PWA Modelling and Co-ordinated Continuous and Logical Control of a Laboratory Scale Plant with Hybrid Dynamics

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Abstract: Many process plants are nonlinear and together with this they include a combination of continuous valued and logical control inputs and subsystems. This paper attempts to explore the potential of hybrid model predictive control (MPC) to cope with both of these problems. It uses a laboratory scale plant that was designed for experiments with hybrid systems. This plant has both continuous and logical control inputs and it is considerably nonlinear. An approximate hybrid model of the plant in the form of a piecewise affine (PWA) system is developed and evaluated in the first part of the paper. After that a hybrid MPC based on PWA model is applied to the control of the plant. While designing hybrid MPC and evaluating its performance, there is a special focus on the following question. Logical and continuous control systems are usually designed separately. This may result in unforeseen interactions between logical and continuous control and in the deterioration of the control performance. However, hybrid MPC is based on hybrid model that captures both logical and continuous dynamics in one unified framework. Hence it can reasonably be expected that hybrid MPC can avoid undesirable interactions and possibly also make use of these interactions in a positive way (e.g. to speed up the control response using logical inputs). Control results obtained with hybrid MPC are indeed fairly good and they show clear improvement over the results achieved with separate design of logical and continuous control.

Keywords: Hybrid systems, model predictive control, piecewise affine systems

1. INTRODUCTION

Model predictive control (MPC) of hybrid systems has recently attracted a considerable research attention. This attention is reflected in the growing number of publications on hybrid MPC. Monographs such as (Christophersen, 2007), (Borrelli, 2003) and survey paper (Morari & Baric, 2006) can be quoted as important examples representing a vast and constantly growing body of literature. The application area of hybrid MPC is twofold. First, many process plants comprise continuous-valued as well as logical/discrete-valued control inputs and components. Such plants are naturally modelled as hybrid systems and this requires the use of control approaches for hybrid systems. Second, non-linearities that are ubiquitous in the models of process plants can often be well approximated by a special class of hybrid systems called piecewise affine (PWA) systems. The result is again a plant model in the form of a hybrid system.

This paper is focused on both of the above mentioned aspects of the hybrid MPC. It uses a case study of a laboratory scale plant. This plant exhibits hybrid phenomena that are found in many process control applications. The plant includes both continuous valued and logical control inputs and its dynamic behaviour abruptly changes at certain operating points. Most continuous components of the plant are nonlinear. This nonlinear behaviour must be approximated by a PWA model. This approximation is necessary for the design of hybrid MPC controller and it is a non-trivial task. PWA approximation, selection of individual affine models, their validity regions and comparison with the responses of the original nonlinear plant are described in detail. Finally a hybrid model is obtained whose hybrid features are both due to the hybrid nature of the plant itself and due to the PWA approximation of plant nonlinearities. Further, the attention is turned to MPC control of this plant. A special emphasis is laid on the ability of hybrid MPC to achieve an integrated design of logical and continuous control.

Typically, logical control is responsible for safety related and limiting functions such as preventing the process variables from leaving safe operation limits, starting and shutdown of process equipment. Logical controllers are also used to manipulate logical control inputs such as on/off valves. On the other hand, the regulatory and supervisory control is performed by continuous controllers. Common design practice relies on separate design of logic and continuous control. As non-trivial and not easily predictable interactions often arise between continuous and logical parts of the control system, this practice may result in a poor control performance. On the other hand, hybrid model describes both continuous and logical (or more generally discrete-valued) parts of the whole system, including continuos/logical interactions, the hybrid controller designed on the basis of this model can be expected to control the whole plant in a co-ordinated manner and avoid the deteriorating effects of interactions between separately designed logical and continuous control systems. However, it is well known that expectations though well founded in the theory and practical reality may be two different worlds. For this reason, this

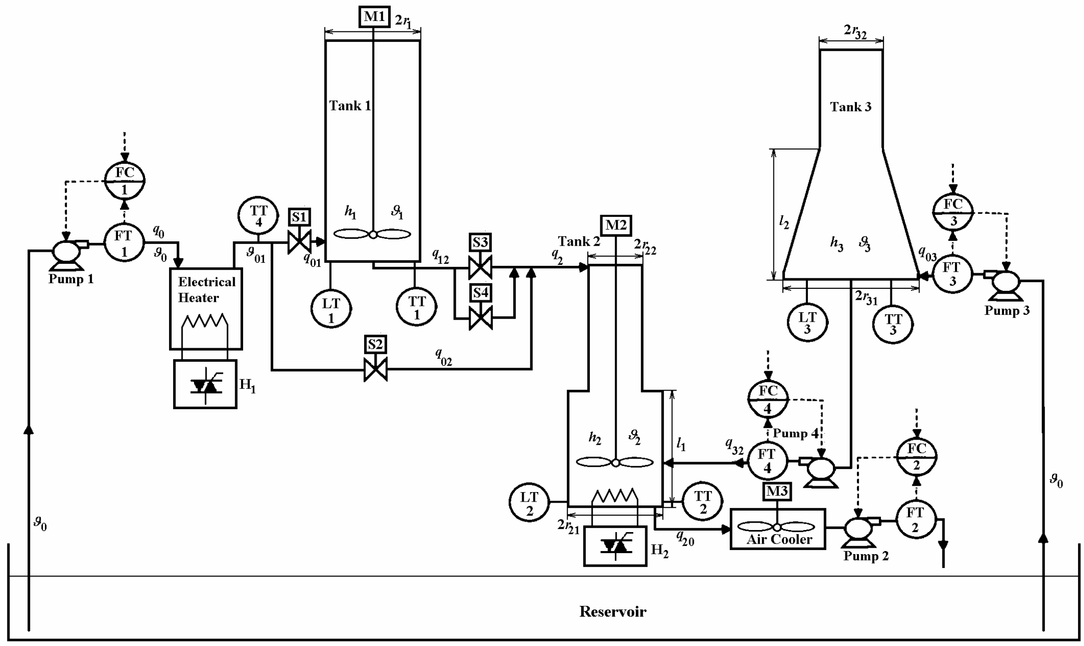

Fig. 1. Structure of the laboratory scale plant, FT, LT, TT are flow, level and temperature transmitters FC – flow controller, S– solenoid valve, M – motor, r_1= 5.64 cm, r_{21}= 5.8 cm, r_{22}= 3 cm, r_{31}= 6 cm, r_{32}= 2.9 cm, tank height l_{max}=80 cm, l_1= l_2=40 cm

paper in its final part attempts to make an experimental comparison of a separate design of logic and continuous control on the one hand and co-ordinated design based on hybrid model on the other hand.

2. EXPERIMENTAL PLANT

A detailed description of the experimental plant has recently been given by the author in (Hlava & Šulc, 2008). As the full text of this paper is available from the IFAC-PapersOnLine website, the description of this plant in the present paper can be short. Plant structure is shown in Fig. 1. Basic components are three water tanks. Tanks 2 and 3 have special shapes that introduce changes in dynamics. The tanks are thermally insulated to make the heat losses negligible. Water from the reservoir mounted under the plant is drawn by Pump 1 and Pump 3 to the respective tanks. The delivery rates can be continuously changed. The flow rates are measured using turbine flow-meters. To compensate for pump non-linearity, it is beneficial to use slave flow rate controllers.

The flow from Pump 3 is fed directly to Tank 3. The flow from Pump 1 goes through heater and it is further controlled by solenoid valve S1. The power output of the heater can be changed continuously and ϑ_{01} can be made to follow a specified function of time. Another continuously controlled heater is mounted on the bottom of Tank 2. The temperatures are measured with Pt1000 sensors at the points shown in Fig. 1. In addition to the pumps, whose delivery flow rates can be changed continuously, the plant includes another way of manipulating the flow: solenoid valves. These discrete valued actuators control the flow from Tank 1 to Tank 2 (valves S3, S4). The flow is changed in three steps: no valve open, one open, both valves open. Tank 1 can be bypassed by closing S1 and opening S2. The air-water heat exchanger with cooling fan at the output from Tank 2 keeps the water temperature in the reservoir roughly constant. Water levels are measured using pressure sensors.

The plant is controlled from a PC using two data acquisition boards (11 analog inputs, 6 analog outputs, 6 digital outputs) and interface hardware (power amplifiers, solid state relays, signal conditioning devices). The basic software tool for identification and control experiments is the Real Time Toolbox. It allows an easy connection of Matlab/Simulink environment with the real world. Alternatively WinCon-8000 industrial control system produced by ICP DAS can be used. This control system makes it possible to experiment with the implementation of advanced control algorithms using real industrial hardware. The changeover from PC to WinCon-8000 and vice versa is simple: two connectors with analog and digital inputs/outputs have to be reconnected.

3. CONTROL OBJECTIVE

Many control scenarios can be defined with this plant. Some examples are given in (Hlava & Šulc, 2008). The scenario considered in this paper uses Tank 1 and 2. This scenario is inspired in part by (Slupphaug *et al.*, 1997) and it can be formulated as follows. Tank 1 serves as a buffer that receives water from an upstream process. Water flow rate and temperature are disturbances. The main control objective is to deliver the water to a downstream process at a desired temperature (temperature ϑ_2), while the flow demand of the downstream process is variable and hence it also acts as a disturbance. Power output of heater H_2 is a continuous manipulated variable. Valves S3 and S4 are used as a discrete valued manipulated variable to control the flow from Tank 1 to Tank 2 in three steps. Valve S1 is used to close water flow to Tank 1, if tank overflow is to be avoided. There is no valve at the output of Tank 2, but an effect equivalent to closing an output valve is achieved by switching off Pump 2.

The main control objective necessarily includes several auxiliary objectives. Tank levels must be kept within specified limits, and overflow as well as emptying of the tanks must be avoided. It is also necessary to avoid the necessity to close valve S1 in order to prevent Tank 1 overflow. In a real control situation, closing S1 would mean that water from the upstream process cannot flow to the buffer but must be re-routed to the environment. Similarly it is necessary to avoid the necessity to switch off Pump 2 in order to prevent Tank 2 from underflow. The standard way to satisfy these auxiliary objectives would be to use separately designed control logic.

4. MATHEMATICAL MODEL OF THE PLANT

Plant model is derived using mass and energy balances. The reader is referred to (Hlava & Šulc, 2008) for details. In this paper just the part of the plant model will be given that is relevant to the specified control objective (i.e. excluding Tank 3). Assuming liquid incompressibility and constant heat capacity c, negligible heat losses and ideal mixing, the following model is obtained

$$\dot{h}_1(t) = (1/A_1)\left(q_0(t)\sigma_0(t) - 0.1k_v\sigma_1(t)\sqrt{gh_1(t)}\right) \tag{1}$$

$$\dot{h}_2(t) = \begin{cases} (1/A_{21})\left(0.1k_v\sigma_1(t)\sqrt{gh_1(t)} - q_{20}(t)\sigma_2(t)\right) & h_2(t) \le l_1 \\ (1/A_{22})\left(0.1k_v\sigma_1(t)\sqrt{gh_1(t)} - q_{20}(t)\sigma_2(t)\right) & h_2(t) > l_1 \end{cases} \tag{2}$$

$$\dot{\vartheta}_1(t) = q_0(t)\sigma_0(t)\left(\vartheta_{01}(t) - \vartheta_1(t)\right)/A_1h_1(t) \tag{3}$$

$$\dot{\vartheta}_2(t) = \begin{cases} \dfrac{\left(0.1k_v\sigma_1(t)\sqrt{gh_1(t)}\left(\vartheta_1(t) - \vartheta_2(t)\right) + \dfrac{H(t)}{\rho c}\right)}{A_{21}h_2(t)} & h_2(t) \le l_1 \\ \dfrac{\left(0.1k_v\sigma_1(t)\sqrt{gh_1(t)}\left(\vartheta_1(t) - \vartheta_2(t)\right) + \dfrac{H(t)}{\rho c}\right)}{A_{21}l_1 + A_{22}(h_2(t) - l_1)} & h_2(t) > l_1 \end{cases} \tag{4}$$

where $A_i = \pi r_i^2$, discrete valued input σ_0 assumes values 0,1 (S1 closed, S1 open), σ_1 assumes values 0,1,2 (no valve open, S3 open, S3 and S4 open), σ_2 assumes values 0,1 (Pump 2 off, Pump 2 running with flow rate q_{20} depending on the flow demand of the downstream process), H is power output of heater H_2, k_v is flow coefficient of valves S3 and S4.

5. APPROXIMATE PLANT MODEL IN A PWA FORM

Plant model (1)-(4) includes continuous and discrete valued inputs, dynamics switching depending on operating point in (2), (4) and non-linear elements. It must first be approximated by a PWA model. The general form of a discrete-time PWA system is given by

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{M}_i\boldsymbol{x}(k) + \boldsymbol{N}_i\boldsymbol{u}(k) + \boldsymbol{f}_i \\ \boldsymbol{y}(k) &= \boldsymbol{C}_i\boldsymbol{x}(k) + \boldsymbol{D}_i\boldsymbol{u}(k) + \boldsymbol{g}_i \end{aligned} \tag{5}$$

where each dynamics i=1,2..N_D is active in a polyhedral partition $Đ$ that is defined by guard lines described by

$$\boldsymbol{G}_i^x\boldsymbol{x}(k) + \boldsymbol{G}_i^u\boldsymbol{u}(k) \le \boldsymbol{G}_i^c \tag{6}$$

That means, the dynamics i represented by matrices and vectors [$\boldsymbol{M}_i$, $\boldsymbol{N}_i$, $\boldsymbol{f}_i$, $\boldsymbol{C}_i$, $\boldsymbol{D}_i$, $\boldsymbol{g}_i$] is active in the region of state-input space which satisfies constraints (6). Unlike some other approaches to hybrid MPC that use probabilistic Bayesian approach to combine weighted local linearized models (e.g. Nandola & Bhartiya, 2008), the approach considered in this paper is deterministic and local models are just switched depending on the region in state-input space.

It has been noted already in the well known seminal paper on PWA systems (Sonntag, 1981) that nonlinear systems can be globally approximated arbitrarily close by PWA systems and this claim has often been repeated. This claim is certainly true. However, it is also true that although there are several methods of experimental identification of PWA models (see Paoletti *et al.*, 2007 for an overview), there is no general systematic procedure to find a PWA approximation of a given non-linear system described by analytical state equations. The route to the PWA approximation is always closely connected with a particular system to be approximated. In this section, model described by (1)- (4) will be considered. Its approximation by a set of affine models of the form (5), (6) can proceed as follows.

An obvious source of partial models is logical control inputs σ_0, σ_1, σ_2. The best way to handle these inputs is to associate one partial model with each combination of their values. This results in 12 partial models. Equations (2) and (4) include dynamics switching at water level l_1. That means the number of 12 must be doubled and 24 partial models are obtained as an absolute minimum for modeling this plant. These 24 partial models must further be linearized. To achieve an acceptable precision, each model is approximated by a set of linearized models. The linearization is done in two steps.

1. Obtain linearizations around general operating points characterized by a vector of input and state variables (σ_{0P}, σ_{1P}, σ_{2P}, q_{0P}, q_{20P}, ϑ_{0P}, H_p, h_{1P}, h_{2P}, ϑ_{1P}, ϑ_{2P}). If possible, steady state operating points should be preferred.
2. Find a suitable set of operating points together with adequate partitioning of state-input space that will be well representative of the dynamics of the original system.

This procedure can be most simply illustrated for (1). If S1 is open (σ_{0P}=1) the respective steady state characteristics is

$$q_{0P} = \frac{1}{\sigma_{0P}}0.1k_v\sigma_{1P}\sqrt{gh_{1P}}\ , \tag{7}$$

and (1) can be linearized around a steady state operating point (h_{1P}, q_{0P}, σ_{0P}, σ_{1P}). This linearization is given by

$$\dot{h}_1(t) = \frac{\sigma_{0P}}{A_1}(q_0(t) - q_{0P}) - \frac{0.1k_v\sigma_{1P}}{2A_1}\sqrt{\frac{g}{h_{1P}}}\,(h_1(t) - h_{1P}) \tag{8}$$

Substituting for q_{0P} from (7), linearization can be modified to

$$\dot{h}_1(t) = \frac{\sigma_{0P}}{A_1}q_0(t) - \frac{0.1k_v\sigma_{1P}\sqrt{g}}{2A_1\sqrt{h_{1P}}}h_1(t) - \frac{0.1k_v\sigma_{1P}\sqrt{gh_{1P}}}{2A_1} \tag{9}$$

It can be seen that (9) holds even if σ_{1P}=0. What is less obvious is the case of σ_{0P}=0. Equation (7) cannot be used and (1) is autonomous system that has just zero steady state. However, it can be linearized around a non-steady state

$$\dot{h}_{1P}(t) = -0.1 k_v \sigma_{1P} \sqrt{g h_{1P}} \Big/ A_1 \tag{10}$$

and this linearization has the same form as (9). Thus (9) is general linearized approximation of (1). As actual values of the variables and not deviations from operating point are used, (9) is affine and not linear.

The next step is to find a suitable set of representative operating points. The simplest approach would be to divide the whole range of h_1 into several intervals of identical length and to take midpoints of these intervals as selected nominal operating points. A better way is to modify (9) to the form

$$\frac{2A_1\sqrt{h_{1P}}}{0.1k_v\sigma_{1P}\sqrt{g}}\dot{h}_1(t) + h_1(t) = \frac{2\sigma_{0P}\sqrt{h_{1P}}}{0.1k_v\sigma_{1P}\sqrt{g}} q_0(t) - h_{1P} \tag{11}$$

Dynamics of (11) can be characterised by time constant

$$\tau = 2A_1\sqrt{h_{1P}} \Big/ \left(0.1k_v\sigma_{1P}\sqrt{g}\right) \tag{12}$$

and the partitioning of the whole range of h_1 can be done in such a way as to keep the ratio of maximum and minimum time constant within each interval the same. The nominal operating point in each interval is then again selected so that the nominal time constant would have the same ratio to the minimum and maximum value of time constant within this interval. If the whole range is divided into four intervals, this reasoning leads to the following

$$\begin{aligned} &h_{1m1} = h_{1min}^{\frac{3}{4}} h_{1max}^{\frac{1}{4}}; \quad h_{1m2} = h_{1min}^{\frac{2}{4}} h_{1max}^{\frac{2}{4}}; \quad h_{1m3} = h_{1min}^{\frac{1}{4}} h_{1max}^{\frac{3}{4}} \\ &h_{1P1} = \sqrt{h_{1min} h_{1m1}}; \quad h_{1P2} = \sqrt{h_{1m1} h_{1m2}}; \quad h_{1P3} = \sqrt{h_{1m2} h_{1m3}} \\ &h_{1P4} = \sqrt{h_{1m3} h_{1max}} \end{aligned} \tag{13}$$

where $h_{1\min}$ and $h_{1\max}$ are the minimum and maximum values of h_1 respectively, h_{1m1}, h_{1m2}, h_{1m3} are limiting points of subintervals, h_{1P1}, h_{1P2}, h_{1P3}, h_{1P4} are nominal operating points.

Other parts in plant model can be approximated by PWA systems in a similar way. The dynamics of water level in Tank 2 as described by (2) depends mainly on h_1, while h_2 just governs switching between two partial models. Linearization of (1) as described by (9) actually means that square root was replaced by the linear part of Taylor series. Using the same method for (2) results in

$$\dot{h}_2(t) = \left\langle \begin{array}{ll} \dfrac{1}{A_{21}}\left(\dfrac{0.1k_v\sigma_{1P}\sqrt{g}}{2\sqrt{h_{1p}}} h_1(t) + \dfrac{0.1k_v\sigma_{1P}\sqrt{gh_{1p}}}{2} - q_{20}(t)\sigma_{2P} \right) & h_2(t) \le l_1 \\ \dfrac{1}{A_{22}}\left(\dfrac{0.1k_v\sigma_{1P}\sqrt{g}}{2\sqrt{h_{1p}}} h_1(t) + \dfrac{0.1k_v\sigma_{1P}\sqrt{gh_{1p}}}{2} - q_{20}(t)\sigma_{2P} \right) & h_2(t) \le l_1 \end{array} \right. \tag{14}$$

Steady state characteristics of (3) are unity gain for any nonzero h_1. Its linearization is therefore also very simple as all terms that include the difference ϑ_{0P}-ϑ_{1P} equal zero.

$$\dot{\vartheta}_1(t) = \left(q_{0P} / A_1 h_{1P}\right)\left(\vartheta_{01}(t) - \vartheta_1(t)\right) \tag{15}$$

Using (7) this equation can be modified to

$$\dot{\vartheta}_1(t) = \left(0.1k_v\sigma_{1P}/A_1\right)\sqrt{\left(g/h_{1P}\right)}\left(\vartheta_{01}(t) - \vartheta_1(t)\right) \tag{16}$$

This equation has time constant

$$\tau = A_1\sqrt{h_{1P}} \Big/ \left(0.1k_v\sigma_{1P}\sqrt{g}\right) \tag{17}$$

Apart from multiplicative constant, this is the same expression as (12). Thus, the same partitioning of the range h_1 is obtained.

Equation (4) poses a more difficult problem. However, the effect of power output of the heater $H(t)$ on the dynamics of ϑ_2 is linear and the equation can be linearized around a point where $H(t)$=0. Then the steady state relation between ϑ_2 and ϑ_1 is again unity gain and the linearized equation is

$$\dot{\vartheta}_2(t) = \left\langle \begin{array}{ll} \dfrac{\left(0.1k_v\sigma_{1P}\sqrt{gh_{1P}}\left(\vartheta_1(t) - \vartheta_2(t)\right) + \dfrac{H(t)}{\rho c}\right)}{A_{21}h_{2P}} & h_2(t) \le l_1 \\ \dfrac{\left(0.1k_v\sigma_{1P}\sqrt{gh_{1P}}\left(\vartheta_1(t) - \vartheta_2(t)\right) + \dfrac{H(t)}{\rho c}\right)}{A_{21}l_1 + A_{22}(h_{2P} - l_1)} & h_2(t) > l_1 \end{array} \right. \tag{18}$$

Selection of nominal operating points is partly given by dynamics switch at level l_1. To achieve good approximation subranges $<h_{2\min}, l_1>$ and $(l_1, h_{2\max}>$ are further partitioned in a similar way as it was done with Tank 1. In this paper, the partitioning of both subranges into four intervals is used. Total number of partial models is then 384 (=2*3*2*4*8). The result is a continuous time PWA model with four states and seven inputs

$$\begin{aligned} &\dot{\mathbf{x}}(t) = \mathbf{A_i}\mathbf{x}(t) + \mathbf{B_i}\mathbf{u}(t) + \mathbf{o_i} \quad i = 1,\ldots,384 \\ &\mathbf{x}(t) = \left[h_1(t)\ h_2(t)\ \vartheta_1(t)\ \vartheta_2(t)\right]^T \\ &\mathbf{u}(t) = \left[\sigma_0(t)\ \sigma_1(t)\ \sigma_2(t)\ H(t)\ q_0(t)\ \vartheta_{01}(t)\ q_{20}(t)\right]^T \end{aligned} \tag{19}$$

Most elements of matrices in (19) are zeros. The expressions for the few nonzero elements can be written according to (9)-(18). Partial models are valid in regions that are defined by specified values of logical inputs σ_0, σ_1, σ_2 and minimum and maximum limits on state variables. These specifications are formulated in the form of guard lines (6). Due to the high dimensions and great number of variants the expressions for matrices $\boldsymbol{G}_i^x, \boldsymbol{G}_i^u, \boldsymbol{G}_i^c$ cannot be given here.

System (19) must finally be converted to discrete time. This is done by discretizing each partial model separately assuming zero order hold at the inputs. Control Systems Toolbox for Matlab has no routine for discretization of affine systems. However it can be easily derived that the formulae for $\boldsymbol{N}_i$ and $\boldsymbol{f}_i$ in (5) have the same form

$$\boldsymbol{N}_i = e^{A_i T_v}\int_0^{T_v} e^{-A_i\tau}\boldsymbol{B}_i d\tau; \; \boldsymbol{f}_i = e^{A_i T_v}\int_0^{T_v} e^{-A_i\tau}\boldsymbol{o}_i d\tau \tag{20}$$

Hence the computation of discretized model is possible by using c2d command first with arguments ($\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $\boldsymbol{C}_i$, $\boldsymbol{D}_i$) to obtain $\boldsymbol{N}_i$ and then with arguments ($\boldsymbol{A}_i$, $\boldsymbol{o}_i$, $\boldsymbol{C}_i$, 0) to obtain $\boldsymbol{f}_i$.

The comparison of responses of the original model (1)–(4) and its PWA approximation is shown in Fig. 2. To evaluate the PWA model in a wide range of changes, the following step changes are used: q_0 changes at t=1000 *s* from 1 to 1.2 l/min, σ_1 changes at t=2000 s from 1 to 0 and at t= 2100 s back to 1, q_{20} changes at t=2400 s from 1 to 1.4 l/min and at t=3000 s to 1.2 l/min, ϑ_0 changes at t=3000 s from 40 to 50°C, H changes at t=4500 s from 0 to 500 W. Initial conditions are: h_{10}=0.6 m, h_{20}=0.12 m, ϑ_{10}=30°C, ϑ_{20}=55°C, sampling period T_s=1 s. The responses of PWA system were simulated using the Simulink block included in Multi-Parametric (MPT) Toolbox (Kvasnica *et al.*, 2004). There is generally a good agreement between the responses of the original system and its PWA approximation. Any comparison using a specified set of input signals has naturally a limited value because the agreement depends also on how close is the actual response to the selected set of representative operating points. However, it can be said that in most cases the maximum peak error does not exceed 1 cm or 1°C and normally the difference is in the range of tenths of cm and °C most of the time. It is also possible to decrease the number of partial models. Figure 2 was obtained with 384 partial models. If the range of h_2 is divided in just four sub-ranges, the number of models is reduced to 192 and the precision of PWA approximation remains good. However, any further reduction of the number of partial models results in a marked decrease of approximation precision.

6. CONTROL DESIGN AND EXPERIMENTS

Standard procedure to design a control system satisfying the objectives specified above is divided into two separate tasks: design of logical control and design of continuous control (in this paper the term continuous relates to control where the variables are continuous-valued regardless of whether the controller is designed in continuous or discrete time setting). Logic part of the control system is described by a set of simple rules. Normal and desirable state of the logical control inputs is σ_0=1; σ_1=1, σ_2=1. That means, water from the upstream process flows to the buffer (Tank 1) and further to the supply (Tank 2) and the supply is able to meet the demand of the downstream process while water levels of both tanks remain within acceptable ranges $w_{1\min} \le h_1 \le w_{1\max}$, $w_{2\min} \le h_2 \le w_{2\max}$. If water levels deviate from these ranges, the following rules apply

A. If $h_1 < w_{1\min}$, set σ_1 to zero to avoid Tank 1 emptying
B. If $h_1 > w_{1\max}$, set σ_0 to zero to avoid Tank 1 overflow, if also $h_2 < w_{1\min}$, set σ_1 to 2 to accelerate the recovery of both water levels to normal ranges
C. If $h_2 < w_{2\min}$, set σ_2 to zero to avoid Tank 2 emptying
D. If $h_2 > w_{2\max}$, set σ_1 to zero to avoid Tank 2 overflow

Continuous control system is designed as SISO control loop, where $H(t)$ is manipulated variable and temperature ϑ_2 is a controlled variable. It can be seen from (4) that logical control inputs act as disturbances and they may have adverse effects on control performance.

The situation changes when logical and continuous control is designed in a unified way. System described by (1)-(4) is treated as a hybrid system and model predictive controller can be designed based on its PWA approximation. Hybrid MPC controller is designed using MPT Toolbox. To compare performance of the systems using separate design and hybrid model predictive control, the following control experiment was performed. Starting from the state q_0=q_{20}=1 l/min, h_1=0.5 m, h_2=0.3 m, ϑ_1=50°C, ϑ_2=40°C, the setpoint was increased from 40°C to 60°C. Separate design used logic rules defined in the beginning of this section. The continuous control algorithm was MPC with linear performance function and control horizon 2. The unified design used MPC with the same performance function and control horizon, however this MPC algorithm could make use of the logical control inputs. The responses are shown in the following figures.

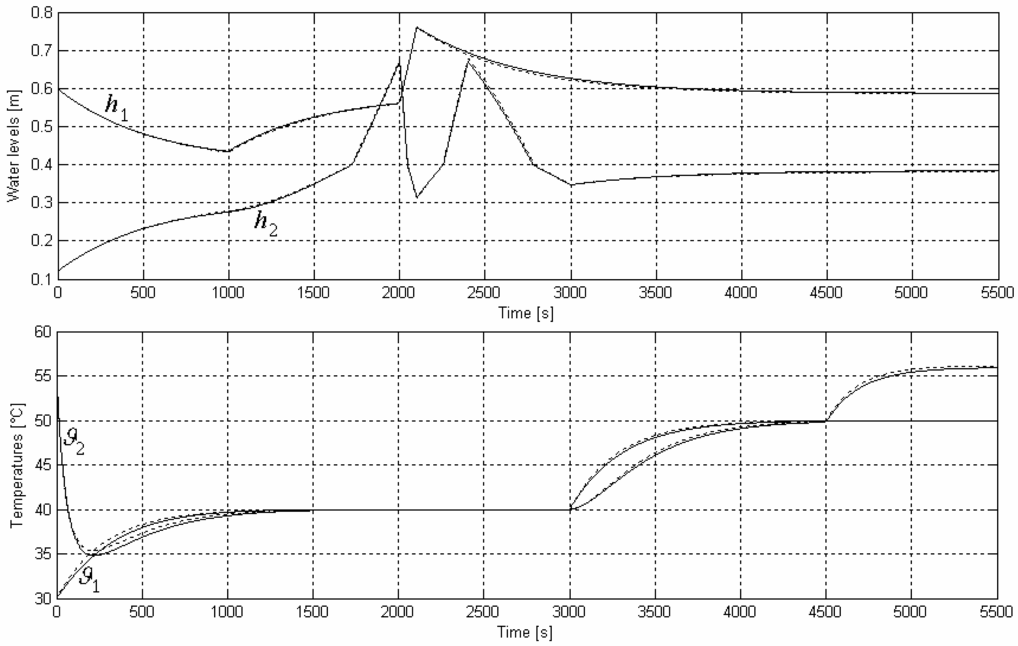

Fig. 2 Comparison of responses of the original plant model and its PWA approximation (in all responses original plant models is plotted with solid line and PWA approximation with dotted line).

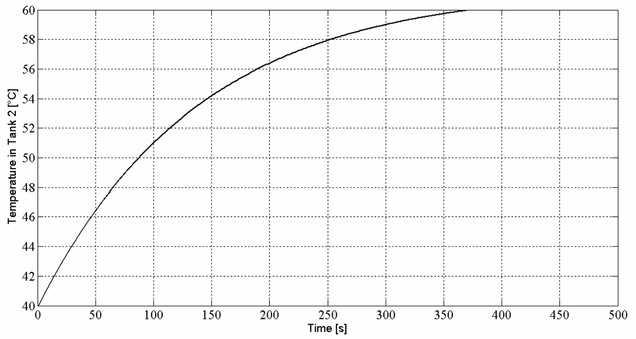

Fig. 3 Setpoint response Separate design

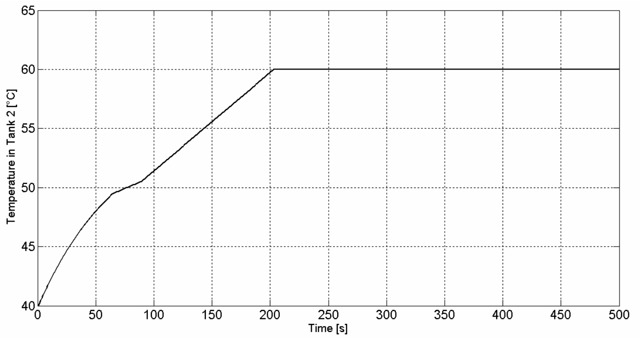

Fig. 4 Setpoint response – Unified design

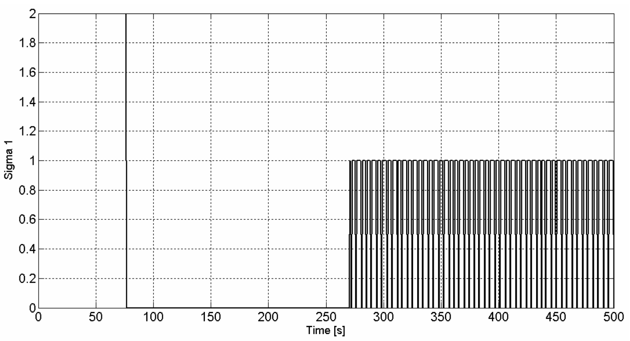

Fig. 5 Logical manipulated variable σ_1

It can be seen that unified design achieves better results. The control time is much shorter. Fig. 5 shows that this improvement is due to the ability of the unified design to make use of logical manipulated variables. Unlike separate design, logical control inputs can be used not only to keep the water levels within specified limits but also to accelerate control responses. In the beginning σ_1 is used to accelerate the setpoint response by increasing the inflow of warmer water to Tank 2. When controlled variable reaches setpoint, σ_1 is used just to keep the water levels within specified range.

7. CONCLUSION

This paper was focused on the possibilities that hybrid model predictive control can offer for unified design of logical and continuous control. As the character of this paper is mainly experimental, its results are connected with the particular laboratory plant considered and they cannot be regarded as completely general. In spite of that, some conclusions can be made. It could be seen that the application of hybrid MPC is not particularly easy. The development of the PWA model that is necessary for hybrid MPC takes up a substantial part of the paper. The complexity of the PWA model expressed by several hundreds of partial systems is also quite high even in the case of this laboratory scale plant that has still quite simple structure compared with real industrial process plants.

On the other hand, it has been shown that hybrid MPC can make use of the information how the controlled variable is affected by logical control inputs to improve control responses. The setpoint response was improved by adding the effect of opening valve S4 to the effect of increasing heater power output. Thus, the control results achieved with hybrid MPC were better than the results obtained with separate design of logical and continuous control.

Given paper length allowed to present one selected control experiment. Other control scenarios can be devised and tested with similar results and there is still a large open space for further experiments with this plant focused on exploring the possibilities offered by hybrid MPC for co-ordinated design of logical and continuous control. A particular attention will also be paid to the real time implementation of explicit MPC using industrial control system WinCon-8000 that can be used with this plant as an alternative to the academic experimental setting using Matlab/Simulink, Real Time Toolbox and PC data acquisition boards.

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