Soft Constraints for Robust MPC of Uncertain Systems

Guru Prasath *,** **John Bagterp Jørgensen** **,[1]

* *FLSmidth Private Limited. Automation Division Chennai, India (e-mail: mgp-in@flsaindia.co.in)*

** *Informatics and Mathematical Modelling Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark (e-mail: jbj@imm.dtu.dk).*

Abstract: In this paper we develop a robust constrained predictive controller for linear systems. The controller is equipped with soft output constraints that are used in a novel way to have robustness against model plant mismatch. By simulation we compare the performance of the new robust constrained predictive controller to a nominal predictive controller. In the nominal case, the performance of the robust predictive controller is comparable to the performance of the nominal predictive controller. In the case of plant model mismatch, the robust predictive controller performs significantly better than the nominal predictive controller.

Keywords: Linear Model Predictive Control, Robust Predictive Control, Soft Constraints

1. INTRODUCTION

Model predictive control has become a standard technology in high level control of chemical processes. However, little advice is available regarding tuning methodologies of such controllers in the face of the inevitable plant model mismatch. The closed-loop performance of nominal linear model predictive control can be quite poor when the models are uncertain. Consequently, some years after commissioning, many high-level control systems are turned off due to bad closed-loop performance. This is often due to changes in the plant dynamics caused by wear and tear combined with lack of the necessary human resources at the plant to re-tune and maintain the MPC. Model predictive controllers with robust performance against model-plant mismatch is therefore crucial in long-term maintenance and success of MPC system. Using soft output constraints in a novel way, we demonstrate by simulation that the poor performance of predictive control in the case of plant model mismatch can be improved significantly. Therefore, we suggest use of the soft constraints to tune and improve the performance of linear model predictive control.

Specifically, we investigate the effect of uncertain models on the performance of a regularized l_2 model predictive controller with input constraints, input-rate constraints and soft output constraints (Maciejowski, 2002; Goodwin et al., 2005; Qin and Badgwell, 2003). Previously, the soft output constraints have been used to replace hard output constraint and guarantee feasibility (Scokaert and Rawlings, 1999). We use the soft output constraints to create a dead zone around the set point and demonstrate by simulation that the performance of such an MPC does not degrade much in the nominal case but improves significantly in the case of plant model mismatch. This technique is similar but not identical to the funnels used by Honeywell in RMPC (Qin and Badgwell, 2003; Havlena and Lu, 2005; Havlena and Findejs, 2005). Compared to classical process control, our use of the soft constraints has some similarities to PID control with dead zones (Shinskey, 1988).

We use a finite impulse response (FIR) model for prediction of the process outputs. In contrast to state space parameterizations, the FIR model is in a form that can easily be applied in robust predictive control, i.e. predictive control based on robust linear programming or second-order cone programming (Hansson, 2000; Vandenberghe et al., 2002; Boyd and Vandenberghe, 2004). To facilitate comparative performance studies of l_2 and robust MPC, a FIR based l_2-MPC benchmark has been established (Prasath and Jørgensen, 2008). The soft output constraint included in the MPC acts as a dead zone to the controller to reduce its sensitivity to noise and uncertainty when the process output is close to its target. This use of soft constraints for robustness is new, simple, and gives good performance. Bemporad and Morari (1999) provide an excellent survey of methodologies for robust model predictive control.

This paper is organized as follows. We derive the predictive controller consisting of a regulator and an estimator with soft output constraints in Section 2. Section 3 illustrates by simulation the performance of MPC with and without soft constraints for both deterministic and stochastic processes. Conclusions are given in Section 4.

2. FIR MODEL BASED MPC

Model predictive control systems consist of an estimator and a regulator as illustrated in Figure 1. The inputs to

[1] Corresponding author.

the MPC are the target values, r, for the process outputs, z, and the measured process outputs, y. The output from the MPC is the manipulated variables, u.

2.1 Plant and Sensors

The plant is assumed to be a linear state space system

$$\boldsymbol{x}_{k+1} = A\boldsymbol{x}_k + Bu_k + B_d d_k + G\boldsymbol{w}_k \tag{1a}$$

$$\boldsymbol{z}_k = C\boldsymbol{x}_k \tag{1b}$$

with x being the states, u being the manipulated variables (MVs), d being unmeasured disturbances, and w being stochastic process noise. z denotes the controlled variables (CVs). The measured outputs, y, are the controlled outputs, z, corrupted by measurement noise, v. Consequently

$$\boldsymbol{y}_k = \boldsymbol{z}_k + \boldsymbol{v}_k \tag{1c}$$

The initial state, the process noise, and the measurement noise are assumed to be normally distributed stochastic vectors

$$\boldsymbol{x}_0 \sim N(\bar{x}_0, P_0) \tag{2a}$$

$$\boldsymbol{w}_k \sim N_{iid}(0, Q) \tag{2b}$$

$$\boldsymbol{v}_k \sim N_{iid}(0, R) \tag{2c}$$

The measured output, y, is the signal available for feedback and used by the estimator. u is the signal generated by the control system and implemented on the plant.

2.2 Regulator

Stable processes can be represented by the finite impulse response (FIR) model

$$z_k = b_k + \sum_{i=1}^{n} H_i u_{k-i} \tag{3}$$

in which $\{H_i\}_{i=1}^n$ are the impulse response coefficients (Markov parameters). b_k is a bias term generated by the estimator. b_k accounts for discrepancies between the predicted output and the actual output. In this paper, the output predictions used by the regulator are based on the FIR model (3). Consequently, using the FIR model (3), the regularized l_2 output tracking problem with input and soft output constraints may be formulated as

$$\min_{\{z,u,\eta\}} \phi = \frac{1}{2}\sum_{k=0}^{N-1} \|z_{k+1} - r_{k+1}\|_{Q_z}^2 \; + \|\Delta u_k\|_{S_u}^2 + \sum_{k=1}^{N} \frac{1}{2}\|\eta_k\|_{S_\eta}^2 + s_{\eta_k}'\eta_k \tag{4a}$$

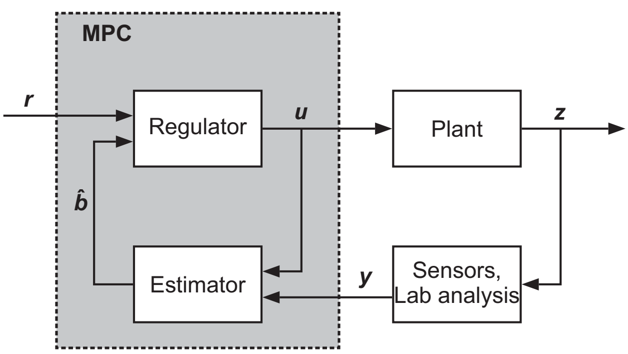

Fig. 1. Generic model predictive control system.

subject to the constraints

$$
\begin{aligned}
z_k &= b_k + \sum_{i=1}^{n} H_i u_{k-i} & k &= 1,\dots,N & \text{(4b)}\\
u_{\min} &\le u_k \le u_{\max} & k &= 0,\dots,N-1 & \text{(4c)}\\
\Delta u_{\min} &\le \Delta u_k \le \Delta u_{\max} & k &= 0,\dots,N-1 & \text{(4d)}\\
z_k &\le z_{\max,k} + \eta_k & k &= 1,\dots,N & \text{(4e)}\\
z_k &\ge z_{\min,k} - \eta_k & k &= 1,\dots,N & \text{(4f)}\\
\eta_k &\ge 0 & k &= 1,\dots,N & \text{(4g)}
\end{aligned}
$$

in which $\Delta u_k = u_k - u_{k-1}$. In this formulation, the control and the prediction horizon are identical. If desired, a prediction horizon longer than the control horizon could be included in the formulation. However, we prefer instead to select the control horizon sufficiently long such that any boundary effects at the end of the horizon has no influence on the solution in the beginning of the horizon. (4) can be converted to a constrained linear-quadratic optimal control problem. Efficient algorithms exists for the solution of such problems with long prediction horizons, N. In this paper we adopt another approach and formulate a dense quadratic program in standard form that is equivalent with (4).

Define the vectors Z, R, U and η as

$$Z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{bmatrix} \quad R = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix} \quad U = \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{N-1} \end{bmatrix} \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_N \end{bmatrix} \tag{5}$$

Then the predictions by the impulse response model (4) may be expressed as

$$Z = c + \Gamma U \tag{6}$$

For the case $N = 6$ and $n = 3$, Γ is assembled as

$$\Gamma = \begin{bmatrix} H_1 & 0 & 0 & 0 & 0 & 0 \\ H_2 & H_1 & 0 & 0 & 0 & 0 \\ H_3 & H_2 & H_1 & 0 & 0 & 0 \\ 0 & H_3 & H_2 & H_1 & 0 & 0 \\ 0 & 0 & H_3 & H_2 & H_1 & 0 \\ 0 & 0 & 0 & H_3 & H_2 & H_1 \end{bmatrix}$$

and c is

$$c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} = \begin{bmatrix} b_1 + (H_2 u_{-1} + H_3 u_{-2}) \\ b_2 + (H_3 u_{-1}) \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

Similarly, for the case $N = 6$, define the matrices Λ and I_0 by

$$\Lambda = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ -I & I & 0 & 0 & 0 & 0 \\ 0 & -I & I & 0 & 0 & 0 \\ 0 & 0 & -I & I & 0 & 0 \\ 0 & 0 & 0 & -I & I & 0 \\ 0 & 0 & 0 & 0 & -I & I \end{bmatrix} \quad I_0 = \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Define $s_\eta = \begin{bmatrix} s_{\eta_1}' & s_{\eta_2}' & \dots & s_{\eta_N}' \end{bmatrix}'$ and

$$\mathcal{Q}_z = \begin{bmatrix} Q_z & & & \\ & Q_z & & \\ & & \ddots & \\ & & & Q_z \end{bmatrix} \quad \mathcal{S}_i = \begin{bmatrix} S_i & & & \\ & S_i & & \\ & & \ddots & \\ & & & S_i \end{bmatrix}$$

with $i = \{u, \eta\}$. Then the objective function (4) may be expressed as

$$
\begin{aligned}
\phi &= \frac{1}{2}\sum_{k=0}^{N-1} \|z_{k+1} - r_{k+1}\|^2_{\mathcal{Q}_z} + \|\Delta u_k\|^2_{\mathcal{S}_u} \\
&\qquad\qquad + \frac{1}{2}\|\eta_{k+1}\|^2_{\mathcal{S}_\eta} + s'_{\eta_{k+1}}\eta_{k+1} \\
&= \frac{1}{2}\|Z - R\|^2_{\mathcal{Q}_z} + \frac{1}{2}\|\Lambda U - I_0 u_{-1}\|^2_{\mathcal{S}_u} \\
&\qquad\qquad + \frac{1}{2}\|\eta\|^2_{\mathcal{S}_\eta} + s'_\eta \eta \\
&= \frac{1}{2}\|c + \Gamma U - R\|^2_{\mathcal{Q}_z} + \frac{1}{2}\|\Lambda U - I_0 u_{-1}\|^2_{\mathcal{S}_u} \\
&\qquad + \frac{1}{2}\|\eta\|^2_{\mathcal{S}_\eta} + s'_\eta \eta \qquad\qquad (7) \\
&= \frac{1}{2}U'\left(\Gamma'\mathcal{Q}_z\Gamma + \Lambda'\mathcal{S}_u\Lambda\right)U \\
&\qquad + \left(\Gamma'\mathcal{Q}_z(c - R) - \Lambda'\mathcal{S}_u I_0 u_{-1}\right)' U \\
&\qquad + \left(\frac{1}{2}\|c - R\|^2_{\mathcal{Q}_z} + \frac{1}{2}\|I_0 u_{-1}\|^2_{\mathcal{S}_u}\right) \\
&\qquad + \frac{1}{2}\eta'\mathcal{S}_\eta\eta + s'_\eta\eta \\
&= \frac{1}{2}U'HU + g'U + \rho + \frac{1}{2}\eta'\mathcal{S}_\eta\eta + s'_\eta\eta \\
&= \frac{1}{2}x'\bar{H}x + \bar{g}'x + \rho
\end{aligned}
$$

with

$$H = \Gamma'\mathcal{Q}_z\Gamma + \Lambda'\mathcal{S}_u\Lambda \tag{8a}$$

$$g = \Gamma'\mathcal{Q}_z(c - R) - \Lambda'\mathcal{S}_u I_0 u_{-1} \tag{8b}$$

$$\rho = \frac{1}{2}\|c - R\|^2_{\mathcal{Q}_z} + \frac{1}{2}\|u_{-1}\|^2_{\mathcal{S}_u} \tag{8c}$$

$$x = \begin{bmatrix} U \\ \eta \end{bmatrix} \quad \bar{H} = \begin{bmatrix} H & 0 \\ 0 & \mathcal{S}_\eta \end{bmatrix} \quad \bar{g} = \begin{bmatrix} g \\ s_\eta \end{bmatrix} \tag{8d}$$

Consequently, we may solve the FIR based MPC regulator problem (4) by solution of the following convex quadratic program

$$\min_x \quad \psi = \frac{1}{2}x'\bar{H}x + \bar{g}'x \tag{9a}$$

$$s.t. \quad x_{\min} \leq x \leq x_{\max} \tag{9b}$$

$$b_l \leq \bar{A}x \leq b_u \tag{9c}$$

in which

$$x_{\min} = \begin{bmatrix} U_{\min} \\ 0 \end{bmatrix} \qquad x_{\max} = \begin{bmatrix} U_{\max} \\ \infty \end{bmatrix} \tag{10a}$$

$$b_l = \begin{bmatrix} \Delta U_{\min} \\ -\infty \\ Z_{\min} - c \end{bmatrix} \; A = \begin{bmatrix} \Lambda & 0 \\ \Gamma & -I \\ \Gamma & I \end{bmatrix} \; b_u = \begin{bmatrix} \Delta U_{\max} \\ Z_{\max} - c \\ \infty \end{bmatrix} \tag{10b}$$

In a model predictive controller only the first vector, u_0^*, of $U^* = \left[(u_0^*)' \; (u_1^*)' \; \ldots \; (u_{N-1}^*)'\right]'$, is implemented on the process. At the next sample time the open-loop optimization is repeated with new information due to a new measurement.

2.3 Soft Constraint Principle

Figure 2 illustrates the stage cost function for l_2 model predictive control (nominal MPC) and l_2 model predictive control with a dead zone (soft MPC). The stage cost

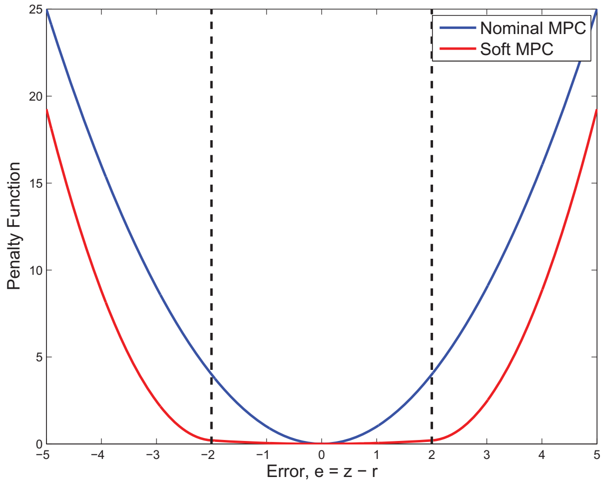

Fig. 2. The set point deviation penalty function for nominal MPC and soft MPC.

function, or penalty function, is plotted as function of the set-point error, $e = z - r$. The penalty function of the nominal MPC is a quadratic function. The penalty function of the soft MPC is constructed such that it is zero or almost zero within the dead-zone between the soft limits and growths quadratically when the set-point error exceeds the soft limits. The small penalty within the soft limits ensures that the controller produces a steady state offset free response. By having the penalty small within the soft constraints, the controller does not react much to small errors. In this way we avoid that the controller introduces significant real disturbances to the process because it reacts to say measurement noise or plant-model mismatch. Outside the soft limits, it is assumed that the deviation from target is due to a real process disturbance, and the soft MPC may be designed to react in the same way as the nominal MPC.

2.4 Simple Estimator

To have offset free steady state control when unknown step responses occur, we must have integrators in the feedback loop. This may be achieved using a FIR model in difference variables. Assume that the relation between the inputs and outputs may be represented as

$$\Delta y_k = \Delta z_k = e_k + \sum_{i=1}^{n} H_i \Delta u_{k-i} \tag{11}$$

in which Δ is the backward difference operator, i.e. $\Delta y_k = y_k - y_{k-1}$, $\Delta z_k = z_k - z_{k-1}$, and $\Delta u_k = u_k - u_{k-1}$. This representation is identical with the FIR model (3)

$$y_k = z_k = \hat{b}_k + \sum_{i=1}^{n} H_i u_{k-i} \tag{12}$$

if $\hat{b}_k$ is computed by

$$e_k = \Delta y_k - \sum_{i=1}^{n} H_i \Delta u_{k-i} \tag{13a}$$

$$\hat{b}_k = \hat{b}_{k-1} + e_k \tag{13b}$$

Note that in the regulator optimization problem $b_1 = b_2 = \ldots = b_N = \hat{b}_k$ at each time instant. This is based on the assumption that the disturbances enter the process as constant output disturbances. Of course this may not be how the disturbances enter the process in practice, and significant performance deterioration may result as a consequence of this representation.

3. SIMULATIONS

In this Section we consider plants of the form

$$\boldsymbol{Z}(s) = G(s)U(s) + G_d(s)\left(D(s) + \boldsymbol{W}(s)\right) \tag{14a}$$
$$\boldsymbol{y}(t_k) = \boldsymbol{z}(t_k) + \boldsymbol{v}(t_k) \tag{14b}$$

with the transfer functions

$$G(s) = \frac{K(\beta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\tau s} \tag{15a}$$
$$G_d(s) = \frac{K_d(\beta_d s + 1)}{(\tau_{d1} s + 1)(\tau_{d2} s + 1)} e^{-\tau_d s} \tag{15b}$$

The disturbance model, $G_d(s)$, is kept fixed at its nominal value, while the transfer function, $G(s)$, from $U(s)$ varies around its nominal value, $G_0(s)$. This is used to illustrate the consequence of model uncertainty on the MPC closed-loop performance. The nominal system is $K = K_d = 1$, $\tau_1 = \tau_2 = \tau_{d1} = \tau_{d2} = 5$, $\beta = \beta_d = 2$, and $\tau = \tau_d = 5$. The system is converted to a discrete time state space model (1) using a sample time of $T_s = 1$ and a zero-order-hold assumption on the inputs.

The predictive controller is based on the impulse response coefficients of the following system

$$Z(s) = \hat{G}(s)U(s) \tag{16}$$

in which $\hat{G}(s)$ is identical to the nominal plant $G_0(s)$.

The simple estimator described in Section 2.4 is used for bias estimation. The input limits are $u_{\min} = -1$, $u_{\max} = 1$, $\Delta u_{\min} = -0.2$, and $\Delta u_{\max} = 0.2$. The horizon of the impulse response model is $n = 40$ and the control horizon is $N = 120$. The MPC is tuned with $Q_z = 1$ and $S = 10^{-3}$.

The unknown deterministic process disturbance, $D(s)$, the stochastic process disturbances, $\boldsymbol{W}(s)$ or $\boldsymbol{w}_k$, and the measurement noise, $\boldsymbol{v}(t_k) = \boldsymbol{v}_k$, used in the simulations are illustrated in Figure 3. The stochastic process disturbances is $\boldsymbol{w}_k \sim N(0, 0.01)$, and the stochastic measurement noise is $\boldsymbol{v}_k \sim N(0, 0.01)$.

3.1 Nominal Stochastic System

We consider the case when the model used by the controller is identical to the deterministic part of the plant model. However, the plant has in addition to the deterministic part stochastic process disturbances and stochastic measurement noise as illustrated in Figure 3.

Consider the case with no determistic disturbance, i.e. $D(s) = 0$. The performances of the nominal MPC and the soft MPC applied to this system are compared in Figure 4. The output variances produced by the two controllers are almost identical, while the input variance of the soft MPC is much smaller than the input variance of the nominal MPC. Due to the low penalties within the soft limits, the soft MPC does not react to measurement noise and do not need to compensate such previous erroneous measurement noise induced input moves.

Figure 5 illustrates the performance of the nominal MPC and the soft MPC when the model is identical to the

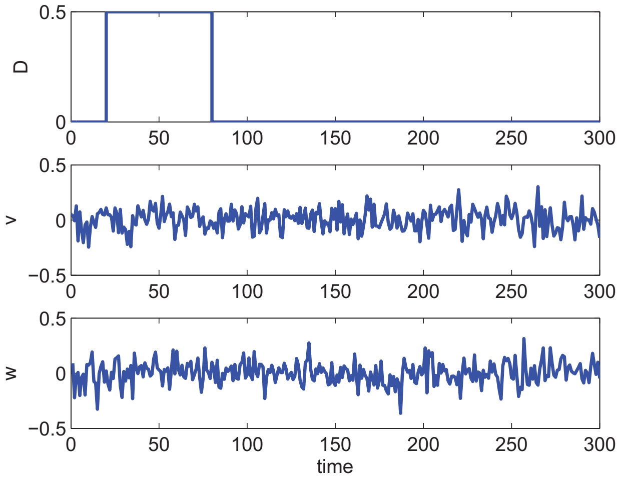

Fig. 3. External signals used in the closed loop simulations. $D(s)$ or d_k is the unknown deterministic disturbance, $\boldsymbol{v}_k$ is the stochastic measurement noise, and $\boldsymbol{w}_k$ is the stochastic process noise,

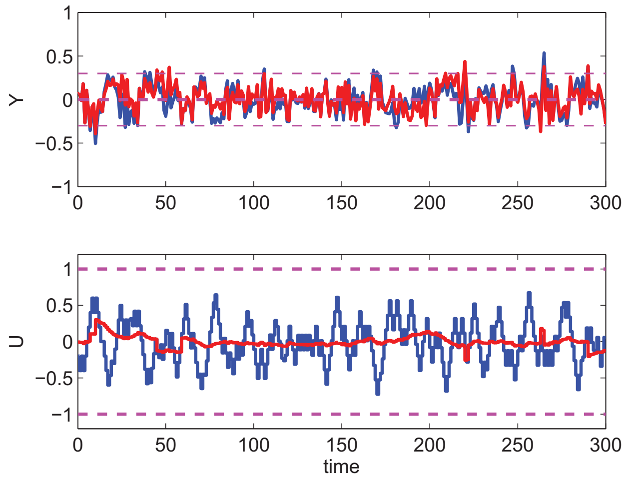

Fig. 4. Comparison of normal and soft MPC with nominal models applied to a stochastic system with no deterministic disturbance (Nominal MPC = blue, Soft MPC = red).

plant model and the external signals illustrated in Figure 3 are applied to the model (14). Also in this case, the controlled variable, Y (or Z), of the two controllers are similar while the manipulated variable, U, of the soft MPC has significantly less variance than the manipulated variable, U, of the nominal MPC.

3.2 Uncertain Determistic System

We consider a deterministic system without stochastic process noise nor stochastic measurement noise. However, the model used by the controllers is different from the plant model. The process is perturbed by an unknown deterministic disturbance, $D(s)$, as illustrated in figure 3.

We compare the performance of the nominal MPC and the soft MPC for model-plant mismatches defined by the time delay, τ, the gain K, the time constant τ_1, and the zero β.

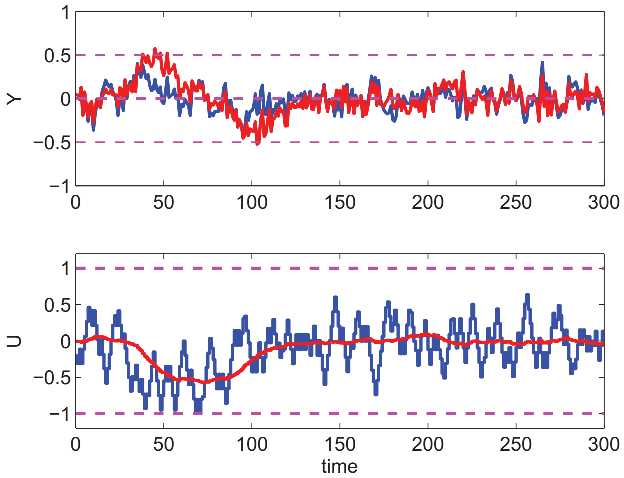

Fig. 5. Comparison of normal and soft MPC with nominal models applied to a stochastic system with an unknown deterministic disturbance (Nominal MPC = blue, Soft MPC = red). The external signals are shown in Figure 3.

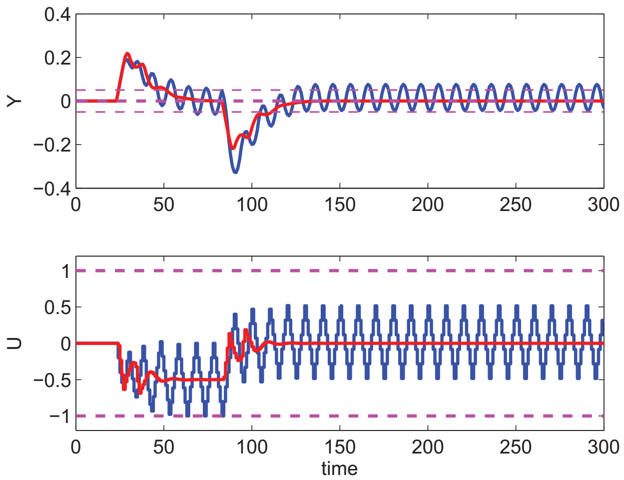

Fig. 6. Closed-loop MPC performance with time delay uncertainty. The plant delay is $\tau = 3$ and the model delay is $\hat{\tau} = 5$ (Nominal MPC = blue, Soft MPC = red).

Figure 6 and Figure 7 illustrate closed-loop performances achieved by the nominal and soft MPC when there is time delay plant-model mismatch. The soft MPC has smaller input variation than the nominal MPC, and the soft MPC provies better control than the nominal MPC in terms of set point deviations.

Figures 8-10 illustrate the performances of the nominal MPC and the soft MPC in the case of model-plant mismatch in the gain, the time constant, and the zero, respectively. In all cases, the soft MPC has significantly less input variation than the nominal MPC. Furthermore, the outputs are significantly better controlled by the soft MPC than by the nominal MPC.

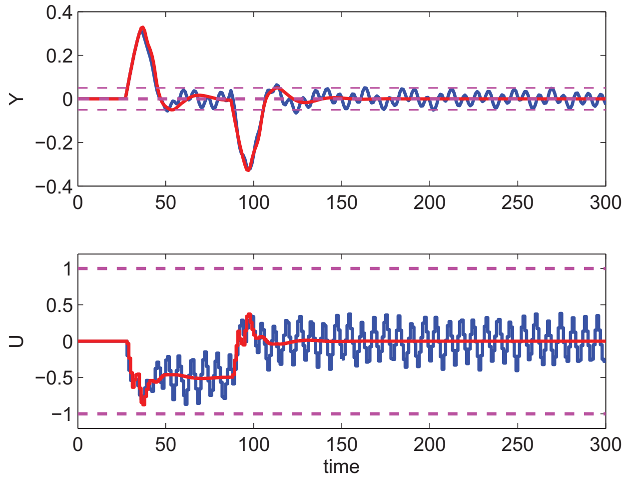

Fig. 7. Closed-loop MPC performance with time delay uncertainty. The plant delay is $\tau = 7$ and the model delay is $\hat{\tau} = 5$ (Nominal MPC = blue, Soft MPC = red).

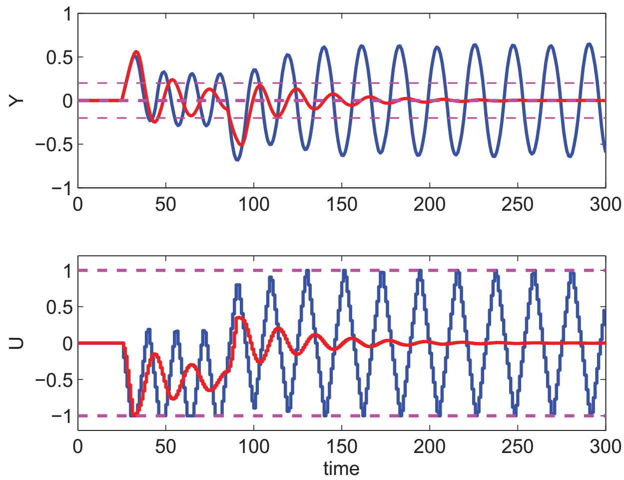

Fig. 8. Closed-loop MPC performance with gain uncertainty. The plant gain is $K = 2$ and the model gain is $\hat{K} = 1$ (Nominal MPC = blue, Soft MPC = red).

3.3 Uncertain Stochastic System

Figure 11 illustrates the closed loop performance of a nominal MPC and a soft MPC applied to the system (14) with the external signals in Figure 3 and a plant-model mismatch in the gain. The plant gain is $K = 2$ and the model gain is $\hat{K} = 1$. By inspection, it is obvious that the performance of the soft MPC is significantly better than the performance of the nominal MPC. The superior performance is achieved by having a small set point deviation penalty within the soft constraints such that the controller does not react aggressively when close to the set point. In this way it avoids perturbing the system due to stochastic measurement noise and plant-model mismatch.

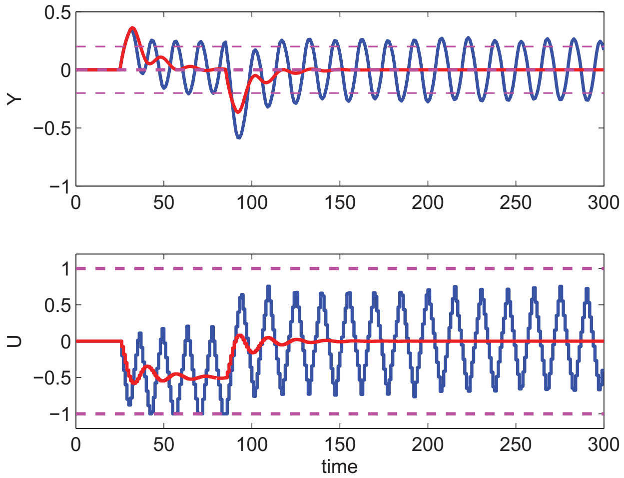

Fig. 9. Closed-loop MPC performance with time constant uncertainty. The plant time constant is $\tau_1 = 2$ and the model time constant is $\hat{\tau}_1 = 5$ (Nominal MPC = blue, Soft MPC = red).

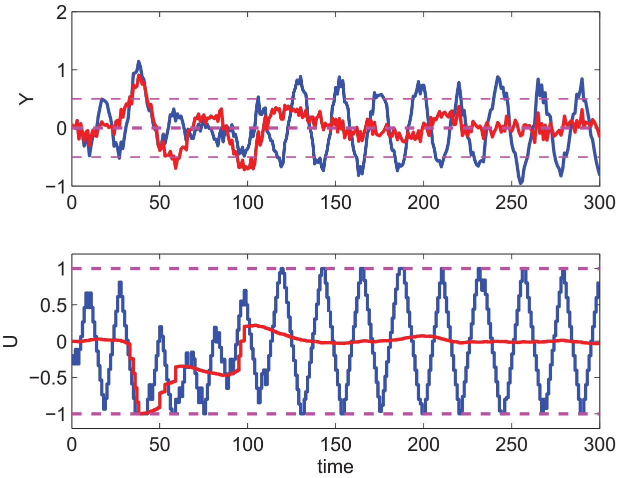

Fig. 11. Closed-loop MPC performance for a system with the external signals in Figure 3 and gain uncertainty. The plant gain is $K = 2$ and the model gain is $\hat{K} = 1$. Nominal MPC = blue, Soft MPC = red.

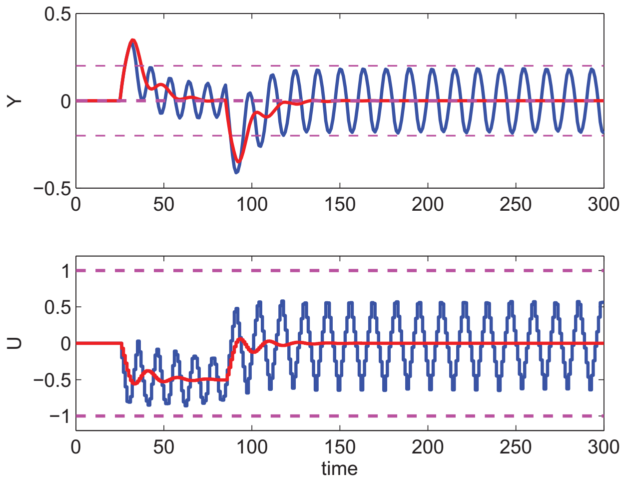

Fig. 10. Closed-loop MPC performance with zero uncertainty. The plant zero is $\beta = 4.5$ and the model zero is $\hat{\beta} = 2$ (Nominal MPC = blue, Soft MPC = red).

4. CONCLUSION

We have developed a l_2 regularized predictive controller with soft constraints and demonstrated efficient application of this controller to uncertain stochastic systems. We call this controller soft MPC. It is illustrated and verified by simulations that this soft MPC provides significantly better closed loop performance than nominal MPC. The soft MPC also provides much better performance degradation in the face of plant-model mismatch than nominal MPC. These features are expected to contribute to better closed loop performance, easier maintenance, easier tuning, and longer lifetime of model predictive controllers for chemical processes.

REFERENCES

Bemporad, A. and Morari, M. (1999). Robust model predictive control: A survey. In A. Garulli, A. Tesi, and A. Vicino (eds.), *Robustness in Identification and Control*, volume 245 of *Lecture Notes in Control and Information Sciences*, 207–226. Springer-Verlag.

Boyd, S. and Vandenberghe, L. (2004). *Convex Optimization*. Cambridge University Press, Cambridge, UK.

Goodwin, G.C., Seron, M.M., and De Dona, J.A. (2005). *Constrained Control and Estimation. An Optimization Approach*. Springer, New York.

Hansson, A. (2000). A primal-dual interior-point method for robust optimal control of linear discrete-time systems. *IEEE Transactions on Automatic Control*, 45, 1639–1655.

Havlena, V. and Lu, J. (2005). A distributed automation framework for plant-wide control, optimisation, scheduling and planning. In *IFAC World Congress 2005*. IFAC, Prague, Czech Republic.

Havlena, V. and Findejs, J. (2005). Application of model predictive control to advanced combustion control. *Control Engineering Practice*, 13, 671–680.

Maciejowski, J.M. (2002). *Predictive Control with Constraints*. Prentice Hall, Harlow, England.

Prasath, G. and Jørgensen, J.B. (2008). Model predictive control based on finite impulse response models. In *ACC 2008*. American Control Conference 2008, Seattle, Washington.

Qin, S.J. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, 11, 733–764.

Scokaert, P.O.M. and Rawlings, J.B. (1999). Feasibility issues in linear model predictive control. *AIChE Journal*, 45, 1649–1659.

Shinskey, F.G. (1988). *Process Control Systems. Application, Design, and Tuning*. McGraw-Hill, New York.

Vandenberghe, L., Boyd, S., and Nouralishadi, M. (2002). Robust linear programming and optimal control. In *15th IFAC World Congress 2002*. IFAC, Barcelona, Spain.