A nonlinear control strategy for a Bidirectional Flow Process

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Abstract: A nonlinear control strategy based on Interconnection Damping Assignment Passivity Based Control (IDA-PBC) is proposed for a process with bidirectional flow. The bidirectional flow condition introduces singularities in the control action under certain operation conditions. A solution to this problem is proposed such that operation through the singular points is possible and the stability conditions around the desired operation point are exactly preserved. In addition, a passivity based integral action is included in order to take into account the effects of model uncertainties and unknown step like disturbances. A description of the process and the controller design methodology is presented along with some numerical simulations illustrating the closed-loop behavior of the proposed controller.

1. INTRODUCTION

There are many applications where the characteristic of the process changes and there is the possibility of having singular points associated to the control variable. This condition means that the states of the process are not controllable at the singular point and the control variable became unbounded. This condition can be found for instance in chemical reactors (E.J. McColm and M. T. Tham, 1995) and in electromechanical systems (F. Zhang and B. Fernandez, 2006). In most cases this problem is overcome by modifying the control law such that the singular point is eliminated. There are two approaches for dealing with this problem: the first one is based on differentiation (E. J. McColm and Ming T. Tham, 1995), and the second one on a modification of the control law (H. Xu and P.A. Ioannou, 2004). In this work, the second option is used to design an IDA-PB controller that can deal with singular points without affecting the closed loop stability when the system is outside the set of singular points.

Port-Hamiltonian (PH) systems and Interconnection Damping Assignment Passivity Based Control (IDA-PBC) are two powerful approaches for modeling and control of nonlinear systems. PH representations are physical motivated, since they are based on models representing mass and energy balances; where the structure of the model takes into account the interaction between the system and it environment.

IDA-PBC control approach relies on the notions of interconnections, dissipation and energy balance, (Ortega et al., 2001, 2002). The capability to define precisely the interconnections and energy dissipations of non-linear processes makes the use of PH representation an attractive modeling tool and hence, IDA-PBC an effective alternative to design high performance non-linear controllers for complex processes.

The IDA-PBC approach has been shown to be very effective when the system characteristics changes from one operation mode to another. For instance, in (Ramírez et al., 2008) the same IDA-PB controller is used to stabilize a minimum and a non-minimum phase system, and in (Batlle et al., 2005) this approach is used to stabilize a bidirectional power systems, where the direction of power is reversed depending on the operation mode of a power converter. In this work, a nonlinear bidirectional flow process with singular points operation is used as an application example. The process consists of three serials tanks at same height; hence flow inversion between tanks is possible. The process also considers an unknown disturbance.

This paper is organized as follows: Section 2 describes a process comprising three tanks in series with the possibility of having reversing flow, and the model of this system. Section 3 presents the design of a IDA-PBC plus integral action. In section 4, a solution to the singular point operation is proposed and the system stability is analyzed. Some simulations results are presented in section 5 and finally, in section 6, some closing remarks are given.

2. THE THREE TANKS CONTROL PROBLEM

The multi tank serial circuit is a multivariable system, fully actuated and minimum phase. This process has three tanks of the same height in a serial arrangement, as depicted in Fig. 1.

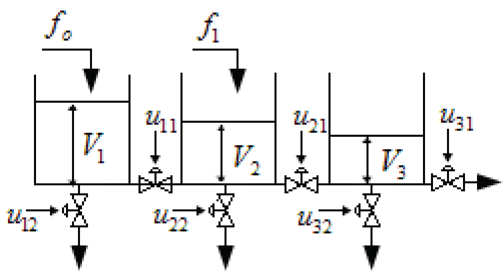

Fig. 1. Proposed system of serial tanks

In Fig. 1 the control valves are u_{11}, u_{21} and u_{31}. The remaining valves, u_{12}, u_{22} and u_{32} are manual valves, whose openings remain constant during the entire operation. On other hand, the feed flow rate into the first tank is measurable and the feed flow rate into the second tank is unknown. The last tank does not have any independent feed flow rate.

The control objective is to operate the tanks at different heights by allowing flow reversing operations.

The tanks are at the same height so the flow direction between tanks depends on the bottom pressure on each of them; i.e. the flow will go from the tank with higher water level to the tank with lower one. This reversing flow phenomenon occurs only in the first and second tanks because they are the only ones with an independent feed flow rate, f_o and f_I respectively. The maximum water level in the third tank is the level of the second tank, because it does not have an independent water flow rate. Another phenomenon, that arises during the flow reversing process, is the lack of controllability, this occurs when the water level in the first and second tank are the same.

Physically, given a set of feed flow rates the processes always operate in one of these modes. In order to model the process using mass balance equations, the following variables are defined: $x_i \in \mathbb{R}_+$ is the volume inside of a tank i, A_i the cross section (they are constant and the same for all tanks) and $k_{ij}(u_{ij}) = u_{ij}$ linear valve opening functions. Thus, the equations representing the system are:

$$\dot{x}_1 = f_o - u_{11} \cdot sign(x_1 - x_2) \cdot a_1 - b_1 \ , \qquad (1)$$

$$\dot{x}_2 = f_1 + u_{11} \cdot sign(x_1 - x_2) \cdot a_1 - u_{21} \cdot sign(x_2 - x_3) \cdot a_2 - b_2 \ , \qquad (2)$$

$$\dot{x}_3 = u_{21} \cdot sign(x_2 - x_3) \cdot a_2 - u_{31} \cdot a_3 - b_3 \ , \qquad (3)$$

where, for notational convenience, we have defined:

$$a_1 = \sqrt{2g\left(\frac{x_1}{A_1} - \frac{x_2}{A_2}\right)} \ ; \ a_2 = \sqrt{2g\left(\frac{x_2}{A_2} - \frac{x_3}{A_3}\right)} \ ; \ a_3 = \sqrt{2g\frac{x_3}{A_3}}$$

$$b_1 = \sqrt{2g\frac{x_1}{A_1}} \cdot u_{12} \ ; \ b_2 = \sqrt{2g\frac{x_2}{A_2}} \cdot u_{22} \ ; \ b_3 = \sqrt{2g\frac{x_3}{A_3}} \cdot u_{32} \ .$$

In order to invert the flow rate direction, between the first and second tank, the feed flow rate in the first tank must satisfy $f_o \le b_1$. This can be obtained by calculating the operation point for u_{11}

$$u_{o11} = \frac{f_o - b_1}{a_1 \cdot sign(x_{o1} - x_{o2})} \ . \qquad (4)$$

Thus, in order to have $u_{o11} \ge 0$, for a operation point $x_{o1} < x_{o2}$ with $x_{o1}, x_{o2} \in \mathbb{R}_+$, the following inequality has to be satisfied that $f_o - b_1 \le 0$. To get a flow from the first to the second tank we need $f_o - b_1 \ge 0$.

We will also define the set of singular points as all the $x_1, x_2 \in \mathbb{R}_+$, such that $x_1 = x_2$.

From the knowledge of the process and given a set of possible combinations of feed flow rates, the following operating modes can be identified:

Operation Mode 1: The feed flows are: $f_o > b_1, f_1 \ge 0$, the initial state $x_1^0 > x_2^0 > x_3^0$, with $x_1^0, x_2^0, x_3^0 \in \prod \subset \mathbb{R}_+$ and references $x_1^* > x_2^* > x_3^*$, with $x_1^*, x_2^*, x_3^* \in \mathbb{R}_+$. The set $\prod$ represents the physical admissible levels. The flow goes from the first to the second tank.

Operation Mode 2: The feed flows are: $f_o \le b_1, f_1 > 0$, the initial state $x_1^0 > x_2^0 > x_3^0$, with $x_1^0, x_2^0, x_3^0 \in \prod \subset \mathbb{R}_+$ and references $x_1^* < x_2^*$ and $x_3^* < x_2^*$, with $x_1^*, x_2^*, x_3^* \in \mathbb{R}_+$. The flow is inverted and goes from the second to the first tank.

Operation Mode 3: The feed flows are: $f_o \le b_1, f_1 > 0$, the initial state $x_1^0 < x_2^0$ and $x_3^0 < x_2^0$, with $x_1^0, x_2^0, x_3^0 \in \prod \subset \mathbb{R}_+$ and references $x_1^* < x_2^*$ and $x_3^* < x_2^*$, with $x_1^*, x_2^*, x_3^* \in \mathbb{R}_+$. The flow goes from the second to the first tank.

Operation Mode 4: The feed flows are: $f_o > b_1, f_1 \ge 0$, the initial state $x_1^0 < x_2^0$ and $x_3^0 < x_2^0$, with $x_1^0, x_2^0, x_3^0 \in \prod \subset \mathbb{R}_+$ and references $x_1^* > x_2^* > x_3^*$, with $x_1^*, x_2^*, x_3^* \in \mathbb{R}_+$. The flow is inverted and goes from the first to the second tank.

3. CONTROLLER DESIGN USING IDA-PBC

It is convenient to represent the system in a PH form, to simplify the application of the IDA-PBC.

Consider a process described by a PH system of the form

$$\dot{x} = \left[J(x) - \Re(x)\right]\frac{\partial H}{\partial x}(x) + g(x)u + q(x)f \qquad (5)$$

Where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the mass (volume) variables and the control, respectively. The smooth function $H(x)$ typically represents the total stored mass, $f = \left[f_O \ \ f_1\right]^T \in \mathbb{R}^m$ represents constant disturbances and $q(x)$ defines the interaction between the system and f. the skew-symmetric matrix $J(x) = -J^T(x)$ represents the interconnection between the different system's components, and $\Re(x) = \Re^T(x) \ge 0$ is the dissipation matrix, while $g(x)$ defines the interconnection of the system with its environment. A detailed overview of PH systems can be found in (van der Shaft, 2004).

To represent the tank processes as PH model, the following storage function is selected, which represents the total mass (volume) in the system

$$H(x) = x_1 + x_2 + x_3 \geq 0\,. \tag{6}$$

The IDA-PBC methodology allows to find a static control feedback $u = \beta(x)$ such that the desired performance is specified by a closed loop dynamic as

$$\dot{x} = \left[J_d(x) - \Re_d(x) \right] \frac{\partial H_d(x)}{\partial x}\,, \tag{7}$$

where $H_d(x)$ is the desired total mass function fixed by the designer and which has a strict minimum in x^*. The matrices $J_d(x) = -J_d^T(x)$ and $\Re_d(x) = \Re_d^T(x) \geq 0$ are the desired interconnection and damping matrices respectively. In order to get decoupled outputs, the closed loop port Hamiltonian system has to have a null interconnection matrix and a diagonal damping matrix. For accomplishing this objective, it is possible to define the open-loop PH system such that it satisfies these characteristics. The PH matrixes are

$$J(x) = -J^T(x) = 0\,, \tag{8}$$

$$\Re(x) = \Re^T(x) = diag\{b_1, b_2, b_3\}\,, \tag{9}$$

$$g(x) = \begin{bmatrix} -sign(x_1 - x_2)\cdot a_1 & 0 & 0 \\ sign(x_1 - x_2)\cdot a_1 & -sign(x_2 - x_3)\cdot a_2 & 0 \\ 0 & sign(x_2 - x_3)\cdot a_2 & -a_3 \end{bmatrix}, \tag{10}$$

$$q(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T. \tag{11}$$

The closed loop interconnection and damping matrices have to be equal to the open loop matrices; i.e. $J_d(x) = J(x)$ and $\Re_d(x) = \Re(x)$. If this is satisfied, the closed-loop process has decoupled outputs. The references levels are constant, so the following desired storage function (Ramírez *et al,* 2008) can be used,

$$\begin{aligned} H_d(x) &= \sum_{i=0}^{n} x_i + \sum_{i=0}^{n} -(1-k_i)x_i - k_i x_i^* \ln(x_i) \\ &= \sum_{i=0}^{n} \left[x_i - k_i x_i^* \ln(x_i) \right]. \end{aligned} \tag{12}$$

For the given desired storage function and for previously defined open and closed loop PH matrices, by matching (5) and (7) the following control law, where $\hat{f}_1$ is an estimation of f_1, is obtained

$$u_{11} = \frac{f_o - b_1 + b_1 \cdot k_1 \left(1 - \dfrac{x_1^*}{x_1}\right)}{sign(x_1 - x_2)\cdot a_1}\,, \tag{13}$$

$$u_{21} = \frac{\hat{f}_1 - b_2 + b_2 k_2 \left(1 - \dfrac{x_2^*}{x_2}\right) + u_{11}\cdot a_1 \cdot sign(x_1 - x_2)}{sign(x_2 - x_3)\cdot a_2}\,, \tag{14}$$

$$u_{31} = \frac{-b_3 + b_3 k_3 \left(1 - \dfrac{x_3^*}{x_3}\right) + u_{21}\cdot a_2 \cdot sign(x_2 - x_3)}{a_3}\,, \tag{15}$$

If the control inputs (13), (14) and (15) are replaced in (5), then the time derivate of the desired storage function, will be negative, thereby the closed-loop system is asymptotically stable.

Equations (13) and (14) require to know the system parameters and flow rates f_o and f_1. In order to compensate the lack of knowledge about the values of these flow rates, an integral action is considered in the final control law. The solution used in this paper was presented in (Ortega and García-Canseco, 2004) and consists in adding an integral term of the passive output to the control.

Let's consider the system presented in (5) in closed loop with $u = \beta(x) + \upsilon$, where υ is an integral action added to the system trough a state variable and defined as

$$\dot{\upsilon} = -K_I g^T(x) \nabla H_d(x) \tag{16}$$

With $K_I = K_I^T > 0$. Then, all stability properties of x^* are preserved. In fact the closed loop clearly takes the PCH form

$$\begin{bmatrix} \dot{x} \\ \dot{\upsilon} \end{bmatrix} = \begin{bmatrix} J_d(x) - \Re_d(x) & g(x)K_I \\ -K_I g^T(x) & 0 \end{bmatrix} \begin{bmatrix} \nabla_x W \\ \nabla_\upsilon W \end{bmatrix}, \tag{33}$$

where

$$W(x,\upsilon) = H_d(x) + \frac{1}{2}\upsilon^T K_I^{-1} \upsilon \tag{17}$$

is the new total storage function which now qualifies as Lyapunov function. For this application it is convenient to use a diagonal gain matrix, i.e. $K_I = diag\{k_{I1}, k_{I2}, k_{I3}\}$, $g(x)$ like in (10) and $H_d(x)$ like in (12).

4. SINGULAR POINT REGULARIZATION

The control action (13), (14) and (15), have singulars points arising when two tanks have the same level. In this case, the flow between these tanks becomes null and the control action becomes inexistent. Operating the system on this singulars point is not required in this application. However, if the controller attempts to invert the flow between two contiguous tanks, it is necessary to pass from a state $x_1 > x_2$ to a state

$x_1 < x_2$. Along the trajectory is necessary to pass trough $x_1 = x_2$, which make the control law unfeasible. This only happens between the first and second tank, hence, the solution is only used in the first control input.

Based on the work of Haojian and Ioannou (2004), a singular point solution is proposed. Assume a function $\eta(x) \in \mathbb{R}$ such that $\eta(0) = 0$. The inverse of $\eta(x)$ is undetermined at zero. To avoid this, the following solution is proposed:

$$\frac{1}{\eta(x)} \Rightarrow \frac{\eta(x)}{\eta^2(x) + \delta(x)}, \tag{18}$$

where $\delta(x)$ is defined as follows:

$$\delta(x) = c_1 \cdot \left(1 - \frac{1}{1 + e^{-c_2 \cdot |x_1 - x_2|}}\right) \cdot \left| e_{ctrl}(x, x^*) \right| \cdot k_{sp}, \tag{19}$$

where k_{sp}, c_1 and c_2 are real positive constants and they are considered as tuning parameters. Equations (19) means that $\delta(x)$ will be zero if x is at the reference x^*. Of course, the references x_1^* and x_2^* must be different, otherwise (18) will be unbounded.

The control input (13) including the singular point solution becomes:

$$u_{11} = \frac{sign(x_1 - x_2) \cdot a_1 \cdot \left(f_o - b_1 + b_1 \cdot k_1 \left(1 - \frac{x_1^*}{x_1} \right) \right)}{a_1^2 + \delta(x)} \; . \tag{20}$$

The control variables u_{21} and u_{31} do not require changes since they do not have singular points.

4.1 Stability Analysis

In this section a brief and simple stability analysis is carried out. Replacing (20), (14) and (15) in (1), (2) and (3), respectively, the closed loop system takes the form:

$$\dot{x}_1 = -\frac{a_1^2}{a_1^2 + \delta(x)} b_1 k_1 \left(1 - \frac{x_1^*}{x_1}\right) + \left(f_o - b_1\right) \frac{\delta_1(x)}{a_1^2 + \delta(x)} \tag{21}$$

$$\dot{x}_2 = -b_2 k_2 \left(1 - \frac{x_2^*}{x_2}\right) + \Delta f_1 \tag{22}$$

$$\dot{x}_3 = -b_3 k_3 \left(1 - \frac{x_3^*}{x_3}\right) \tag{23}$$

where $\Delta f_1 = f_1 - \hat{f}_1$ is the estimation error of the unknown disturbance in the second tank.

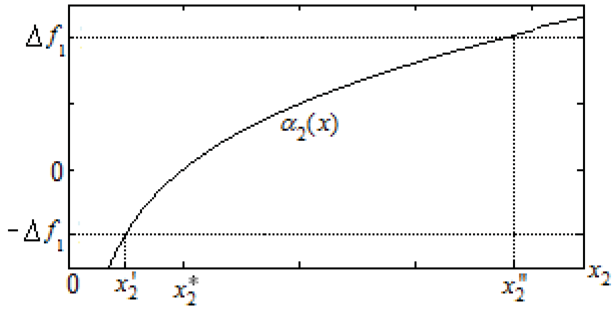

Fig. 2. Right hand terms of equation (22)

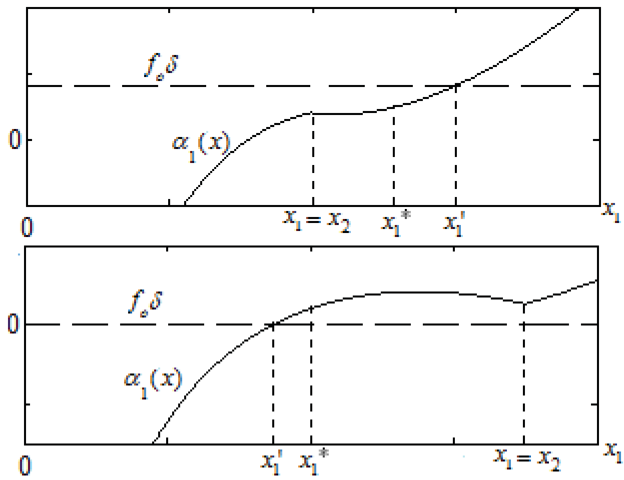

Fig. 3. Right hand terms of equation (21) for a constant δ

Then stability of the closed-loop system can be analyzed as follows: Since equation (23) only depends on x_3 and the term $b_3 k_3$ is positive, x_3^* will be an asymptotically stable equilibrium point. From equation (22) we have that the dynamic only depends on x_2. The stability analysis can be carried out by analyzing the right hand terms, as they are depicted in Fig. 2., where Δf_1 and the term $\alpha_2(x) = b_2 k_2 (1 - x_2^*/x_2)$ have been drawn in terms of x_2. From this plot can be seen that the system will converge to a unique equilibrium point x_2'', so that $\|x_2'' - x_2^*\| < \gamma$, where $\gamma(\Delta f_1) > 0$ is a real positive constant that depends on Δf_1. If $\Delta f_1 = 0$, then x_2^* will be asymptotically stable equilibrium point, as x_3^*. The analysis for x_1 consider the perturbation term $(f_o - b_1)\delta/(a_1^2 + \delta(x))$, which vanishes at the equilibrium, hence x_1 could converge asymptotically to x_1^*. In fact, the expression for the equilibrium point is:

$$\left(f_o - b_1\right) \delta(x) - a_1^2 b_1 k_1 \left(1 - \frac{x_1^*}{x_1}\right) = 0 \,. \tag{24}$$

Equation (24) can be verified at the desired operation point x_1^* or if $(f_o - b_1) = 0$ and $a_1 = 0$. The last case can only be possible if the system is at the singular point; i.e. if $x_1^* = x_2^*$ and $f_o = b_1 = (2gx_1)^{1/2}$. Fig. 3 depicts the right hand terms of (21); $\alpha_1(x)$ defined as:

$$\alpha_1(x) = b_1 \delta(x) + a_1^2 b_1 k_1 \left(1 - \frac{x_1^*}{x_1}\right), \tag{25}$$

and for both condition $(f_o - b_1) > 0$ and $(f_o - b_1) < 0$. From this figure can be seen that there exist only one asymptotically stable equilibrium point x_1^*, which is not the desired reference value, the steady state error will depend on δ and f_o. Fig. 5, shows the effect of making δ dependant on the variable x, as in (20). In this case, the desired reference value is an asymptotically stable equilibrium point.

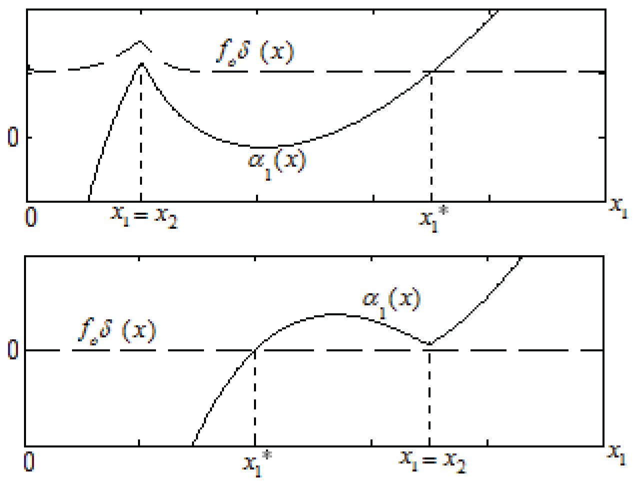

Fig. 4. Right hand terms of equation (21) for a variable δ (x)

5. NUMERICAL SIMULATIONS

In this section, some simulation results illustrating the controller characteristics are presented. The tuning parameters were selected to obtain a closed loop response with overshoots smaller than 15% and small settling times according with the open loop dynamics.

This simulation considers the following: linear control valves, i.e. $k_{ij}(x) = u_{ij}$ and the cross section of all tanks are the same and constant, i.e. $A_1 = A_2 = A_3 = 1731.3$. The tuning parameters for the singular points solution where selected as $C_1 = 500$ y $C_2 = 0.0004$. The feed flows rates have their maximum value at 4000 cm^3/s and are represented in percentage values.

If the flow direction, between the first and second tank, is inverted, then the system go through a singular point. The following simulations show the performance of the system with a flow rate inversion. The parameters were $k_1=4$, $k_2=2$, $k_3=1$ and the integrator parameter were set at $k_{I1}=0.0001$, $k_{I2}=0.01$, $k_{I3}=0.01$.

In the first part, the system is working with a level of 10cm in the first tank and 5 cm in the second one, and the flow between the tanks goes from the first to the second. Fig.5 depicts the closed loop behavior. At 4250 the both set points were increased at the same time. to 35cm and 25cm respectively. At 5000 sec. the level reference of the first tank is set to 15cm and for the second one is kept at 25cm, leading to an inversion of the flow direction. From Fig. 5 can be seen that while the level in the first tank changes, the control tries to maintain the level in the second tank constant, and it invert the flow direction without discontinuities in the control inputs. The coupling between the outputs is due to the integrator compensation, since the static feedback was designed considering a null interconnection matrix. These good results are achieved due to the joint action of the integral action and the methodology used to deal with the singular point.

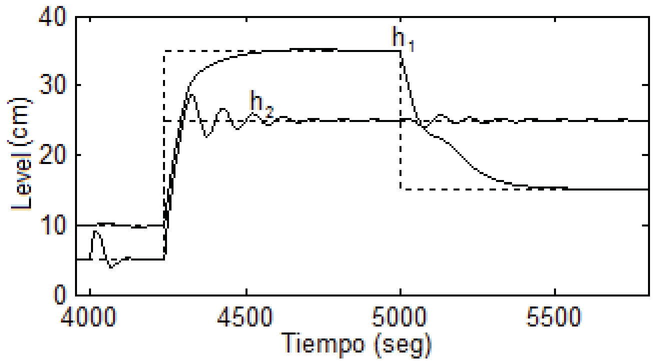

Fig. 5. Levels in first and second tank. Integral action and flow rate inversion

From Fig. 6, can be seen that the control input, in both tanks (1 and 2), are smooth, continuous and bounded. Beside, Fig. 7 shows the flow rate inversion (5070 seconds approx.), and the sudden changes of the flow rate when the control inputs changes their values due to references changes.

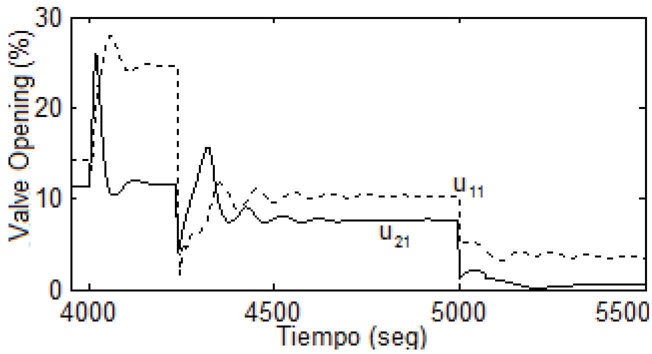

Fig. 6. Control input of the first and second tank. Integral action and flow rate inversion

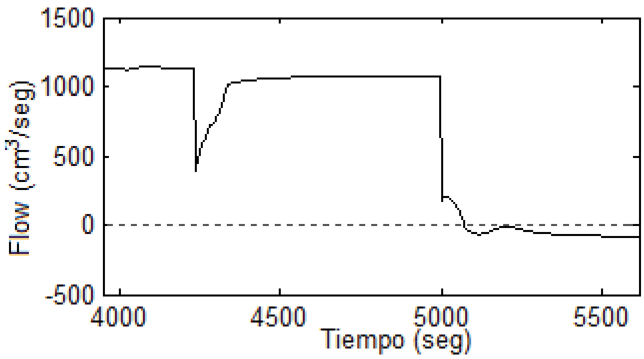

Fig. 7. Flow rate between the first and second tank. Integral action and flow rate inversion

A simple PI controller can not deal with reversing flows, since the process open-loop gain changes sign when the flow changes direction.

6. CONCLUDING REMARKS

This paper presents a novel nonlinear control strategy based on IDA-PBC for a non-linear process with bidirectional flow. The process was modeled as PH model, and by a proper selection of the process closed-loop matrices. A passivity based strategy was designed, and in order to deal with model uncertainties and unknown disturbances, integral action was also considered in the control law. Since the process exhibits uncontrollable operation conditions; i.e. singular points in the control law, a singular point solution was proposed without compromising the stability conditions of the closed-loop process. A nice feature of the proposed singular point solution is that outside the set of singular points the desired closed-loop interconnection and damping specifications are preserved, hence no special considerations must be taken into account when selecting the desired closed-loop PH system in the IDA-PBC design. The closed-loop behavior of the proposed controller has been illustrated by numerical simulations.

Future works will consider a more detailed stability analysis for the general case, including integral actions. Implementation of the controller in a laboratory application is also part of the future work to be carried out.

ACKNOWLEDGEMENTS

This work was supported by Fondecyt Project 1070491

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