Preprints of the
9th International Symposium on Advanced Control of Chemical Processes
The International Federation of Automatic Control
June 7-10, 2015, Whistler, British Columbia, Canada

TuPoster2.6

A survey of guaranteeing feasibility and stability in MPC during target changes

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Abstract: This paper gives a concise review of work on ensuring feasibility and stability within linear MPC during target changes. A summary of similarities and weaknesses of the various proposals in the literature is presented and this forms a useful baseline for establishing where improvements can be made.

Keywords: Model predictive control; Tracking control; Feasibility; Constraints

1. INTRODUCTION

Model Predictive Control (MPC) is one of the most successful advanced control approaches in the industrial field. This is, in part, due to its ability to handle input and state constraints. The basic concepts of model predictive control are well illustrated and understood e.g. Rossiter (2003); Wang (2009). However, it noted that in the literature MPC tracking problems have often been considered as regulating problems, about a steady-state operating point, rather than full tracking problems.

One of the challenges within a tracking scenario is the need to ensure feasibility, that is to guarantee that the class of predictions available to the MPC algorithm can indeed satisfy all the constraints simultaneously. However, even putting to one side issues linked to model uncertainty, feasibility can easily be lost during rapid or large set point changes and during disturbance changes, both of which have a strong impact on terminal constraints Limón et al. (2008); Rossiter (2003). Consequently, there is a strong link between set point tracking and feasibility; feasibility of the controller may be lost and the controller ill-defined, or not defined at all Rawlings et al. (2008), in the case of some set point changes.

A convenient and essential component to enable stability guarantees of MPC algorithms is to ensure feasibility, that is, to ensure the existence of a set of future controls which ensure predictions meet all constraints, including the terminal constraint. For a suitable underlying MPC approach such as dual-mode Scokaert and Rawlings (1998); Rossiter et al. (1998), a feasibility guarantee is often sufficient to enable a simple guarantee of nominal (and at times robust) closed loop stability for the controller. Some authors have tackled the potential loss of feasibility during target changes by developing modified formulations for the MPC algorithm, but nevertheless, the range of solutions and approaches in the literature is still quite limited.

1.1 Overview of MPC approaches for tracking

Many results have been obtained for feasibility and stability of MPC for tracking scenarios. In Rossiter et al. (1996), a Constrained Stable Generalised Predictive controller (CSGPC) for SISO plants is presented; the proposed controller ensures feasibility by deploying, temporarily, an artificial reference as a degree of freedom (d.o.f) and convergence is ensured by means of a contractive constraint based on the artificial reference. In Rossiter (2006), it is demonstrated that changes to the loop target can be a very effective mechanism for increasing the volume of feasible regions; consequently an artificial target can be a more useful degree of freedom within MPC than the more normal choice of future control increments. A conceptually equivalent approach for dealing with temporary infeasibility due to changes in the target is a command governor (CG), whose action is based on the current state, set point and prescribed constraints Bemporad et al. (1997). One example of this approach is the addition to the system of a non-linear low-pass filter, which is selected to ensure the satisfaction of constraints while retaining offset free tracking behaviour. Command governor approaches have a strong synergy with closed loop paradigm implementations of MPC. Similar ideas have been applied in conjunction with non-square techniques Shead and Rossiter (2007) which investigates the impact of non-square systems on feasibility in tracking problem in linear MPC.

In Limón et al. (2008); Ferramosca et al. (2009), a slight variation on MPC for tracking changing constant references for both constrained linear and non-linear systems was presented. These controllers ensure feasibility by means of adding an artificial steady state and input as a degree of freedom to the optimization problem. Convergence to an admissible target steady state is ensured by using a modified cost function and a stabilizing extended terminal constraint. Optimality is ensured by means of an offset cost function which penalizes the difference between the artificial reference and the real one. A subtlety of interest here is that the recovery of local optimality Ferramosca et al. (2009) despite the use of a bias term in the performance index. It is proved that the proposed con-

troller steers the system to the target if this is admissible. If not, the controller converges to an admissible steady state optimum according to the modified performance index. The approach was extended in Ferramosca et al. (2011) by considering an alternative offset cost function based on the infinity norm as it was show this enabled the recovery of a local optimality property while retaining all the key properties of the original formulation.

In Rao and Rawlings (1999), a new infinite horizon MPC formulation for the case of the active steady state constraints is implemented and discussed. This new formulation is based on an iterative algorithm that determines the optimal solution of the control problem within a user specified tolerance. In Rawlings et al. (2008), the case of unreachable set points is considered; here the authors propose that the performance index should be based on the distance from the unreachable set point rather than the artificial reachable one. The authors proved the asymptotic stability and the convergence to the steady state values with a desired tracking response, despite the performance index being technically unbounded.

1.2 Contributions of this paper

This paper reviews some existing methods discussed above which are related to the feasibility and stability in linear MPC for different tracking scenarios and from this proposes avenues for future work. Section 2 demonstrates the basic MPC algorithm components, section 3 describes different methods for retaining feasibility, section 4 presents the summary of similarities and weaknesses of each method and section 5 gives the conclusions and future work.

2. BASIC MPC ALGORITHM COMPONENTS

In this section, we summarise the main MPC algorithm components such as models, performance indices, and the degrees of freedom. These are used as foundation for the comparisons given later.

2.1 State space Model

For convenience, the model type used in the reviewed methods is the state space model represented as follows:

$$x_{k+1} = Ax_k + Bu_k, y_k = Cx_k + Du_k \qquad (1)$$

where x, y, u are states, process output and process inputs respectively and A, B, C, D are matrices defining the model; here take $D = 0$. Input and output disturbances d and p can be incorporated with small modifications:

$$x_{k+1} = Ax_k + B(u_k + d), y_k = Cx_k + p \qquad (2)$$

In this case, the observer models are augmented to include the disturbance into the system dynamics. For conciseness model parameter uncertainty is not discussed in this paper.

2.2 Performance index or cost function

For a typical MPC algorithm, the control law is based on the optimisation of predicted performance based on a performance index. The most common performance index e.g. Rossiter (2003); Rossiter et al. (1996) penalises the

weighted squares of both predicted tracking errors and the control increments/deviations (weights W, W_d), that is:

$$J = \sum_{i=1}^{n_y} \|r_{k+i} - y_{k+i}\|_2^2 + \sum_{i=0}^{n_u-1} \|W(u_k - u_{ss})\|_2^2 + \|W_d\Delta u_k\|_2^2$$

$$(3)$$

where u_{ss}, x_{ss} are the expected steady-states of the input and states which enable $y \to r$ asymptotically, r being the notional true output target.

Nevertheless, the treatment of tracking may require minor changes to this popular index where horizons n_y, n_u are large (or infinite) and it is not possible for the output prediction to reach the desired target while satisfying constraints (which would imply J is unbounded (Rawlings et al. (2008)). A common adjustment, presented in detail in the next section, is to penalise the deviations from a *well selected* reachable but artificial steady state target and in addition penalising the deviation of the artificial steady state target from the true steady state target e.g. Rao and Rawlings (1999); Limón et al. (2008); Rossiter et al. (1996); Rossiter (2006).

2.3 Constraints.

Many processes contain constraints such as upper and lower limits on the input $\underline{u}_i \le u_i \le \overline{u}_i$, or on the input rates $\triangle \underline{u}_i \le \triangle u_i \le \triangle \overline{u}_i$. One can also contain constraints on outputs and states such as $\underline{y}_i \le y_i \le \overline{y}_i$ and $\underline{x}_i \le x_i \le \overline{x}_i$ respectively, and indeed more complex constraints can also exist. Input constraints are usually refereed to hard constraints which must be satisfied while output/state constraints may be refereed to as soft constraints which should be satisfied if possible. Assuming a linear model, the constraints above can be captured as linear inequalities in the assumed d.o.f and hence combine with the performance index to give a quadratic programming optimisation which defines the control law.

Constraints are a key factor because it is often inconsistency between these and the requirement that the predicted output reaches the target asymptotically ($\lim_{k\to\infty} y_k \to r$) while using just a few control d.o.f. that causes so called infeasibility. In such cases MPC is undefined and so something has to be changed. One could relax so called soft constraints, but the approaches discussed in this paper consider the alternative mechanism of temporary, or permanent, changes to the target r.

2.4 Degrees of freedom (d.o.f)

It is common to define the degrees of freedom as the first n_u control increments (or moves), that is $u_k, \cdots, u_{k+n_u-1}$. For convenience, with infinite horizon algorithms, the d.o.f can be equivalently parametrised Rossiter et al. (1998) as perturbations c_k about a nominal stabilising control law.

$$\begin{aligned} u_k - u_{ss} &= -K(x_k - x_{ss}) + c_k; \ i < n_u \\ u_k - u_{ss} &= -K(x_k - x_{ss}); \qquad i \ge n_u \end{aligned} \qquad (4)$$

However, in cases where the change in the steady-state x_{ss}, u_{ss} is too rapid, the prediction class (4) is not sufficiently large to meet constraints and therefore either more (larger n_u) or alternative d.o.f. are required. A simple solution Rao and Rawlings (1999) could be to increase the

control horizon n_u, but this is computationally expensive so not pursued here. It is common in the existing work for guaranteeing feasibility of MPC during tracking to add alternative d.o.f in the optimisation problem to the normal MPC d.o.f mentioned above:

(1) The extra d.o.f proposed in Rossiter et al. (1996) is a slack variable, essentially a deviation from the ideal steady-state.
(2) In Shead et al. (2008); Limón et al. (2008) the d.o.f are two free variables that capture changes in the steady state as well as transient control moves.

This paper does not discuss alternative parameterisations of the degrees of freedom Khan and Rossiter (2013) as that is a conceptually different approach.

3. FEASIBILITY IN MPC FOR TRACKING

Common causes of infeasibility are set point and disturbance changes which cause a conflict between the input parameterisation of (4) (in effect the terminal mode requirements of a dual-mode approach) and the constraints. Several authors have proposed approaches to maintain the feasibility and guarantee asymptotic stability in dual-mode MPC hence the aim of this section is to give a brief review of some of those approaches.

Remark 1. It should be noted that single mode finite horizon algorithms often have better feasibility properties due to the lack of a terminal constraint, but there can also be corresponding performance/stability weaknesses and consequently this paper focusses on dual-mode approaches.

3.1 Slack Variable Endpoint Constraint (SVEC)

This approach was first presented by Rossiter et al. (1996) for guaranteeing feasibility and stability of dual mode MPC during rapid set point changes; further insights and options were presented in Rossiter (2006). The key idea is that constrained stable generalised predictive control (CSGPC[1]) is applied while it remains feasible, but to switch to a new strategy which implements a modified terminal constraint at times when CSGPC is infeasible; this is done by replacing the terminal mode in (4) by:

$$u_k - u_{ss} = -K(x_k - x_{ss}) + c^\infty; \ i \geq n_u \qquad (5)$$

Convergence is achieved by ensuring the implied slack variable c^∞ converges to zero and thus ultimately, the system converges to the correct set point.

(1) The original proposal of Rossiter et al. (1996) enforced a contractive constraint on the slack variable c^∞, that is

$$\|c^\infty_{k+1}\| \leq \|c^\infty_k\| \qquad (6)$$

and minimised the true cost J subject to this constraint and using predictions (5).
(2) Later work in Rossiter (2006) noted that the underlying concept could easily be applied for general terminal conditions and also replaced the constraint (6) by modifying the cost as follows:

$$\min_{\vec{c}, c^\infty} \|c_\infty\|_P + \sum_\infty \|r - y - \alpha c_\infty\|_2^2 + \lambda \|\Delta u\|_2^2 \qquad (7)$$

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[1] Readers should note that CSGPC had deadbeat terminal constraints, although the principal concepts can be applied to more general terminal constraints.

Readers will note the tracking performance is based on the artificial target (namely $r - \alpha c^\infty$) and not the true target and the addition of a term penalising the magnitude of the slack variable.

(3) It can be shown that, assuming $c^\infty = 0$ is feasible asymptotically, then $c^\infty \to 0$ and thus the approach will both retain feasibility and converge as, at each sample instant, there is an implied terminal constraint $\lim_{k \to \infty} y_k = r - \alpha c^\infty$.

Readers may also note that, in cases where the true set point is not reachable, the proposed algorithm will cause the slack variable and thus the output to settle at the nearest (determined by any scaling in J for the MIMO case) boundary.

3.2 MPC for tracking

Although in principle applicable to the MIMO case, the approach of Rossiter (2006) was limited in scope and detail. Subsequently, the approach was extended by Limón et al. (2008) to more generic scenarios such as non-square systems and to take up an flexibility in the implied steady-state for a given target. The authors proposed to use a cost function which is explicitly linked to the states, rather than the outputs, and based on deviations in the states and inputs from their target steady-state values (x_{ss}, u_{ss}). The 'artificial' steady-state is denoted here as x_{sa}, u_{sa} and x_N is the terminal state (N steps ahead):

$$J = \sum_{i=0}^{N-1} (\|x_i - x_{sa}\|_Q^2 + \|u_i - u_{sa}\|_R^2) \\ + \|x_N - x_{sa}\|_P^2 + \|x_{ss} - x_{sa}\|_T^2 \qquad (8)$$

The key contribution of this paper is to define and exploit the full flexibility in the implied steady-state x_{sa}, u_{sa} to meet an implied steady-state output y_{sa}.

$$\begin{bmatrix} A - I_n & B & 0 \\ C & D & -I_p \end{bmatrix} \begin{bmatrix} x_{sa} \\ u_{sa} \\ y_{sa} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_{sa} \\ u_{sa} \\ y_{sa} \end{bmatrix} = X\theta \quad (9)$$

The control input predictions of (4) then take the form

$$u = K(x - x_{sa}) + u_{sa} + c_k = Kx + L\theta + c_k. \qquad (10)$$

where flexibility in the artificial target is embedded in the d.o.f. θ; the definition of L follows immediately from X and (10). It is noted that this new controller provides a large domain of attraction and of course guaranteed feasibility/convergence. However, the additional degree of freedom and the use of artificial targets will result in loss of local optimality.

3.3 Recovering local optimality with modified performance indices

It is clear that the performance indices of (7,8) include what could be considered two alterations from what is ideal.

(1) The predictions are costed on the basis of an artificial (or incorrect) target.
(2) The performance index has a transient bias due to the inclusion of the additional term weighting the distance between the true and artificial targets.

The problem of local optimality loss, that is the inclusion of the bias term, was addressed in Ferramosca et al. (2011).

They showed that one could retain the core property that the artificial target converged to the true target. However, more importantly, by appropriate selection of this bias term, one could also recover local optimality, which is to say the bias term does not impact on the main performance aspects of the optimisation.

To what extent this issue, that is local optimality, is significant is probably open to debate. Clearly the desire is that the artificial target keeps changing until it returns to the true value, and therefore any 'optimality' in the predictions with respect to a temporary artificial target is to some extent meaningless as the future parts of the performance are being assessed against a target which will no longer be valid in the next samples. Ideally, one would want performance to be measured against the most representative infinite horizon performance index possible, and that of course is with respect to the true target.

3.4 Infinite variable horizon regulator

This approach presented in Rao and Rawlings (1999) is also focused on implementing an infinite horizon MPC algorithm for the scenarios where constraints are active at steady state due to set point/disturbance changes. It uses a slightly more complex disturbance model but otherwise is conceptually similar to the previous section. The key difference is that the step used to determine the artificial target deploys a cost which includes a linear term. First capture the offset between the ideal target r and the artificial target y_{sa} as follows:

$$\|r - y_{sa}\| \le \eta; \quad \eta \ge 0 \tag{11}$$

Next, utilise the following minimisation to determine the artificial steady-state; this assumes equivalent equations to (9).

$$\min_{x_{sa}, u_{sa}, \eta} [\eta^T Q_{ss} \eta + (u_{ss} - u_{sa})^T R_{ss} (u_{ss} - u_{sa})] + q_{ss}^T \eta \tag{12}$$

It can be shown that, if feasible, then $\eta = 0$ will result. The authors argue that the combination of linear and quadratic terms on η gives clear benefits in the resulting solution. There are some minor nuances required to ensure the target steady-state x_{sa} is unique.

Hereafter, the control law derivation is a standard dual-mode predictive control law Scokaert and Rawlings (1998), but based on the artificial steady-states. Nevertheless, there is some discussion of the fact (often overlooked in papers) that when the steady-state lies on an active constraint then computation of the admissible set Gilbert and Tan (1991) may not converge!

3.5 Set point-tracking MPC (sp-MPC)

This approach was proposed by Rawlings et al. (2008) for scenarios where the true set points are unreachable, for example due to a large disturbance. This approach is also quite similar conceptually to section 3.1 in that it implicitly uses the same input parametrisation of eqns.(4,5,10) but with some minor differences: (i) there is a two stage optimisation equivalent to algorithm 2.1 in Rossiter et al. (1996) and (ii) the true set point was retained in J rather than the artificial one as in eqns.(7,8).

(1) First find the 'artificial target' $(x_{sa}, u_{sa}, $ or equivalently slack c^∞) as close as possible to the true target

(x_{ss}, u_{ss}) such that one can determine a feasible input trajectory.

$$\min_{\overrightarrow{c}, c_\infty} (x_{sa} - x_{ss})^T Q(x_{sa} - x_{ss}) + (u_{sa} - u_{ss})^T R(u_{sa} - u_{ss}) \tag{13}$$

(2) Use input predictions (4,5) with the c^∞ from optimisation (13) and then optimise the 'true' performance index (for example):

$$J = \sum_{i=0}^{\infty} (\|x_i - x_{ss}\|_Q^2 + \|u_i - u_{ss}\|_R^2) \tag{14}$$

The main contribution of this paper was not so much the choice of degrees of freedom, the steps or the choice of cost, but rather the observation that even with general terminal conditions (CSGPC used dead-beat conditions), despite the cost function (14) being unbounded in cases where the artificial target does not match the true target, it is still possible to derive a rigorous proof of convergence (recursive feasibility is obvious). It is suggested that using the true target rather than the artificial target within J modifies transient behaviour and is expected in general to lead to better performance although this observation is somewhat subjective as one could equally argue that a cost based on an unreachable target is an ill-posed one.

3.6 Robust offset free tracking with active constraints

A robust approach was presented by Shead et al. (2008) and extends the work of Limón et al. (2008) and others by analysing how the decision making alters in the case of parameter uncertainty and where the final target is unreachable. It was shown that optimisation algorithms already in the literature may result in poor choices for the artificial steady-state in the case where there is model uncertainty, that is what appears to be the closest point in space within the feasible region to the real target may in fact be be far from the best artificial target to aim for.

Parameter uncertainty alters the shapes of the feasible regions and implicitly the equalities implicit in (9). It is easy to show that even small changes in a few parameters can lead to large changes in the implied 'best' steady-state (x_{sa}, u_{sa}); clearly the designer wants to be able to monitor, as far as possible, whether the artificial steady-state is indeed close to the best position. The paper Shead et al. (2008) demonstrated that an algorithm such as that in Limón et al. (2008) which does not explicitly allow for unreachability could (not must) converge to the wrong steady-state; for readers looking at these two papers the performance indices and d.o.f. look slightly different but in essence are equivalent.

The original observations of Shead et al. (2008) were extended in Shead et al. (2010) where an algorithm was proposed which iteratively improved the choice of the artificial steady-state to take account of parameter uncertainty. The technique was based on an analysis of the underlying KKT conditions for optimality and compensates for the relaxation of condition of optimality in Kvasnica (2009). Using KKT conditions of optimality, it is shown that the necessary condition for correct target selection for SISO systems is that the sign of the determinant of both the plant and the model's steady state gains and must be the same. For multivariable systems, the necessary condition

for a single active constraint, is that the rows of the inverse matrices of the plant and the model's steady state gains corresponding to the active constraints should be linearly dependent. For multiple active constraints a more general sufficient condition for constrained offset free control is that model gain matrix differs from plant gain matrix by only a scalar gain.

In summary, the key contribution is to make a case for a separate SSTO (steady-state target optimisation Muske and Rawlings (1993),Rao and Rawlings (1999)) rather than embedding into the performance index as in (8,7).

(1) The proposed design is a two stage process with stage 1 being the iterative estimate of the artificial steady-state to improve the choice.
(2) The performance index to be minimised is based solely on the artificial target and takes no account of the real, unreachable, target (as in Rao and Rawlings (1999)).

Nevertheless, while this SSTO is very insightful, it could be difficult to use effectively on MIMO examples as the implied KKT conditions for convergence to the correct point are quite restrictive.

4. DISCUSSION

The predominant approach for dealing with unreachable targets so to determine an an artificial target that can be used in the performance index rather than the true set point. This artificial set point is updated such that it converges to the true set point if possible, and if not, to a point which in some sense is closest. Nevertheless, it would be useful to summarise what the key differences and similarities are and use these as a foundation for considering what issues have yet to be tackled effectively. A convenient comparison considers the steady-state computation, the assumed terminal mode and the choice of performance index. All the approaches discussed in this paper implicitly use a dual-mode prediction structure of the form (4,5).

4.1 Steady state calculation

The main divide is between computations which are done as part of a separate optimisation and those which are embedded within the prediction optimisation. Also, any separate optimisations are typically based on a quadratic or linear programme.

(1) For SVEC, the steady-state is either computed from a linear programme (LP) or included as an inequality (contraction) constraint into the performance optimisation. The work of Rawlings et al. (2008) also deploys a separate SSTO although which in some sense could be viewed as closer to a 1-norm.
(2) Limón et al. (2008),Ferramosca et al. (2011) and Rossiter (2006) embed the steady-state into the performance index as an additional objective and thus have a single optimisation, although with extra d.o.f. to deal with the flexibility in the steady-state.
(3) The works of Rao and Rawlings (1999), Shead et al. (2010) have a comprehensive SSTO which identifies the best (typically via a 2-norm measure) target.

4.2 Terminal modes and constraints

The terminal constraints, or terminal mode of a dual-mode strategy, play a key role in the obtainable performance, the volume of the feasible regions, the computational load and so forth. For the algorithms discussed here, the implied terminal mode is identical and governed by an input prediction of the form of (5), that is a fixed control law with an offset term. Only SVEC differs in that the choice of the underlying feedback K is a dead-beat one although this is not necessary. Asymptotic constraint handling is possible using conventional invariant set approaches.

4.3 MPC performance index, optimisation and d.o.f.

As implied above, the degrees of freedom deployed in each algorithm are essentially equivalent, albeit the original works used different notation and expressions. One can easily show one-to-one mappings between the parametrisations selected. For example one can choose perturbations to an nominal input trajectory or the values themselves, but in essence one has the same flexibility and the preferred option depends upon algebraic convenience. The user can choose the deviations c_k in the transient inputs of (4) and an offset term c^∞ or equivalently u_{sa}, x_{sa} (discounting special cases). The degrees of freedom can be deployed in a single optimisation or determined in a 2-stage process.

With the exception of SVEC which deployed dead-beat terminal constraints, each of the proposed methods implicitly deploys infinite horizon performance indices but there are some minor differences. Conceptually the original choices made of whether to penalise input deviations or input increments or indeed both is not relevant as all algorithms can equally include both terms as in (3) with minor algebraic modifications.

(1) SVEC and Shead et al. (2010) optimise performance with respect to the artificial target only.
(2) The papers of Rossiter (2006), Limón et al. (2008) Ferramosca et al. (2011) optimise performance with respect to the artificial target but add an additional term to penalise the distance from the artificial target to the true target. As a consequence, in some sense the optimisation has mixed objectives which could be argued not to match an ideal criteria.
(3) The work of Rawlings et al. (2008) penalises the distance from the true target, but has a performance index that is strictly unbounded and thus there are questions over how meaningful this really is.

4.4 Summary

Table I summarises the similarities and differences of the algorithms discussed. It is clear that some decisions are viewed as objectively reasonable whereas others are still more open to debate.

(1) It is unclear whether it is better to have a 2-stage or 1-stage approach; obviously this is excluding the large scale problems where a separate optimisation would be automatic anyway.

(2) More papers seems to favour the use of the artificial target within the performance index, but an objective comparison of the pros and cons of this is outstanding.

(3) The use of input parametrisations built on dual mode structures with a feasible terminal mode seem to be used throughout.

(4) Very little work has studied the repercussions of parameter uncertainty whereas all the work implicitly caters for unknown disturbances.

(5) Less attention has considered MPC algorithms without terminal modes/constraints although these often give better feasibility.

Concepts	Techniques	Approaches					
		[1]	[2]	[3]	[4]	[5]	[6]
SSTO	Separate LP	*		*			
	Separate QP				*	*	
	Single stage QP		*				*
Cost	True target			*			
	Artificial target	*			*	*	
	Mixed		*				*
Terminal constraint	Contraction const.	*					
	Invariant set		*	*	*	*	*

Table 1. Comparison of algorithms: [1] SVEC, [2] Limón et al. (2008), [3] Rawlings et al. (2008), [4] Rao and Rawlings (1999), [5] Shead et al. (2010), [6]Rossiter (2006)

5. CONCLUSIONS AND FUTURE WORK.

Most of the existing work is focussed on a fixed scenario, that is one whereby a single change in the disturbance and or target leads to infeasibility in transients. In assessing the efficacy of the algorithms it is tacitly assumed that thereafter the target and/or disturbance remains fixed. There is no real consensus on what would constitute a fair global comparison of performance and the choices made are usually pragmatic. There is no clear divide between algorithms that deal with permanent as opposed to temporary infeasibility, that is targets which are unreachable in the long term; consequently a comparison of performance under different scenarios has not really been provided. There is no real consensus over whether one or two stage optimisation is to be preferred. Finally, very little work has really looked at the robust case with any rigour.

Notwithstanding the apparent gaps highlighted above, there are also some other areas which seem to have been largely ignored and indeed attract little attention in the predictive control literature as a whole. One of these is how to make more effective use of feed forward information. All the works above assume that there is no feed forward information of either target changes, or indeed disturbances. In practice, in many cases, some feed forward will be available and that should enable the designer to give much more effective handling or even avoidance of infeasibility. Moreover, this has obvious parallels with reference governor approaches. A second issue that has attracted some interest of late is the potential to use alternative parametrisations for the d.o.f. in the input predictions as opposed to just taking the individual values of future inputs for N samples.

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