Preprints of the
9th International Symposium on Advanced Control of Chemical Processes
The International Federation of Automatic Control
June 7-10, 2015, Whistler, British Columbia, Canada

Stable Adaptive Predictive Control System Design via Adaptive Output Predictor for Multi-rate Sampled Systems ⋆

Ikuro Mizumoto * **Masataka Ikejiri** * **Taro Takagi** **

* *Kumamoto University, 2-39-1 Kurokami, Kumamoto, 860-8555, JAPAN(e-mail: ikuro@gpo.kumamoto-u.ac.jp).*
** *National Institute of Technology, Maizuru College, Shiroya 234, Maizuru, Kyoto 625-8511, JAPAN(e-mail: t.takagi@maizuru-ct.ac.jp)*

Abstract: This paper deals with a design problem of an adaptive predictive control for uncertain multi-rate sampled systems. In the proposed method, an adaptive predictive control using a simple adaptive output estimator, which has been previously proposed for single-rate sampled system, will be expanded to a multi-rate sampled system. A robust and model-free design method of feedforward compensators for designing the adaptive output estimator for predictive control and for setting an input constraint, "almost strictly positive real (ASPR) constraint" for stable control system will also be provided for the considered multi-rate systems. The effectiveness of the proposed method will be confirmed through an experiments of two-tank process control.

Keywords: Adaptive control, predictive control, multi-rate sampled systems, output estimator

1. INTRODUCTION

Predictive control, including MPC and GPC, has been recognized as one of the advanced control scheme on practical process control and it has been widely used in industry due to its understandable underlying idea and powerful control performance than standard PID controls (Clarke et al., 1987; Garcia et al., 1989; Mayne et al., 2000; Maciejowski, 2002). However, in most practical cases, unfortunately, it might be difficult to obtain an exact model of the controlled system because of the existence of some kind or another uncertainty in the practical systems. Since the predictive control schemes require the accurate model of the considered system, the performance of the obtained predictive control system is significantly affected by the accuracy of the given model. The adaptive controls have attracted a great deal of interest as a method to solve the problem on model uncertainties and adaptive type predictive controls have been investigated as in Yoon and Clarke (1994); Nicolao et al. (1996); Fukushima et al. (2007), including an adaptive GPC with a recursive least squares parameter estimator(Yoon and Clarke, 1994), a constrained receding horizon predictive control based adaptive predictive regulator(Nicolao et al., 1996) and an adaptive model predictive control based on a robust MPC method with the comparison model(Fukushima et al., 2007). In those methods, however, in order to attain good parameter estimation for uncertain systems, the structure and the order of the controlled system had to be known. In many practical cases, it is also difficult to obtain the information of the structure and the order of the controlled system, and in the case where the order of the system was relatively higher, the adaptive algorithm might become complex with a large number of parameters to be estimated. With this in mind, a novel adaptive predictive control using an adaptive output predictor with a simple structure for uncertain controlled systems has recently been proposed(Mizumoto and Fujimoto, 2012). In this adaptive predictive control, one can design an adaptive output estimator with a simple structure provided that the system is minimum-phase and has a relative degree of 1, and the stability can be guaranteed by considering a control input constraint based on the virtual ASPR (almost strictly positive real) based output feedback input. However, this method was only applicable to single-rate sampled systems.

In digital control systems, a system with different output sampling periods and input updating rate is recognized as a multi-rate system and one can see such a multi-rate system in many industries because hardware limitations on sensoring and actuating result in different sampling periods. It is valuable to expand the method to multi-rate systems.

In this paper, we expand the method in Mizumoto and Fujimoto (2012) to multi-rate sampled systems. By expanding the adaptive output estimator provided in Mizumoto et al. (2010c, 2011) to adaptive output predictor for multi-rate systems, an adaptive predictive controller with the adaptive output predictor will be proposed. Furthermore, by setting an "almost strictly positive real (ASPR) input constraint", it will be shown that a stable adaptive predictive control can be designed for multi-rate sampled systems as well as single-rate sampled systems. The difficulty is to realize a virtual fast-rate control system which satisfies the given assumptions using the information from the multi-rate system. We will also provide an approximated and model-free design method of compensators which realize

⋆ This work was partially supported by KAKENHI, the Grant-in-Aid for Scientific Research (C) 25420444, from the Japan Society for the Promotion of Science (JSPS).

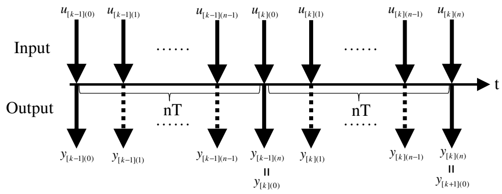

Fig. 1. Considered multi-rate system

the given assumptions on the virtual fast-rate system. Finally, the efffectiveness of the proposed method will be confirmed through numerical simulations.

2. PROBLEM STATEMENT

Consider a multi-rate system for which the input is updated with a fast uniform updating period of T and the output is slowly sampled with a uniform sampling period of nT.

For this multi-rate system, suppose that a virtual fast-rate sampled system with a virtual output sampling period of T can be represented as

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + \boldsymbol{b}u(k) \\ y(k) &= \boldsymbol{c}^T\boldsymbol{x}(k) \end{aligned} \tag{1}$$

where $\boldsymbol{x}(k) := \boldsymbol{x}(kT)$ and $y(k) := y(kT)$ denote the state and virtual fast-rate output of the multi-rate system at a time instant kT.

Now define $y_{[k]} := y(nk)$, and denote virtual fast-rate outputs with sampling period of T within the sampling interval of $y_{[k]}$ and $y_{[k+1]}$ by (see Fig. 1)

$$y_{[k](i)} = y(nk+i) \quad (i = 0, 1, \cdots, n) \ , \tag{2}$$

$$\begin{aligned} y_{[k](0)} &= y_{[k]} \\ y_{[k](n)} &= y_{[k+1]} = y_{[k+1](0)}. \end{aligned} \tag{3}$$

Note that the output $y_{[k](i)}$ for $i = 1, 2, \cdots, n-1$ can not be measured practically. Further, denoting the state and the input at a time instant $(nk+i)T$ by $\boldsymbol{x}_{[k](i)}$ and $u_{[k](i)}$, we have the following virtual fast-rate system representation of the considered multi-rate sampled system:

$$\begin{aligned} \boldsymbol{x}_{[k](i+1)} &= A\boldsymbol{x}_{[k](i)} + \boldsymbol{b}u_{[k](i)} \\ y_{[k](i)} &= \boldsymbol{c}^T\boldsymbol{x}_{[k](i)} \end{aligned} \tag{4}$$

The objective of this paper is to provide an adaptive output predictor for the given multi-rate system and propose an stable adaptive predictive control design method with the adaptive output predictor.

3. BASIC DESIGN OF ADAPTIVE PREDICTIVE CONTROL SYSTEM

3.1 Adaptive output estimator for a multi-rate system

Firstly, we consider an adaptive output estimator which estimate unmeasured outputs $y_{[k](i)},\ i = 1, \cdots, n-1$ within the output sampling interval based on the output estimators proposed in Mizumoto et al. (2010c),Mizumoto et al. (2011),Mizumoto and Fujimoto (2013).

We impose the following assumption on the virtual fast-rate system (1) (or (4)).

Assumption 1. For the virtual fast-rate system (1) (or (4)), there exists a known stable parallel feedforward compensator (PFC):

$$\begin{aligned} \boldsymbol{x}_{fe}(k+1) &= A_{fe}\boldsymbol{x}_{fe}(k) + \boldsymbol{b}_{fe}u(k) \\ y_{fe}(k) &= \boldsymbol{c}_{fe}^T\boldsymbol{x}_{fe}(k) \end{aligned} \tag{5}$$

such that the resulting augmented system:

$$\begin{aligned} \boldsymbol{x}_{ae}(k+1) &= A_{ae}\boldsymbol{x}_{ae}(k) + \boldsymbol{b}_{ae}u(k) \\ y_{ae}(k) &= \boldsymbol{c}_{ae}^T\boldsymbol{x}_{ae}(k) = y(k) + y_{fe}(k) \end{aligned} \tag{6}$$

$$\boldsymbol{x}_{ae}(k) = \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}_{fe}(k) \end{bmatrix}, A_{ae} = \begin{bmatrix} A & \boldsymbol{0} \\ \boldsymbol{0} & A_{fe} \end{bmatrix}, \boldsymbol{b}_{ae} = \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{b}_{fe} \end{bmatrix},$$
$$\boldsymbol{c}_{ae}^T = \begin{bmatrix} \boldsymbol{c}^T & \boldsymbol{c}_{fe}^T \end{bmatrix}$$

is minimum-phase and has relative degree of 1.

Under Assumption 1, there exists an appropriate nonsingular transformation $[y_{ae}(k), \boldsymbol{\eta}_{ae}(k)^T]^T = \Phi\boldsymbol{x}_{ae}(k)$ such that the augmented virtual fast-rate system (6) can be transformed into the following canonical form (Isidori, 1995):

$$\begin{aligned} y_{ae}(k+1) &= a_y^* y_{ae}(k) + b_y^* u(k) + \boldsymbol{c}_\eta^T\boldsymbol{\eta}(k) \\ \boldsymbol{\eta}_{ae}(k+1) &= A_\eta\boldsymbol{\eta}(k) + \boldsymbol{b}_\eta y_{ae}(k) \end{aligned} \tag{7}$$

The zero dynamics of the system (7) is stable from Assumption 1, i.e. A_η is a stable matrix.

Using the expression defined in (2), the system's output in (7) can be represented by

$$\begin{aligned} y_{ae[k](0)} = a_y^* y_{ae[k-1](n-1)} &+ b_y^* u_{[k-1](n-1)} \\ &+ \boldsymbol{c}_\eta^T\boldsymbol{\eta}_{[k-1](n-1)} \end{aligned} \tag{8}$$

$$\begin{aligned} y_{ae[k-1](i)} = a_y^* y_{ae[k-1](i-1)} &+ b_y^* u_{[k-1](i-1)} \\ &+ \boldsymbol{c}_\eta^T\boldsymbol{\eta}_{[k-1](i-1)} \end{aligned} \tag{9}$$

Furthermore, from (8) and (9), the sampled output $y_{ae[k](0)}$ can be expressed by using measured outputs and inputs as

$$\begin{aligned} y_{ae[k](0)} = b_y^* u_{[k-1](n-1)} &+ b_y^* \sum_{i=1}^{n-1} a_{yi}^* u_{[k-1](n-1-i)} \\ &+ a_{yn}^* y_{ae[k-1](0)} + \boldsymbol{c}_{\eta n}^T\boldsymbol{\eta}_{[k-1](0)} \end{aligned} \tag{10}$$

with

$$\begin{aligned} a_{yj}^* &= a_{y(j-1)}^* a_y^* + \boldsymbol{c}_{\eta(j-1)}^T\boldsymbol{b}_\eta \ , \ a_{y0}^* = 1 \\ \boldsymbol{c}_{\eta j}^T &= a_{y(j-1)}^*\boldsymbol{c}_\eta^T + \boldsymbol{c}_{\eta(j-1)}^T A_\eta \ , \ \boldsymbol{c}_{\eta 0}^T = \boldsymbol{0}^T \\ & (j = 1, \cdots, n) \end{aligned}$$

Taking the expressions in (9) and (10) in to account, the output estimator is designed as follows:

$$\begin{aligned} \hat{y}_{ae[k](0)} &= \hat{b}_{y[k]} u_{[k-1](n-1)} \\ &\quad + \hat{b}_{y[k-1]} \sum_{i=1}^{n-1} \hat{a}_{i[k]} u_{[k-1](n-1-i)} + \hat{a}_{n[k]} y_{ae[k-1](0)} \\ \hat{y}_{ae[k](1)} &= \hat{a}_{1[k]} y_{ae[k](0)} + \hat{b}_{y[k]} u_{[k](0)} \\ \hat{y}_{ae[k](i)} &= \hat{a}_{1[k]} \hat{y}_{ae[k](i-1)} + \hat{b}_{y[k]} u_{[k](i-1)} \quad , \\ & \qquad (i = 2, \cdots, n-1) \end{aligned} \tag{11}$$

by neglecting the signal $\boldsymbol{\eta}_{[k-1](n-1)}$ from the stable zero dynamics. Where $\hat{a}_{i[k]}$ and $\hat{b}_{y[k]}$ are estimated values of a^*_{yi} and b^*_y respectively, and are estimated by the following parameter adjusting law with a period of nT.

$$\hat{b}_{y[k]} = \bar{\sigma}\hat{b}_{y[k-1]} - \bar{\sigma}\gamma_b u_{[k-1](n-1)}\epsilon_{ae[k](0)} + p_{b[k]} \quad (12)$$

$$\hat{a}_{i[k]} = \bar{\sigma}\hat{a}_{i[k-1]} - \bar{\sigma}\gamma_{ai}\hat{b}_{y[k-1]}u_{[k-1](n-1-i)}\epsilon_{ae[k](0)} + p_{ai[k]} \quad (i = 1, \cdots, n-1) \quad (13)$$

$$\hat{a}_{n[k]} = \bar{\sigma}\hat{a}_{n[k-1]} - \bar{\sigma}\gamma_{an}y_{ae[k-1](0)}\epsilon_{ae[k](0)} + p_{an[k]} \quad (14)$$

$$\gamma_{ai}, \gamma_b > 0 \ , \ \bar{\sigma} = \frac{1}{1+\sigma} \ , \ 0 < \sigma$$

where $p_{b[k]}, p_{ai[k]}$ are parameter projections which are given by

$$p_{b[k]} = \begin{cases} 0 & \text{if } \underline{b}_y \le \hat{b}_{y[k]} \le \bar{b}_y \\ \bar{\sigma}\gamma_b u_{[k-1](n-1)}\epsilon_{ae[k](0)} & \text{otherwise} \end{cases}$$

$$p_{ai[k]} = \begin{cases} 0 & \text{if } i \ne 1 \text{ or } \underline{a}_y \le \hat{a}_{1[k]} \le \bar{a}_y \\ \bar{\sigma}\gamma_{a1}\hat{b}_{y[k-1]}u_{[k-1](n-2)}\epsilon_{ae[k](0)} & \text{otherwise} \end{cases} \quad (15)$$

$$p_{an[k]} = \begin{cases} 0 & \text{if } n \ne 1 \text{ or } \underline{a}_y \le \hat{a}_{1[k]} \le \bar{a}_y \\ \bar{\sigma}\gamma_{a1}y_{ae[k-1](0)}\epsilon_{ae[k](0)} & \text{otherwise} \end{cases}$$

with $\underline{a}_y$, $\bar{a}_y$, $\underline{b}_y$ and $\bar{b}_y$ such that

$$\underline{a}_y \le a^*_y \le \bar{a}_y \ , \ \underline{b}_y \le b^*_y \le \bar{b}_y$$

and $\epsilon_{ae[k](0)} = \hat{y}_{ae[k](0)} - y_{ae[k](0)}$ is an estimate error which can be generated using the available signals as follows without causality problem:

$$\begin{aligned}\epsilon_{ae[k](0)} = &\Big\{\bar{\sigma}\hat{b}_{y[k-1]}u_{[k-1](n-1)} \\ &+\hat{b}_{y[k-1]}\sum_{i=1}^{n-1}\bar{\sigma}\hat{a}_{i[k-1]}u_{[k-1](n-1-i)} \\ &+\bar{\sigma}\hat{a}_{n[k-1]}y_{ae[k-1](0)} - y_{ae[k](0)}\Big\} \\ &/\Big\{1 + \bar{\sigma}\gamma_b u^2_{[k-1](n-1)} \\ &+\hat{b}_{y[k-1]}\sum_{i=1}^{n-1}\bar{\sigma}\gamma_{ai}\hat{b}_{y[k-1]}u^2_{[k-1](n-1-i)} \\ &+\bar{\sigma}\gamma_{an}y^2_{ae[k-1](0)}\Big\}\end{aligned}$$

Concerning the boundedness of the obtained output estimator, the following lemma is obtained (Mizumoto and Fujimoto, 2013).

Lemma 1. Under Assumption 1, all the signals in the designed output estimator are bounded with bounded inputs and outputs.

3.2 Adaptive Output Predictor for a multi-rate system

Based on the proposed output estimator (11), we design i-step adaptive output predictor, at a time instant knT, as follows:

$$\hat{y}_{[k](1)} = \hat{a}_{1[k]}y_{ae[k](0)} + \hat{b}_{y[k]}u_{[k](0)} - y_{fe[k](1)}$$

$$\begin{aligned}\hat{y}_{[k](i)} &= \hat{a}_{1[k]}\hat{y}_{ae[k](i-1)} + \hat{b}_{y[k]}u_{[k](i-1)} - y_{fe[k](i)} \\ &= \hat{a}^i_{1[k]}y_{ae[k](0)} + \hat{b}_{y[k]}\sum_{j=1}^{i}\hat{a}^{i-j}_{1[k]}u_{[k](j-1)} \\ &\quad -\boldsymbol{c}^T_{fe}A^j_{fe}\boldsymbol{x}_{fe[k](0)} + \sum_{j=1}^{i}\boldsymbol{c}^T_{fe}A^{i-j}_{fe}\boldsymbol{b}^*_{fe}u_{[k](j-1)}\end{aligned} \quad (16)$$

Therefore, 1-step to αn-step future predicted output from a time instant knT can be obtained by

$$\begin{aligned}\hat{\boldsymbol{y}}_{[k]} &= \left[\ \hat{y}_{[k](1)} \ \cdots \ \hat{y}_{[k](\alpha n)}\ \right]^T \\ &= \left(y_{ae[k]}\hat{\boldsymbol{a}}_{[k]} - A_{est}\boldsymbol{x}_{fe[k](0)}\right) \\ &\quad + \left(\hat{b}_{y[k]}\hat{A}_{[k]} - B_{est}\right)\boldsymbol{u}_{[k]}\end{aligned} \quad (17)$$

with

$$\hat{\boldsymbol{a}}_{[k]} = \begin{bmatrix} \hat{a}_{1[k]} \\ \vdots \\ \hat{a}^{\alpha n}_{1[k]} \end{bmatrix} \ , \ \boldsymbol{u}_{[k]} = \begin{bmatrix} u_{[k](0)} \\ \vdots \\ u_{[k](\alpha n-1)} \end{bmatrix}$$

$$\hat{A}_{[k]} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \hat{a}_{1[k]} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \hat{a}^{\alpha n-1}_{1[k]} & \cdots & \hat{a}_{1[k]} & 1 \end{bmatrix} \ , \ A_{est} = \begin{bmatrix} \boldsymbol{c}^T_{fe}A_{fe} \\ \boldsymbol{c}^T_{fe}A^2_{fe} \\ \vdots \\ \boldsymbol{c}^T_{fe}A^{\alpha n}_{fe} \end{bmatrix}$$

$$B_{est} = \begin{bmatrix} \boldsymbol{c}^T_{fe}\boldsymbol{b}_{fe} & 0 & \cdots & 0 \\ \boldsymbol{c}^T_{fe}A_{fe}\boldsymbol{b}_{fe} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \boldsymbol{c}^T_{fe}A^{\alpha n-1}_{fe}\boldsymbol{b}_{fe} & \cdots & \boldsymbol{c}^T_{fe}A_{fe}\boldsymbol{b}_{fe} & \boldsymbol{c}^T_{fe}\boldsymbol{b}_{fe} \end{bmatrix}$$

3.3 Basic Design of Adaptive Predictive Control via Adaptive Output Predictor

We consider designing a predictive control based on the designed adaptive output predictor by minimizing the following cost function over a finite prediction horizon.

$$\begin{aligned}J_{[k]} &= \sum_{i=1}^{\alpha n}\left(\hat{y}_{[k](i)} - y_{m[k](i)}\right)^2 + \sum_{i=1}^{\alpha n}\lambda_{i-1}u^2_{[k](i-1)} \\ &= \left(\hat{\boldsymbol{y}}_{[k]} - \boldsymbol{y}_{m[k]}\right)^T\left(\hat{\boldsymbol{y}}_{[k]} - \boldsymbol{y}_{m[k]}\right) \\ &\quad + \boldsymbol{u}^T_{[k]}\boldsymbol{\Lambda}\boldsymbol{u}_{[k]}\end{aligned} \quad (18)$$

$$\boldsymbol{y}_{m[k]} = \left[\ y_{m[k](1)} \ \cdots \ y_{m[k](\alpha n)}\ \right]^T$$

$$\boldsymbol{\Lambda} = \mathrm{diag}[\lambda_0, \lambda_1, \cdots, \lambda_{\alpha n-1}] \ , \ \lambda_i > 0$$

where $y_{m[k](i)}$ is a reference signal which the output is required to track.

Taking the expression given in (17) into account, the optimal control inputs $\boldsymbol{u}^*_{[k]} = [u^*_{[k](0)}, \cdots, u^*_{[k](\alpha n-1)}]$ which minimizes the cost function over the given prediction horizon is given by

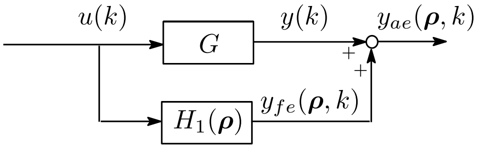

Fig. 2. Augmented systems for adaptive output predictor

$$\begin{aligned} \boldsymbol{u}^*_{[k]} &= -\left\{W^T_{[k]}W_{[k]} + \Lambda\right\}^{-1} W^T_{[k]}\boldsymbol{x}_{u[k]} \qquad (19) \\ W_{[k]} &= \hat{b}_{y[k]}\hat{A}_{[k]} - B_{est} \\ \boldsymbol{x}_{u[k]} &= y_{a[k](0)}\hat{\boldsymbol{a}}_{[k]} - A_{est}\boldsymbol{x}_{fe[k](0)} - \boldsymbol{y}_{m[k]} \end{aligned}$$

The actual control input is then given by using the optimal inputs in first frame periods of output, i.e. $[u_{[k](0)}, \cdots, u_{[k](n-1)}] = [u^*_{[k](0)}, \cdots, u^*_{[k](n-1)}]$.

In practice, it may be difficult to obtain the virtual fast-rate system model from the available information about the considered multi-rate sampled system. In the following, we propose an model-free design scheme of a PFC using the input/output data of the considered multi-rate sampled system.

We suppose that one can obtain an input/output data set $\{y^0_{[k](0)}, u^0_{[k](0)}, u^0_{[k](1)}, \cdots, u^0_{[k](n-1)}\}$ and define the approximated fast-rate output data within the sampling interval by

$$\bar{y}^0_{[k](i)} = \frac{y^0_{[k+1](0)} - y^0_{[k](0)}}{n} i + y^0_{[k](0)}, \ i = 0, \cdots, n-1 \qquad (20)$$

with

$$\bar{y}^0_{[k](0)} = y^0_{[k](0)}, \quad \bar{y}^0_{[k](n)} = \bar{y}^0_{[k+1](0)} = y^0_{[k+1](0)}$$

Considering an augmented system with a PFC $H(\boldsymbol{\rho})$ as shown in Fig. 2, the approximated fast-rate augmented system's output is obtained by

$$\bar{y}_{ae[k](i,\ \boldsymbol{\rho})} = \bar{y}^0_{[k](i)} + H_1(\boldsymbol{\rho})[u^0_{[k](i)}] \qquad (21)$$

The objective here is to find a parameter $\boldsymbol{\rho}$ such that the approximated fast-rate augmented system's output $\bar{y}_{ae[k](i,\ \boldsymbol{\rho})}$ identical to the ideal model output $y^*_{a1[k](i)}$ obtained by

$$y^*_{a1[k](i)} = G^*_{a1}(z)[u^0_{[k](i)}] \qquad (22)$$

That is, to find a parameter $\boldsymbol{\rho}$ which minimizes the following performance function is the objective.

$$J(\boldsymbol{\rho}) = \sum_{k=0}^{N}\sum_{i=0}^{n-1}\left(\bar{y}_{ae[k](i,\ \boldsymbol{\rho})} - y^*_{a1[k](i)}\right)^2 \qquad (23)$$

Practical design of the PFC is provided as follows. Firstly define the ideal PFC output by

$$y^*_{fe[k](i)} = y^*_{a1[k](i)} - \bar{y}^0_{[k](i)} \qquad (24)$$

Suppose that the ideal PFC $H^*_1(z)$ is given as the following compensator of order m.

$$H^*_1(z) = \frac{N^*_H(z)}{D^*_H(z)} = \frac{b^*_1 z^{m-1} + \cdots + b^*_m}{a^*_0 z^m + a^*_1 z^{m-1} + \cdots + a^*_m} \qquad (25)$$

THus the ideal PFC output y^*_fcan be expressed by

$$y^*_{fe[k](i)} = \frac{N^*_H(s)}{D^*_H(s)}[u^0_{[k](i)}] \qquad (26)$$

Then, by introducing a stable filter:

$$F(z) = \frac{1}{z^m + f_1 z^{m-1} + \cdots + f_m} \qquad (27)$$

of order m to the PFC given in (26), we have

$$y^*_{fe[k](i)} + \frac{D^*_H(z) - F(z)}{F(z)}\left[y^*_{fe[k](i)}\right] = \frac{N^*_H(z)}{F(z)}\left[u^0_{[k](i)}\right] \qquad (28)$$

and thus $y^*_{fe[k](i)}$ can be expressed by

$$y^*_{fe[k](i)} = \frac{S^*(z)}{F(z)}\left[y^*_{fe[k](i)}\right] + \frac{N^*_H(z)}{F(z)}\left[u^0_{[k](i)}\right] = \boldsymbol{\rho}^{*T}\boldsymbol{z}_{[k](i)}$$
$$(S^*(z) = F(z) - D^*_H(z)) \qquad (29)$$

where,

$$\boldsymbol{\rho}^* = [s^*_0\ s^*_1\ \cdots\ s^*_m\ b^*_1\ \cdots\ b^*_m]^T, \quad (z^*_i = f_i - a^*_i)$$

$$\boldsymbol{z}_{[k](i)} = \left[\frac{z^m}{F(z)}[y^*_{fe[k](i)}]\ \frac{z^{n-1}}{F(z)}[y^*_{fe[k](i)}] \cdots \frac{1}{F(z)}[y^*_{fe[k](i)}]\ \frac{z^{m-1}}{F(z)}[u^0_{[k](i)}] \cdots \frac{1}{F(z)}[u^0_{[k](i)}]\right]^T$$

Taking this expression into consideration, denote a PFC output for any parameter $\boldsymbol{\rho}$ as

$$y_{fe[k](i,\ \boldsymbol{\rho})} = \boldsymbol{\rho}^T\boldsymbol{z}_{[k](i)} \qquad (30)$$

and consider minimizing the following performance function.

$$J_f(\boldsymbol{\rho}) = \sum_{k=0}^{N}\sum_{i=0}^{n-1}\left(\boldsymbol{\rho}^T\boldsymbol{z}_{[k](i)} - y^*_{fe[k](i)}\right)^2 \qquad (31)$$

The parameter $\boldsymbol{\rho}$ which minimizing the performance function (31) can be obtained by

$$\boldsymbol{\rho} = \left(Z^T Z\right)^{-1} Z^T Y^*_{fe} \qquad (32)$$

with $Z = [\boldsymbol{z}_{[0](0)}\ \boldsymbol{z}_{[0](1)}\ \cdots\ \boldsymbol{z}_{[N](n-1)}]^T$ and $Y^*_{fe} = [y^*_{fe[0](0)}\ y^*_{fe[0](1)}\ \cdots\ y^*_{fe[N](n-1)}]^T$.

With the designed PFC $H_1(z, \boldsymbol{\rho})$, since the augmented system $G_{a1}(z)$ can be expressed by

$$\begin{aligned} G_{a1}(z) &= G(z) + H_1(z, \boldsymbol{\rho}) \\ &= G(z) + H^*_1(z) + (H_1(z, \boldsymbol{\rho}) - H^*_1(z)) \\ &= G^*_{a1}(z)(1 + \Delta_2(z)) \qquad (33) \end{aligned}$$

where

$$\Delta_2(z) = G^{*-1}_{a1}(z)\left(H_1(z, \boldsymbol{\rho}) - H^*_1(z)\right)$$

Thus we have the following Lemma concernig the robustness of the obtained PFC.

Lemma 2. The resulting augmented system $G_{a1}(z) = G(z) + H_1(z, \boldsymbol{\rho})$ with the PFC $H_1(z, \boldsymbol{\rho})$ is minimum-phase and has relative degree of 1 provided that

(1) $G^*_{a1}(z)$ is minimum-phase and has relative degree of 1.
(2) $\Delta_2(z) \in RH_\infty$ and $\|\Delta_2(z)\|_\infty < 1$.

Proof) The results can be easily confirmed through the same argument on Theorem 1 in Mizumoto et al. (2010a).

4. STABLE ADAPTIVE PREDICTIVE CONTROL SYSTEM DESIGN

4.1 Adaptive Predictive Control with Constraints based on System's ASPR-ness

One may control the system stably with the designed ideal control input $\boldsymbol{u}^*_{[k]}$, however, one can not guarantee the stability of the obtained adaptive predictive control system by direct use of $\boldsymbol{u}^*_{[k]}$ as mentioned in Mizumoto and Fujimoto (2012). We consider the following input constraint associated with ASPR property of the system on the ideal input $\boldsymbol{u}^*_{[k]}$ in order to guarantee the boundedness of all the signals in the control system under Assumptions 2 and 3. The proposed constraint has the same structure of the one proposed in Mizumoto and Fujimoto (2012), but for the multi-rate systems, the estimated output is utilized for the constraint.

Assumption 2. For system (1), there exists a stable PFC:

$$\begin{aligned} \boldsymbol{x}_f(k+1) &= A_f \boldsymbol{x}_f(k) + \boldsymbol{b}_f u(k) \\ y_f(k) &= \boldsymbol{c}_f^T \boldsymbol{x}_f(k) + d_f u(k) \end{aligned} \tag{34}$$

such that the resulting augmented system:

$$\begin{aligned} \boldsymbol{x}_a(k+1) &= A_a \boldsymbol{x}_a(k) + \boldsymbol{b}_a u(k) \\ y_a(k) &= \boldsymbol{c}_a^T \boldsymbol{x}_a(k) + d_a u(k) \end{aligned} \tag{35}$$

with $\boldsymbol{x}_a(k) = [\boldsymbol{x}(k), \boldsymbol{x}_f(k)]^T$ and

$$A_a = \begin{bmatrix} A & \mathbf{0} \\ \mathbf{0} & A_f \end{bmatrix}, \boldsymbol{b}_a = \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{b}_f \end{bmatrix}, \boldsymbol{c}_a^T = \begin{bmatrix} \boldsymbol{c}^T \\ \boldsymbol{c}_f^T \end{bmatrix}, d_a = d_f$$

is ASPR (or strongly ASPR Mizumoto et al. (2007),Mizumoto et al. (2010b)). That is, the augmented system (35) is minimum-phase and has relative degree of 0 with $d_a > 0$.

Assumption 3. The minimum value of the output feedback gain $\theta_{p\min}$, such that the resulting closed-loop system is SPR, is known.

Remark 1. Under this Assumption 2, there exists a static output feedback gain $\theta_{p\min}$ such that for all output feedback $u(k) = -\theta_p y_a(k) + v(k)$ with $\theta_p \geq \theta_{p\min}$, the resulting closed-loop system is SPR (Mizumoto et al., 2007, 2010b).

Under Assumptions 2 and 3, with the output feedback:

$$\begin{aligned} u_{k(i)} &= -\theta_{p\min} y_{a[k](i)} + v_{[k](i)} \\ &= -\tilde{\theta}_{p\min} \tilde{y}_{a[k](i)} + \tilde{v}_{k(i)} \\ \tilde{y}_{ak(i)} &= \boldsymbol{c}_a^T \boldsymbol{x}_{a[k](i)}, \ \tilde{\theta}_{p\min} = \frac{\theta_{p\min}}{1 + d_a \theta_{p\min}}, \\ \tilde{v}_{k(i)} &= \frac{1}{1 + d_a \theta_{p\min}} v_{k(i)} \end{aligned} \tag{36}$$

the resulting closed-loop system:

$$\begin{aligned} \boldsymbol{x}_{a[k](i+1)} &= A_{ac} \boldsymbol{x}_{a[k](i)} + \boldsymbol{b}_{ac} \tilde{v}_{[k](i)} \\ y_{a[k](i)} &= \boldsymbol{c}_{ac}^T \boldsymbol{x}_{a[k](i)} + d_{ac} \tilde{v}_{[k](i)} \end{aligned} \tag{37}$$

is SPR. Where

$$A_{ac} = A_a - \tilde{\theta}_{p\min} \boldsymbol{b}_a \boldsymbol{c}_a^T \ , \ \boldsymbol{b}_{ac} = \boldsymbol{b}_a$$
$$\boldsymbol{c}_{ac} = \frac{1}{1 + d_a \theta_{p\min}} \boldsymbol{c}_a \ , \ d_{ac} = d_a$$

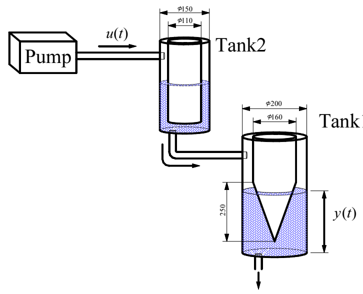

Fig. 3. Outline installation drawing of the two-tank system

Then, adaptive predictive control with ASPR constraints is designed as follows (Mizumoto and Fujimoto, 2012):

$$u_{[k](i)} = \begin{cases} \psi_{A[k](i)} & \text{if } |\hat{e}_{[k](i)}| > \delta \\ \psi_{B[k](i)} & \text{if } |\hat{e}_{[k](i)}| \leq \delta \end{cases} \tag{38}$$

$$\psi_{A[k](i)} = \begin{cases} \min\left\{\underline{\psi}_{[k](i)}, \overline{\psi}_{[k](i)}\right\} \\ \qquad \text{if } u^*_{[k](i)} \leq \min\left\{\underline{\psi}_{[k](i)}, \overline{\psi}_{[k](i)}\right\} \\ \max\left\{\underline{\psi}_{[k](i)}, \overline{\psi}_{[k](i)}\right\} \\ \qquad \text{if } u^*_{[k](i)} \geq \max\left\{\underline{\psi}_{[k](i)}, \overline{\psi}_{[k](i)}\right\} \\ u^*_{[k](i)} \ \text{otherwise} \end{cases} \tag{39}$$

$$\psi_{B[k](i)} = \begin{cases} u_{max} & \text{if } u^*_{[k](i)} \geq u_{max} \\ -u_{max} & \text{if } u^*_{[k](i)} \leq -u_{max} \\ u^*_{[k](i)} & \text{otherwise} \end{cases} \tag{40}$$

$$\begin{aligned} \overline{\psi}_{[k](i)} &= -\tilde{\theta}_{p\max} \hat{\tilde{e}}_{ak(i)}, \ \underline{\psi}_{k(i)} = -\tilde{\theta}_{p\min} \hat{\tilde{e}}_{ak(i)} \\ \hat{\tilde{e}}_{ak(i)} &= \hat{\tilde{y}}_{ak(i)} - y_{mk(i)} \ , \ \hat{\tilde{y}}_{ak(i)} = \hat{y}^*_{[k](i)} + \boldsymbol{c}_f^T \boldsymbol{x}_{f[k](i)} \\ \hat{y}^*_{[k](i)} &= \begin{cases} y_{[k](i)} \\ \hat{y}_{[k](i)} \ \ (i = 1, 2, \cdots, n-1) \end{cases} \end{aligned}$$

where $u_{\max}$ is a maximum value of the input for an input saturation constraint and $\tilde{\theta}_{p\max}$ is any upper bound of the gain such as $\tilde{\theta}_{p\min} < \tilde{\theta}_{p\max} < 1/d_a$. $\psi_{A[k](i)}$ represents ASPR feedback constraints on the designed predictive control.

Then, we have the following theorem concerning the boundedness of all the signals in the proposed adaptive predictive control system.

Theorem 1. Under Assumptions 1 to 3, designing the adaptive predictive controller as in (38) with (19) and adaptive predictor (16) with parameter adjusting laws (12), (13) and (14), all the signals in the obtained control system are bounded.

Proof: The proof can be done by following the results in Mizumoto and Fujimoto (2012, 2013).

5. VALIDATION THROUGH EXPERIMENTS

The effectiveness of the proposed method is confirmed through experiments of the two-tank system (See Fig. 3).

The step response of the controlled system is shown in Fig. 4, and we suppose that the system is unknown but the input and output data of the step response given in Fig. 4 is available.

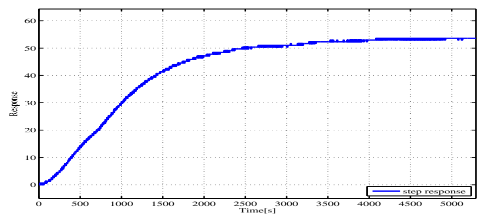

Fig. 4. Step response of the controlled system

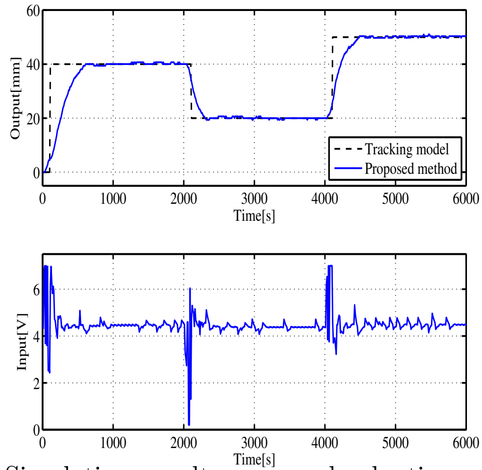

Fig. 5. Simulation result: proposed adaptive predictive controller with model-free PFCs for multirate system with $n = 5$

Using the given input/output data, we designed PFCs for output predictor and ASPR realization by setting ideal augmented systems as

$$G^*_{a1}(z) = \frac{0.4}{z - 0.99} \quad \text{for output predictor} \tag{41}$$

$$G^*_{a2}(z) == \frac{2.036z - 1.5}{z - 0.99} \quad \text{for ASPRness} \tag{42}$$

The obtained PFCs are as follows:

$$H_1(s) = \frac{0.3734z^2 - 0.7275z + 0.3535}{z^3 - 2.870z^2 + 2.744z - 0.974} \tag{43}$$

$$H_2(s) = \frac{2.036z^3 - 5.224z^2 + 4.411z - 1.223}{z^3 - 2.814z^2 + 2.636z + 0.8211} \tag{44}$$

The design parameters in the adaptive predictive controller were set as follows:

$$\alpha = 2,\ \gamma_{ai} = 10^{-5},\ \gamma_b = \times 10^{-5}$$
$$\sigma = 10^{-7},\ \delta = 3,\ u_{\max} = 7$$
$$\Lambda = I,\ \tilde{\theta}_{\min} = 0.3929,\ \tilde{\theta}_{\max} = 0.4912$$

Figure 5 shows the simulation results by proposed method with model-free designed PFCs for a multirate system with $n = 5$. Pretty good control performance was shown.

6. CONCLUSIONS

In this paper, we proposed an adaptive predictive control scheme for multi-rate sampled systems. By designing adaptive output predictor for multi-rate systems based on the first order representation of the system, a simple adaptive predictive controller with the adaptive output predictor was designed so as to minimize the given cost function. Furthermore, by setting an "almost strictly positive real (ASPR) input constraint", it was shown that a stable adaptive predictive control can be designed for multi-rate sampled systems.

REFERENCES

Clarke, D., Mohtadi, C., and Tuffs, P. (1987). Generalized predictive control - part 1. the basic algorithm. *Automatica*, 23(2), 137–148.

Fukushima, H., Kim, T.H., and Sugie, T. (2007). Adaptive model predictive control for a class of constrained linear systems based on the comparison model. *Automatica*, 43(2), 301–308.

Garcia, C., Prett, D., and Morari, M. (1989). Model predictive control: Theory and practice -a survey. *Automatica*, 25(3), 335–348.

Isidori, A. (1995). *Nonlinear Control Systems*. Springer, 3rd edition.

Maciejowski, J. (2002). *Predictive Control with constraintss*. Prentic Hall.

Mayne, D., Rawlings, J., Rao, C., and Scokaert, P. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, 36(6), 789–814.

Mizumoto, I., Chen, T., Ohdaira, S., Kumon, M., and Iwai, Z. (2007). Adaptive output feedback control of general mimo systems using multirate sampling and its application to a cart-crane system. *Automatica*, 43(12), 2077–2085.

Mizumoto, I. and Fujimoto, Y. (2012). Adaptive predictive control system design with an adaptive output estimator. *Proc. of 51st IEEE Conference on Decision and Control, Maui, Hawaii, USA*, 5434–5441.

Mizumoto, I. and Fujimoto, Y. (2013). Fast-rate output feedback control system design with adaptive output estimator for nonuniformly sampled multirate systems. *Int. J. Adapt. Control Signal Process., Published online in Wiley Online Library. DOI: 10.1002/acs.2407.*

Mizumoto, I., Fujimoto, Y., Watanabe, N., and Iwai, Z. (2011). Fast rate adaptive output feedback control of multirate sampled systems with an adaptive output estimator. *International Journal of Innovative Computing, Information and Control*, 7(7), 4377–4395.

Mizumoto, I., Ikeda, D., Hirahata, T., and Iwai, Z. (2010a). Design of discrete time adaptive pid control systems with parallel feedforward compensator. *Control Engineering Practice*, 18(2), 168–176.

Mizumoto, I., Ohdaira, S., and Iwai, Z. (2010b). Output feedback strictly passivity of discrete-time nonlinear systems and adaptive control system design with a pfc. *Automatica*, 46(9), 1503–1509.

Mizumoto, I., Ohdaira, S., Watanabe, N., Tanaka, H., Harada, H., Fujimoto, Y., Kinoshita, H., and Iwai, Z. (2010c). Output feedback control of multirate sampled systems with an adaptive output estimator and its application to a liquid level process control. *Journal of System Design and Dynamics*, 4(2), 314–330.

Nicolao, G.D., Scattolini, R., and Sala, G. (1996). An adaptive predictive regulator with input saturations. *Automatica*, 32(4), 597–601.

Yoon, T.W. and Clarke, D. (1994). Adaptive predictive control of the benchmark plant. *Automatica*, 30(4), 621–628.

