(z)\alpha(z^{-1})\phi^*(z^{-1})}$$

and

$$s^2_{y,1-dof} = \underset{z=0}{Residue}\frac{\eta(z)\eta(z^{-1})}{z\alpha(z)\phi^*(z)\alpha(z^{-1})\phi^*(z^{-1})}$$

with

$$\eta(z^{-1}) = \frac{\alpha(z^{-1})\theta(z^{-1}) - \omega(z^{-1})\beta(z^{-1})z^{-f-1}}{(1-z^{-1})^d}.$$

For this 1-DOF controller, we have

$$\hat{s}^2_{1-dof} = \underset{z=0}{Residue}[\psi(z)\psi(z^{-1})\frac{1}{z} + \lambda\frac{\delta(z)\gamma(z)\delta(z^{-1})\gamma(z^{-1})}{z\alpha(z)\phi^*(z)\alpha(z^{-1})\phi^*(z^{-1})} + \frac{\zeta(z)\zeta(z^{-1})}{z\alpha(z)\alpha(z^{-1})}],$$
$$= 1 + 0.05 \times 2.2030 + 3.9888,$$
$$= 5.0990,$$
$$= 4.9773 + 0.05 \times 2.4336,$$
$$= s^2_{y,1-dof} + \lambda s^2_{u,1-dof}.$$

It has much higher values for the sums of squares of the error and input variables.

5. CONCLUSION

In this paper, we have presented a model predictive controller that can prevent an inverse response of a control system with a nonminimum phase transfer function. A model predictive controller with a long-range prediction can suppress only part of the inverse response because it can see part of the inverse response through its finite control horizon. The predictive controller presented in this paper has an infinite control horizon, so it can completely prevent an inverse response. The paper also demonstrates the fact: Long-range predictive control is only legitimate for tracking control.

REFERENCES

G.E.P. Box and G.M. Jenkins. *Time Series Analysis, Forecasting and Control.* Holden Day, San Francisco, CA., USA, 1976. ISBN 0-816-21104-3.

E.F. Camacho and C. Bordons. *Model Predictive Control in the Process Industry.* Springer, 1995. ISBN 3-540-19924-1 or 978-3-5401992-4-3. 1995.

E.F. Camacho and C. Bordons. *Model Predictive Control.* Second Edition. Spinger, 2003. ISBN 978-1-8523369-4-3.

D.W. Clarke, C. Mohtadi and P.S. Tuffs. *Generalized Predictive Control: Part I & II".* Automatica, Vol 23, No. 2, pp 137-148 & 149-160. 1987.

C.R. Cutler and B.C. Radamaker. *Dynamic Matrix Control: "A Computer Control Algorithm".* Proceedings of the Joint American Control Conference, San Francisco, 1980.

B.C. Ding. *Modern Predictive Control.* CRC Press. Taylor & Francis Group. 2010. ISBN 978-1-4200853-0-3.

M.J. Grimble. *Robust Industrial Control: Optimal Design Approach for Polynomial Systems.* Prentice-Hall International Ltd., U.K., 1994. ISBN 0-136-55283-8.

E. Mosca. *Optimal, Predictive and Adaptive Control.* Prentice-Hall Inc., Englewood Cliffs, NJ, USA, 1995. ISBN 0-13-847609-8.

K. Vu. *The ARIMA and VARIMA Time Series: Their Modelings, Analyses and Applications.* AuLac Technologies Inc., Ottawa, ON, Canada, 2007. ISBN 978-0-9783996-1-0.

K. Vu. *Optimal Discrete Control Theory: The Rational Function Structure Model.* AuLac Technologies Inc., Ottawa, ON, Canada, 2008a. ISBN 978-0-9783996-0-3.

K. Vu. *ISO, MPC and PID: The Good, The Bad and The Ugly Discrete Controllers.* White Paper of AuLac Technologies Inc., Ottawa, ON, Canada. 2008b. Downloadable from company's website URL: www.aulactechnologies.com.

