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Parameter estimation for Non-uniformly Sampled Wiener Systems with Dead-zone Nonlinearities ⋆

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Abstract: In this paper, an identification algorithm for the non-uniformly Wiener systems with dead-zone nonlinearities is proposed. Firstly, a uniform model of the Wiener system is reformulated with the help of the lift technique and a switching function. Then, a least squares based iterative recursive algorithm with variable forgetting factor is presented using the auxiliary model and iterative method. Simulation results indicate the effectiveness of the proposed method.

Keywords: parameters estimation, Least-squares algorithm, Wiener systems, nonlinear systems, Iterative methods.

1 INTRODUCTION

Hard nonlinearity is a common phenomenon existing in the industrial plants, which includes preload, dead-zone, saturation, saturation and dead-zone, piecewise-linear and their composition etc.(Bai, 1998). As input output parametric models (I.J. Leontaritis, 1985), how to identify parameters of this system is a challenging issue. Because this kind of system cannot be written as a polynomial form. So the existing methods, such as the maximum likelihood estimation (Hagenblad et al., 2008), the multi-innovation stochastic gradient algorithm (Ding, 2013), gradient-based and least squares-based iterative methods (Ding et al., 2013), the NARMAX methodology (S.A.Billings, 2013), cannot be directly used to estimate the parameters. Furthermore, most of the achievements focus on the single-rate systems and ignore the general multi-rate sampled characteristics except the dual-rate sampled Hammerstein systems with preload nonlinearity (J. Chen, 2013; X. L. Li, 2014).

In order to estimate the non-uniformly sampled Wiener models with dead-zone nonlinearities, an online iteratively recursive algorithm is proposed in this paper. Firstly, the Wiener model with hard nonlinearity is transformed into an analytic form using an appropriated switching function. Then, an iterative recursive least squares algorithm is proposed to estimate the parameters of the system directly. In order to improve the identification accuracy and the ability of anti-jamming, a variable forgetting factor is introduced. The proposed algorithm is formulated as a recursive form with iterating the estimation of parameters into information vector.

The paper is organized as follows. Section 2 depicts the non-uniformly sampled Wiener systems with dead-zone nonlinear characteristic. Then, the proposed algorithm is derived in section 3 and the performance of the algorithm is analyzed in Section 4. Section 5 provides an illustrative example. Finally, conclusions are summarized in Section 6.

2 PROBLEM FORMULATION

Consider a non-uniformly sampled Wiener system with dead-zone nonlinearity (shown in Fig.1)

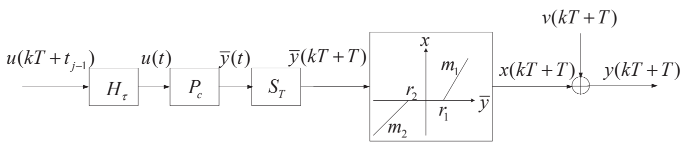

Fig. 1. The structure of non-uniformly sampled-Wiener system

where $u(kT+t_{j-1})$ and $y(kT+T)$ are the input and output of the system, $\bar{y}(kT+T)$ and $x(kT+T)$ are the input and output of the nonlinear part, respectively. $v(kT+T)$ is a white noise with zero mean. H_τ is a non-uniformly zero-order holder with irregularly sampled intervals $\{\tau_1, \tau_2, \ldots, \tau_m\}$, the input updating frequency is set as $kT+t_{j-1}, j = 1, 2, ..., m(t_0 = 0, t_j := \tau_1+\tau_2+...+\tau_j)$. P_c is the linear dynamic block and S_T is the sampler with frame period $T := \tau_1 + \tau_2 + ... + \tau_m = t_m$. Using the lifting technology, H_τ is formulated as follows:

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$$u(t)=\begin{cases} u(kT), \quad kT \le t < kT+t_1 \\ u(kT+t_1),\ kT+t_1 \le t < kT+t_2 \\ \vdots \\ u(kT+t_{m-1}),\ kT+t_{m-1} \le t < kT+T \end{cases} \tag{1}$$

P_c is assumed as the state-space model

$$P_c := \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_c\mathbf{x}(t) + \mathbf{B}_c u(t) \\ \bar{y}(t) = \mathbf{C}x(t) + Du(t) \end{cases} \tag{2}$$

where $\mathbf{x}(t) \in R^n$ is the state vector, $\bar{y}(t) \in R$, $u(t) \in R$ are the output and input of P_c, respectively. $\mathbf{A}_c \in R^{n\times n}$, $\mathbf{B}_c \in R^n$, $\mathbf{C} \in R^{1\times n}$ and $D \in R$ are parameter matrices. Discretizing Eq.(2) with the frame period T (X. L. Li, 2002; F. Ding, 2009; L. Xie, 2010), the linear part is written as

$$\begin{aligned} \bar{y}(kT) &= \frac{\sum_{j=1}^{m} B_j(z^{-1})}{A(z^{-1})} u(kT+t_{j-1}) \\ &= [1-A(z^{-1})]\bar{y}(kT) + \sum_{j=1}^{m} B_j(z^{-1})u(kT+t_{j-1}) \end{aligned} \tag{3}$$

with

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_{n_a} z^{-n_a}$$

$$B(z^{-1}) = b_{10} + b_{11} z^{-1} + b_{12} z^{-2} + ... + b_{1n_b} z^{-n_b}$$

$$B_j(z^{-1}) = b_{j1} z^{-1} + b_{j2} z^{-2} + ... + b_{jn_b} z^{-n_b}, j = 2, 3, \cdots, m$$

where z^{-1} is the backward shift operator: $z^{-1}x(kT) = x(kT-T)$. The hard nonlinear part (shown in Fig.2) is formulated as

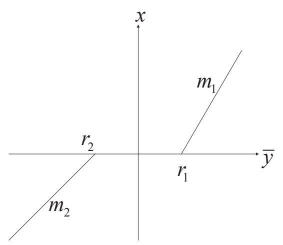

Fig. 2. The dead-zone characteristic

$$x(kT) = \begin{cases} m_1(\bar{y}(kT) - r_1), r_1 \le \bar{y}(kT) \\ 0, r_2 \le \bar{y}(kT) \le r_1 \\ m_2(\bar{y}(kT) - r_2), \bar{y}(kT) \le r_2 \end{cases} \tag{4}$$

where m_1 and m_2 are slopes of the corresponding linear segment. $r_1 > 0$ and $r_2 < 0$ are the constants for the dead-zone points. Since nonlinear parameters is unknown, a switching function $h(\cdot)$ is introduced (M.C. Kung, 1984)

$$h(\bar{y}(t)) := \begin{cases} 0, \bar{y}(t) > 0 \\ 1, \bar{y}(t) \le 0 \end{cases} \tag{5}$$

Based on the switching function, Eq.(4) is re-written as

$$\begin{aligned} x(kT) =& m_1 h(r_1 - \bar{y}(kT))(\bar{y}(kT) - r_1) \\ &+ m_2 h(\bar{y}(kT) - r_2)(\bar{y}(kT) - r_2) \end{aligned} \tag{6}$$

Similarly, $\bar{y}(kT)$ becomes

$$\begin{aligned} \bar{y}(kT) =& h(r_1 - \bar{y}(kT))\bar{y}(kT) + h(\bar{y}(kT) - r_2)\bar{y}(kT) \\ &+ h(\bar{y}(kT) - r_1)h(r_2 - \bar{y}(kT))\bar{y}(kT) \end{aligned} \tag{7}$$

Substituting Eq.(7) into Eq.(6), then

$$\begin{aligned} x(kT) =& \bar{y}(kT) - h(r_1 - \bar{y}(kT))\bar{y}(kT) - h(\bar{y}(kT) \\ &- r_2)\bar{y}(kT) - h(\bar{y}(kT) - r_1)h(r_2 - \bar{y}(kT))\bar{y}(kT) \\ &+ m_1 h(r_1 - \bar{y}(kT))(\bar{y}(kT) - r_1) + m_2 h(\bar{y}(kT) - \\ & r_2)(\bar{y}(kT) - r_2) \end{aligned} \tag{8}$$

The system output is formulated as

$$\begin{aligned} y(kT) =& x(kT) + v(kT) = \bar{y}(kT) - h(r_1 - \\ & \bar{y}(kT))\bar{y}(kT) - h(\bar{y}(kT) - r_2)\bar{y}(kT) - h(\bar{y}(kT) - \\ & r_1)h(r_2 - \bar{y}(kT))\bar{y}(kT) + m_1 h(r_1 - \bar{y}(kT))(\bar{y}(kT) - \\ & r_1) + m_2 h(\bar{y}(kT) - r_2)(\bar{y}(kT) - r_2) + v(kT) \end{aligned} \tag{9}$$

Define an internal variable

$$\begin{aligned} y'(kT) :=& y(kT) + h(r_1 - \bar{y}(kT))\bar{y}(kT) + h(\bar{y}(kT) - \\ & r_2)\bar{y}(kT) + h(\bar{y}(kT) - r_1)h(r_2 - \bar{y}(kT))\bar{y}(kT) \end{aligned} \tag{10}$$

Substituting Eq.(3) and Eq.(9) into Eq.(10), $y'(kT)$ is reformulated as

$$\begin{aligned} y'(kT) =& [1 - A(z^{-1})]\bar{y}(kT) + \sum_{j=1}^{m} B_j(z^{-1})u(kT + t_{j-1}) \\ &+ m_1 h(r_1 - \bar{y}(kT))\bar{y}(kT) + m_2 h(\bar{y}(kT) - r_2)\bar{y}(kT) \\ &- m_1 r_1 h(r_1 - \bar{y}(kT)) - m_2 r_2 h(\bar{y}(kT) - r_2) + v(kT) \end{aligned} \tag{11}$$

Suppose that the model order n_a, n_b are known, and $u(kT + t_{j-1}) = 0$, $y(kT) = 0$, $v(kT) = 0$, $j = 1, 2, \ldots, m$ while $k \le 0$. Define the parameter vector θ and information vector $\varphi(kT)$ as

$$\begin{aligned} \theta :=& [a_1, a_2, \ldots, a_{n_a}, b_{10}, b_{11}, \ldots b_{1n_b}, b_{21}, \ldots, b_{2n_b}, \ldots, \\ & b_{m1}, \ldots, b_{mn_b}, m_1, m_2, r_1, r_2]^T \in R^{n_a + mn_b + 5} \end{aligned}$$

$$\begin{aligned} \varphi(kT) :=& [-\bar{y}(kT - T), -\bar{y}(kT - 2T), \ldots, -\bar{y}(kT - n_a T), \\ & u(kT), u(kT - T), \ldots, u(kT - n_b T), \\ & u(kT - T + t_1), \ldots, u(kT - n_b T + t_1), \\ & \ldots, u(kT - T + t_{m-1}), \ldots, u(kT - n_b T + t_{m-1}), \\ & h(r_1 - \bar{y}(kT))\bar{y}(kT), h(\bar{y}(kT) - r_2)\bar{y}(kT), \\ & -m_1 h(r_1 - \bar{y}(kT)), -m_2 h(\bar{y}(kT) - r_2)]^T \in R^{n_a + mn_b + 5} \end{aligned}$$

where the superscript T denotes the matrix transform, Eq.(11) is rewritten as a concise form

$$y'(kT) = \varphi^T(kT)\theta + v(kT) \tag{12}$$

Once θ is estimated, parameters of the linear and nonlinear parts are estimated simultaneously.

3 THE RECURSIVE ESTIMATION ALGORITHM

The challenge in parameters estimation mentioned in Eq.(12) is that $\varphi(kT)$ contains the unknown inner variables $\bar{y}(kT-iT), i=0,1,\ldots,n_a$ and unknown parameters m_1, m_2, r_1, r_2. So the standard recursive least squares cannot be used to estimate θ directly. Taking the auxiliary model into consideration (F. Ding, 2004), an effective solution is proposed (shown in Fig.3). The auxiliary model

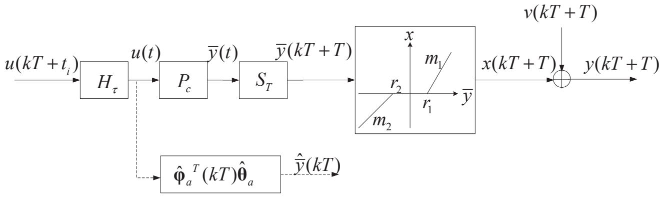

Fig. 3. The Wiener system with an auxiliary model

is

$$\hat{\bar{y}}(kT) = \hat{\varphi}_a(kT)\hat{\theta}_a \tag{13}$$

$$\begin{aligned}\hat{\varphi}_a(kT) :=& [-\hat{\bar{y}}(kT-T), -\hat{\bar{y}}(kT-2T), \ldots, -\hat{\bar{y}}(kT- \\ & n_aT), u(kT), u(kT-T), \ldots, u(kT-n_bT), u(kT-T \\ & +t_1), \ldots, u(kT-n_bT+t_1), \ldots, u(kT-T+t_{m-1}), \\ & \ldots, u(kT-n_bT+t_{m-1})]^T \in R^{n_a+mn_b+1}\end{aligned} \tag{14}$$

$$\begin{aligned}\hat{\theta}_a :=& [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_{n_a}, \hat{b}_{10}, \hat{b}_{11}, \ldots \hat{b}_{1n_b}, \hat{b}_{21}, \ldots, \hat{b}_{2n_b}, \\ & \ldots, \hat{b}_{m1}, \ldots, \hat{b}_{mn_b}]^T \in R^{n_a+mn_b+1}\end{aligned} \tag{15}$$

$\hat{\varphi}_a(kT)$ and $\hat{\theta}_a$ are estimated values of the information vector and parameter vector in the auxiliary model. At time KT, the unknown variables $\bar{y}(kT-iT)$ and m_1, m_2, r_1, r_2 in $\varphi(kT)$ are replaced with the output $\hat{\bar{y}}(kT-iT)$ of the auxiliary model and the estimates $m_1^{k-1}, m_2^{k-1}, r_1^{k-1}, r_2^{k-1}$ at time $KT-T$, respectively. Then the parameter vector is renamed as $\hat{\varphi}(kT)$.

Let $\hat{\theta}(kT)$ and $\hat{\theta}_a(kT)$ be the estimate of θ and θ_a at time KT, separately. Minimizing the following cost function

$$J(\theta) = \sum_{i=1}^{k} [y'(kT) - \hat{\varphi}^T(kT)\theta]^2$$

In order to improve the tracking performance, a variable forgetting factor is introduced. Then, an auxiliary model-based iterative recursive least squares algorithm with variable forgetting factor (VFF-AM-IRLS algorithm for short) is derived

$$\hat{\theta}(kT) = \hat{\theta}(kT-T) + \mathbf{L}(kT)[y'(kT) - \hat{\varphi}^T(kT)\hat{\theta}(kT-T)] \tag{16}$$

$$\begin{aligned}y'(kT) =& y(kT) + h(r_1^{k-1} - \hat{\bar{y}}(kT))\hat{\bar{y}}(kT) + h(\hat{\bar{y}}(kT) - \\ & r_2^{k-1})\hat{\bar{y}}(kT) + h(\hat{\bar{y}}(kT) - r_1^{k-1})h(r_2^{k-1} - \hat{\bar{y}}(kT))\hat{\bar{y}}(kT)\end{aligned} \tag{17}$$

$$\begin{aligned}\mathbf{L}(kT) =& \mathbf{P}(kT-T)\hat{\varphi}(kT) \\ & [\lambda(kT) + \hat{\varphi}^T(kT)\mathbf{P}(kT-T)\hat{\varphi}(kT)]^{-1}\end{aligned} \tag{18}$$

$$\mathbf{P}(kT) = \frac{1}{\lambda(kT)}[\mathbf{I} - \mathbf{L}(kT)\hat{\varphi}^T(kT)]\mathbf{P}(kT-T) \tag{19}$$

$$\begin{aligned}\hat{\varphi}(kT) :=& [-\hat{\bar{y}}(kT-T), -\hat{\bar{y}}(kT-2T), \ldots, -\hat{\bar{y}}(kT-n_aT), \\ & u(kT), u(kT-T), \ldots, u(kT-n_bT), \\ & u(kT-T+t_1), \ldots, u(kT-n_bT+t_1), \ldots, \\ & u(kT-T+t_{m-1}), \ldots, u(kT-n_bT+t_{m-1}), \\ & h(r_1^{k-1} - \hat{\bar{y}}(kT))\hat{\bar{y}}(kT), h(\hat{\bar{y}}(kT) - r_2^{k-1})\hat{\bar{y}}(kT), \\ & -m_1^{k-1}h(r_1^{k-1} - \hat{\bar{y}}(kT)), -m_2^{k-1}h(\hat{\bar{y}}(kT) - r_2^{k-1})]^T\end{aligned} \tag{20}$$

where $\mathbf{I}$ is an identity matrix. $\mathbf{P}(kT)$ is the covariance matrix, and $\lambda(kT)$ avoids the data being saturated. When $\lambda(kT)$ is set a small value, the algorithm has a strongly tracking capability. Meanwhile, the VFF-AM-IRLS is more sensitive to noise interference, and brings large variation of parameter estimation, simultaneously. On the contrary, the large value of $\lambda(kT)$ leads to a small parameter estimation error (PEE), less sensitivity and a slowly convergence rate as well.

In order to increase the convergence rate, improve the anti-jamming performance, and reduce the PEE, a modified form of forgetting factor is proposed

$$\begin{aligned}&\lambda(kT) = \lambda_{\min} + (1-\lambda_{\min})^{2^{R(kT)}} \\ &s.t. \begin{cases} R(kT) = NINT(\rho|\xi(kT)|) \\ \xi(kT) = y'(kT) - \hat{\varphi}^T(kT)\hat{\theta}(kT-T) \end{cases}\end{aligned} \tag{21}$$

where $\lambda(kT)$ is the forgetting factor at time , $\lambda_{\min}$ is the minimum of $\lambda(kT)$, and $0 \le \lambda_{\min} \le 1$. ρ is a gain coefficient, which makes $\lambda(kT)$ tend to 1. $\xi(kT)$ is the error of output between the system and its estimation. $NINT(\cdot)$ is defined as the nearest integer. It can be seen that the magnitude of $\lambda(kT)$ is determined by the prediction error. When the prediction error increases, it is necessary to initiate the data discounting mechanism. The bigger $\xi(kT)$ is, the smaller $\lambda(kT)$ is, the faster tracking ability of the algorithm is. Otherwise, the algorithm is not sensitive to noise and has low PEE simultaneously.

The proposed VFF-AM-IRLS algorithm is summarized as follows.

Step1: Let $k=1$, set $\hat{\theta}(0)$, $\hat{\bar{y}}(0)$ and $\mathbf{P}(0)$ as

$$\hat{\theta}(0) = \mathbf{1}/p_0, \hat{\bar{y}}(0) = \mathbf{1}/p_0, \mathbf{P}(0) = p_0\mathbf{1}$$

where $\mathbf{1}$ is a column vector with appropriate dimension, whose entries are all 1 and $p_0 = 10^6$.

Step2: Collect $\{u(kT+t_{j-1}), y(kT) : j = 1, 2, \ldots m\}$, form $\hat{\varphi}_a(kT)$ using Eq.(14), and calculate $\hat{\bar{y}}(kT)$ as shown in Eq.(13). Then separate parameter of m_1^{k-1}, m_2^{k-1}, r_1^{k-1}, r_2^{k-1} from $\hat{\theta}(kT-T)$ and form $\hat{\varphi}(kT)$ according to Eq.(20).

Step3: Calculate $y'(kT)$, $\lambda(kT)$, $\mathbf{L}(kT)$, $\mathbf{P}(kT)$ using Eq.(17), Eq.(21), Eq.(18), Eq.(19) separately.

Step4: Update $\hat{\theta}(kT)$ on the basis of Eq.(16).

Step5: Let $k = k+1$, and go to step3.

When $\lambda_{\min} = 1$, the algorithm is simplified to the auxiliary model-based iterative recursive least squares (AM-IRLS) algorithm. When $\lambda(kT)$ is constant, the algorithm is deformalized to the auxiliary model-based iterative recursive forgetting least squares (AM-IRFLS) algorithm.

4 CONVERGENCE ANALYSIS

Lemma 1.Given the VFF-AM-IRLS algorithm mentioned in Eq.(16)-Eq.(21) and the system in Eq.(12), if $\hat{\varphi}(kT)$ is persistent excitation (PE), i.e. exist constants $0 < \alpha \leq \beta < \infty$ and an positive integer $N \geq n$ such that the following PE condition holds:

$$(A1)\alpha\mathbf{I} \leq \frac{1}{N}\sum_{i=0}^{N-1}\hat{\varphi}(kT+iT)\hat{\varphi}^T(kT+iT) \leq \beta\mathbf{I}\ a.s., k>0$$

Then, for $0 < \lambda_{\min} < 1$, $\mathbf{P}(kT)$ satisfies

$$\mathbf{P}^{-1}(kT) \geq \frac{\lambda_{\min}^{N-1}}{1-\lambda_{\min}}\alpha\mathbf{I} + \lambda_{\min}^k[\mathbf{P}^{-1}(0) - \frac{\alpha}{1-\lambda_{\min}}\mathbf{I}]$$

Canetti and Espana (R.M. Canetti, 1989), Ding (F. Ding, 2005) proved the Lemma 1.

Let $\mathbf{P}^{-1}(0)$ satisfies

$$\mathbf{P}^{-1}(0) \geq \frac{\alpha\mathbf{I}}{1-\lambda_{\min}}, or p_0 \leq \frac{1-\lambda_{\min}}{\alpha} \tag{22}$$

Then, for $k \geq N$, Eq.(23) is obtained

$$\begin{aligned}\mathbf{P}^{-1}(kT) &\geq \frac{\lambda_{\min}^{N-1}\alpha}{1-\lambda_{\min}}\mathbf{I},\\ or\ \ \mathbf{P}(kT) &\leq \frac{1-\lambda_{\min}}{\lambda_{\min}^{N-1}\alpha}\mathbf{I}, 0<\lambda_{\min}<1\end{aligned} \tag{23}$$

Lemma 1 displays that $\mathbf{P}(kT)$ in the VFF-AM-IRLS algorithm has the finite upper bounds under the PE condition.

Theorem 1. For the system in Eq.(12), $\{v(kT)\}$ is the independent white noise with zero mean and variance σ_v^2 under the condition A1, i.e. $v(kT)$ satisfies:

$$(A2)E[v(kT)] = 0$$
$$(A3)E[v^2(kT)] \leq \sigma_v^2 < \infty$$

Let $E[||\tilde{\theta}(0)||^2] = E[||\hat{\theta}(0) - \theta||^2] = \delta_0 < \infty$, $\hat{\theta}(0)$ is independent of $v(kT)$, $\tilde{\theta}(kT) = \hat{\theta}(kT) - \theta(kT)$ is the parameter error vector. Therefore, for $k \geq N$, the VFF-AM-IRLS algorithm in Eq.(16)-Eq.(21) has the following upper bounds:

$$\begin{aligned}E[||\tilde{\theta}(kT)||^2] \leq \alpha^{-2}p_0^{-2}\prod_{j=1}^{k}\lambda^2(jT)\lambda_{\min}^{-2(N-1)}(1-\lambda_{\min})^2\delta_0\\ +\frac{n(1-\lambda_{\min})}{\alpha\lambda_{\min}^{N-1}}\sigma_v^2 =: f_u(\lambda_{\min}, kT)\end{aligned}$$

The proof of Theorem 1 can be seen in the literature (F. Ding, 2005; Ding, 2014).

Furthermore, if p_0 is set as $p_0 = \frac{1-\lambda_{\min}}{\alpha}$, the $f_u(\lambda_{\min}, kT)$ can be rewritten:

$$f_u(\lambda_{\min}, kT) = \prod_{j=1}^{k}\lambda^2(jT)\lambda_{\min}^{-2(N-1)}\delta_0 + \frac{n(1-\lambda_{\min})}{\alpha\lambda_{\min}^{N-1}}\sigma_v^2$$

The measurement input data $\{u(kT - gT + t_{j-1})\}, g = 0, 1, ..., n_b, j = 1, 2, ..., m, k = 1, 2, ..., L$ is collected from physical systems or experiments, and the output of auxiliary model $\{\bar{y}(kT - hT)\}, h = 1, 2, ..., n_a$ is obtained from auxiliary model, so $\hat{\varphi}(kT)$ is known and α, β are computed according to (A1). Therefore, as $k \to \infty$, the error upper bound $f_u(\lambda_{\min}, kT)$ approximates a finite constant, i.e.

$$f_u(\lambda_{\min}, kT) \to \frac{n(1-\lambda_{\min})}{\alpha\lambda_{\min}^{N-1}}\sigma_v^2$$

where $n = n_a + mn_b + 5$ is the dimension of $\theta(kT)$ which is known, and σ_v^2 cannot be gained in practice. So, σ_v^2 is replaced with its estimation $\hat{\sigma}_v^2$

$$\hat{\sigma}_v^2 = \frac{1}{L}\sum_{i=1}^{L}[y'(iT) - \hat{\varphi}^T(iT)\hat{\theta}(LT)]^2$$

5 AN EXAMPLE

The non-uniformly sampled Wiener system (shown in Fig.1) was expressed as follows:

$$\begin{aligned}\bar{y}(kT) &= \frac{\sum_{j=1}^{m}B_j(z^{-1})}{A(z^{-1})}u(kT+t_{j-1})\\ &= [1-A(z^{-1})]\bar{y}(kT) + \sum_{j=1}^{m}B_j(z^{-1})u(kT+t_{j-1})\end{aligned} \tag{24}$$

where

$$A(z^{-1}) = 1 - 1.529z^{-1} + 0.7408z^{-2}$$
$$B_1(z^{-1}) = 0.1234 + 0.06899z^{-1} + 0.01538z^{-2}$$
$$B_2(z^{-1}) = 0.0421z^{-1} + 0.08506z^{-2}$$

and the parameters of dead-zone nonlinearities were $m_1 = 1.2, m_2 = 1.1, r_1 = 0.8, r_2 = -0.9$. Then the parameter vector was obtained

$$\begin{aligned}\theta =&[a_1, a_2, b_{10}, b_{11}, b_{12}, b_{21}, b_{22}, m_1, m_2, r_1, r_2]^T\\ =&[-1.529, 0.7408, 0.1234, 0.06899, 0.01538, 0.0421,\\ &0.08506, 1.2, 1.1, 0.8, -0.9]^T\end{aligned}$$

Take $m = 2, \tau_1 = 1s, \tau_2 = 1.5s$, then $t_0 = 0s, t_1 = \tau_1 = 1s$, $t_2 = \tau_1 + \tau_2 = 2.5s = T$. In this case, $u(kT)$ and $u(kT+t_1)$ were taken as PE signal sequence with zero mean and unit variance. When the noise variance $\sigma^2 = 0.1$, the VFF-AM-IRLS (with $\rho = 2$, $\lambda_{\min} = 0.95$) algorithm was used to estimate the parameters of system(24). The parameters estimation and their errors $\delta = ||\hat{\theta}(kT) - \theta||/||\theta|| \times 100\%$ were shown in Table 1. It could be seen that the PEE became smaller with quantity of data increasing, and tended to be stable after 1000 periods.

Table **1** The VFF-AM-IRLS estimates and errors with $\sigma^2 = 0.1$

k	100	200	500	1000	2000	3000	4000	true
a_1	-1.21378	-1.24413	-1.49611	-1.50554	-1.54760	-1.53978	-1.53572	-1.52900
a_2	0.48726	0.52605	0.70172	0.71649	0.75210	0.74718	0.74498	0.74080
b_{10}	0.05866	0.13211	0.12628	0.12494	0.12301	0.12239	0.12164	0.12340
b_{11}	-0.01754	0.12587	0.08638	0.06998	0.06579	0.06819	0.06775	0.06899
b_{12}	0.09026	0.09825	0.00070	-0.00571	0.00179	0.00647	0.00804	0.01538
b_{21}	0.01073	0.02086	0.03497	0.03579	0.03919	0.04044	0.04055	0.04210
b_{22}	0.13136	0.16383	0.10563	0.09253	0.09067	0.08808	0.08707	0.08506
m_1	0.75316	0.81348	1.14586	1.22548	1.20082	1.20274	1.21126	1.20000
m_2	0.81880	0.89518	1.08917	1.12283	1.14096	1.12416	1.09914	1.10000
r_1	0.57820	0.84796	0.77022	0.80268	0.77708	0.78057	0.79090	0.80000
r_2	0.08136	-0.55251	-0.86628	-0.88968	-0.90779	-0.89092	-0.86527	-0.90000
$\delta(\%)$	45.85271	25.56411	3.51762	2.05729	2.05862	1.36170	1.48340	0.00000

In order to test the performance of VFF-AM-IRLS algorithm, δ against k were shown in Fig.4 with different $\lambda_{\min}$ and ρ. Without forgetting factor, the method was stable during the estimating process. However the accuracy was lower, as shown in Fig.4 with the blue solid line (PEE was about 9%). Introducing the variable forgetting factor, the algorithm had fast convergence velocity. As a result, the estimated precision was improved greatly, as shown in Fig.4 with the green dosh-dot line (PEE was nearly 2%). Moreover, with ρ increasing, the convergence rate was more sensitive, and there was an evident wavelike at the early state of the estimation (shown in Fig.4 with the purple dot line).

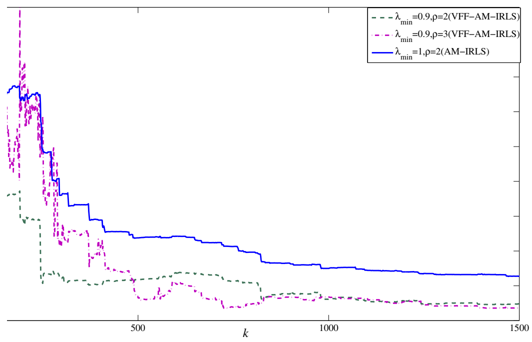

Fig. 4. The parameter estimation errors δ versus k

Set $\sigma^2 = 0.1$ and 0.2, the result was presented in Fig.5. It revealed that the estimated parameters were unstable with a larger σ, while VFF-AM-IRLS algorithm had high ability to eliminate interferences as well as accuracy.

6 CONCLUSIONS

A VFF-AM-IRLS algorithm is developed for a non-uniformly sampled Wiener systems with dead-zone nonlinearities. The proposed algorithm can estimate the system parameters directly by using the auxiliary model method and iterative method. Moreover, a variable forgetting factor is introduced to improve convergence velocity and anti-jamming ability. The simulation results demonstrate the effectiveness of the proposed algorithm. The method can

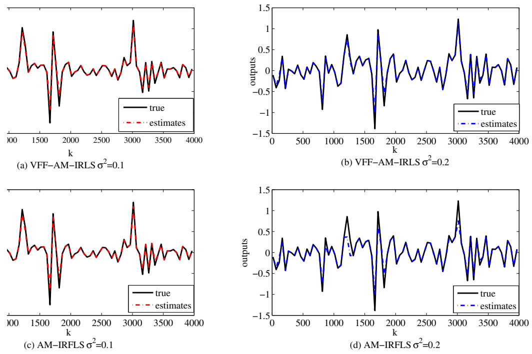

Fig. 5. The comparison of the true and estimates of the VFF-AM-IRLS and AM-IRFLS with different σ

be extended and used for other hard nonlinear Hammerstein, Wiener or Hammerstein-Wiener systems, other non-uniformly sampled systems, and multivariable systems. However, the selection of $\lambda_{\min}$ and ρ is very challenging and worth further investigate.

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