Quasi-decentralized Scheduled Output Feedback Control of Process Systems Using Wireless Sensor Networks ⋆

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Abstract: This paper presents a quasi-decentralized output feedback control structure for multi-unit plants with limited state measurements and distributed control systems that exchange information over a resource-constrained wireless sensor network (WSN). The networked control structure brings together model-based feedback control, state estimation and sensor scheduling to enforce closed-loop stability while simultaneously minimizing the rate of communication over the WSN. Initially, an observer-based output feedback controller is designed for each unit. To conserve the resources of the wireless devices, communication between the local control systems is suspended periodically for extended time periods during which each control system relies on models of the plant units to generate the necessary control action. Communication is then re-established at discrete time instances according to a certain schedule that determines the order and times at which the wireless sensor suites transmit the state estimates needed to update the states of the models embedded in the target units. By analyzing the combined discrete-continuous behavior of the scheduled closed-loop plant, we explicitly characterize the stability of the networked closed-loop system in terms of the communication rate, the sensor transmission schedule, the accuracy of the models, as well as the controller and observer design parameters. The results are illustrated using a chemical plant example where it is shown that by judicious management of the interplays between the control, communication and scheduling design parameters, it is possible to stabilize the plant while simultaneously enhancing the savings in WSN resources beyond what is possible with concurrent transmission configurations.

Keywords: Quasi-decentralized control, wireless sensor networks, model-based control, state estimation, scheduling algorithms, chemical plants.

1. INTRODUCTION

Chemical plants are large-scale dynamical systems that consist of a large number of distributed units which are tightly interconnected through mass and energy flows and recycle. Traditionally, the controller synthesis problem for such plants has been addressed within either the centralized or decentralized control frameworks. Both approaches have been the subject of numerous research studies aimed at understanding their advantages and limitations, as well as the development of strategies to overcome some of those limitations (e.g., see Siljak (1991); Lunze (1992); Sourlas and Manousiouthakis (1995); Katebi and Johnson (1997); Cui and Jacobsen (2002); Camponogara et al. (2002); Huang and Huang (2004); Skogestad (2004); Venkat et al. (2005); Goodwin et al. (2005); Kariwala (2007) and the references therein). Other notable contributions on this problem include the development of plant-wide control strategies based on passivity theory and concepts from thermodynamics (Hangos et al. (1999); Antelo et al. (2007)), the development of agent-based systems to control spatially-distributed reactor networks (Tetiker et al. (2008)), and the analysis and control of integrated process networks using time-scale decomposition and singular perturbations (Baldea et al. (2006)).

An approach that provides a compromise between the complexity of traditional centralized control schemes, on the one hand, and the performance limitations of decentralized control approaches, on the other, is quasi-decentralized control, which refers to a control strategy in which most signals used for control are collected and processed locally, while some signals are transferred between the local units and controllers to adequately account for the interactions and minimize the propagation of process upsets from one unit to another. A key consideration in the design and implementation of quasi-decentralized control systems is the selection of the communication medium over which the local control systems must communicate. While dedicated point-to-point links offer a reliable communication medium, the complexity and costs of installation and maintenance associated with this architecture, as well as the lack of flexibility for real-time reconfiguration, represent major drawbacks especially for large-scale plants with complex interconnections. An alternative solution is the use of wireless communication networks. The viability of this approach stems from the convergence of recent advances in actuator/sensor manufacturing, wireless communications and digital electronics,

⋆ Financial support by NSF CAREER Award, CBET-0747954, is gratefully acknowledged.

which has produced low-cost wireless sensors (e.g., Kumar (2001); Song et al. (2006)) that can be installed for a fraction of the cost of wired devices. Wireless sensor networks (WSNs) offer unprecedented flexibility ranging from high-density sensing capabilities to deployment in areas where wired devices may be difficult or impossible to deploy. Augmenting existing process control and monitoring systems with WSNs has the potential to expand the capabilities of the existing control technology beyond what is feasible with wired architectures alone. These are appealing goals that coincide with the recent calls for expanding the traditional process control and operations paradigm in the direction of smart plant operations (e.g., see Ydstie (2002); Christofides et al. (2007)).

One of the main challenges to be addressed when deploying a low-cost WSN for control is that of handling the inherent constraints on network resources, including the limitations on the computation, processing and communication capabilities. In an effort to address this problem, we developed in Sun and El-Farra (2008a) a quasi-decentralized model-based networked control architecture that enforces closed-loop stability with minimal cross communication between the constituent subsystems. The minimum allowable communication rate was characterized in terms of the plant-models' mismatch for the case when all sensors suites transmit their measurements concurrently and are given simultaneous access to the network. The networked control structure was subsequently generalized in Sun and El-Farra (2008b) to address the problem when only limited state measurements are available (for additional results and references on the design of networked control systems, the reader may refer to Walsh and Ye (2001); Montestruque and Antsaklis (2003); Munoz de la Pena and Christofides (2008) and the references therein).

In addition to transmitting the data at discrete time instances, another important way of conserving the WSN resources is to select and activate only a subset of the deployed sensor suites at any given time to communicate with the rest of the plant. Under this restriction, the stability and performance characteristics of each unit in the plant become dependent not only on the controller design but also on the selection of the scheduling strategy that, at any time, determines the order in which the sensor suites of the neighboring units transmit their data. Forcing the different subsystems to transmit their data at different times creates opportunities for providing a more targeted correction to the models' estimation errors, such that the models with the largest uncertainties can receive more timely updates than is feasible under the simultaneous transmissions configuration.

Motivated by these considerations, we present in this work an integrated approach for model-based control, state estimation and sensor scheduling in plants with limited state measurements and interconnected processing units that communicate over a resource-constrained WSN. The objective is to find a strategy for establishing and terminating communication between the sensors suites of the WSN and the local control systems in a way that minimizes the rate at which each node in the WSN broadcasts data to the rest of the plant without jeopardizing closed-loop stability. The rest of the paper is organized as follows. Following some preliminaries in Section 2, the networked control and scheduling problem is formulated. Section 3 then presents the quasi-decentralized output feedback control structure and describes its implementation over a WSN with the aid of appropriate local state observers, process models and sensor transmission scheduling. The closed-loop system is then formulated and analyzed in Section 4 where precise conditions for closed-loop stability are provided in terms of the communication rate over the WSN, the sensor scheduling strategy, as well as the accuracy of the models and the choice of controller and observer designs. We show how the stability criteria provide systematic tools that can guide the search for optimal transmission schedules that achieve the biggest savings in WSN resource utilization. Finally, the theoretical results are illustrated in Section 5 using a chemical plant example.

2. PRELIMINARIES

2.1 Plant description

We consider a large-scale distributed plant composed of n interconnected processing units, represented by the following state-space description:

$$
\begin{aligned}
\dot{x}_1 &= A_1 x_1 + B_1 u_1 + \sum_{j=2}^{n} A_{1j} x_j, & y_1 &= C_1 x_1 \\
\dot{x}_2 &= A_2 x_2 + B_2 u_2 + \sum_{j=1, j\neq 2}^{n} A_{2j} x_j, & y_2 &= C_2 x_2 \\
&\vdots & &\vdots \\
\dot{x}_n &= A_n x_n + B_n u_n + \sum_{j=1}^{n-1} A_{nj} x_j, & y_n &= C_n x_n
\end{aligned} \tag{1}
$$

where $x_i := [x_i^{(1)} \; x_i^{(2)} \; \cdots \; x_i^{(p_i)}]^T \in \mathbb{R}^{p_i}$ denotes the vector of process state variables associated with the i-th processing unit, p_i is the number of state variables in the i-th unit, $y_i := [y_i^{(1)} \; y_i^{(2)} \; \cdots \; y_i^{(q_i)}]^T \in \mathbb{R}^{q_i}$ and $u_i := [u_i^{(1)} \; u_i^{(2)} \; \cdots \; u_i^{(r_i)}]^T \in \mathbb{R}^{r_i}$ denote the vector of measured outputs and manipulated inputs associated with the i-th processing unit, respectively, x^T denotes the transpose of a column vector x; A_i, B_i, A_{ij} and C_i are constant matrices. The interconnection term $A_{ij}x_j$, where $i \neq j$, describes how the dynamics of the i-th unit are influenced by the j-th unit in the plant. Note from the summation notation in Eq.1 that each processing unit can in general be connected to all the other units in the plant.

2.2 Problem formulation and solution methodology

Referring to plant of Eq.1, we consider a quasi-decentralized control structure in which each unit in the plant has a local control system with its sensors and actuators connected to the local controller through a dedicated wired communication network. An additional suite of wireless sensors is deployed within each unit to transfer data from the local control system to the plant supervisor as well as to the other distributed control systems in the plant. The various sensor suites form a plant-wide WSN through which the plant units and their controllers communicate. The control objective is to stabilize all the plant units at the zero steady-state while simultaneously: (a) keeping the data dissemination and exchange over the WSN to a minimum, and (b) accounting for the lack of full-state measurements within each unit. To address the resource-constraints problem, we develop in the next section an integrated model-based quasi-decentralized output feedback control and scheduling strategy that reduces the exchange of information between the plant units without loss of stability. This is accomplished by: (a) designing for each local control system an appropriate state observer that generate estimates of the local state variables from the measured outputs, (b) including models within each control system to estimate the interaction terms when measurements are not available through the WSN, and (c) limiting the number of WSN nodes that, at any given time, transmit their data to update the corresponding target models. The problem is to find an optimal scheduling strategy for establishing and terminating communication between the sensor suites and the target controllers. To illustrate the main ideas, we will consider as an example the configuration where only one wireless sensor suite is allowed to transmit its data to the appropriate units at any given time, while the other nodes remain dormant until the next suite is allowed to transmit its data.

3. QUASI-DECENTRALIZED STATE ESTIMATION AND CONTROL WITH SCHEDULED SENSOR TRANSMISSIONS

3.1 Synthesis of distributed output feedback controllers

Referring to the plant of Eq.1, we begin by synthesizing for each unit an output feedback controller of the form:

$$\begin{aligned} u_i &= K_i \bar{x}_i + \sum_{j=1, j\neq i}^{n} K_{ij}\bar{x}_j \\ \dot{\bar{x}}_i &= (A_i - L_i C_i)\bar{x}_i + \sum_{j=1,j\neq i}^{n} A_{ij}\bar{x}_j + B_i u_i + L_i y_i, \end{aligned} \tag{2}$$

where $\bar{x}_i$ is an estimate of the state of the i-th unit generated by an observer embedded within the local control system of the i-th unit, K_i is the local feedback gain responsible for stabilizing the i-th subsystem in the absence of interconnections, K_{ij} is a gain that compensates for the effect of the j-th neighboring subsystem on the dynamics of the i-th unit, and L_i is the observer gain (chosen such that $A_i - L_i C_i$ is Hurwitz). Note that, in addition to $\bar{x}_i$ which is supplied continuously by the local observer, the implementation of the controller of Eq.2 requires the availability of observer-generated state estimates from the other units in the plant, $\bar{x}_j$, which can be transmitted only through the WSN. A copy of the local observer must therefore be included within the wireless sensor suite of each unit in order to generate the state estimates which are then broadcast to the rest of the plant. This setup is possible given the computational capabilities of wireless sensors. An alternative approach, which avoids having the wireless sensors carry the computational load of the observer, is to have the WSN nodes transmit only the output measurements, but include within each control system an observer of the full plant instead (not just an observer of the local subsystem) which then generates the required state estimates of the full plant state. In addition to the complexity of designing a centralized observer for the entire plant, another difficulty with this approach is that the observer must be designed to have hybrid dynamics since the WSN data are transmitted only at discrete time instances while the local measurements are supplied continuously (or at least more frequently).

It should also be noted that the choice to use a Luenberger observer is made only to illustrate the design and implementation of the quasi-decentralized output feedback control architecture. This choice, however, is not unique and any other explicit observer design can be used instead. The only requirement is that the observer possess an explicit evolution equation that relates the dynamics of the state estimate explicitly to the plant matrices, the output and the observer design parameters. As we will see in the next section, this feature permits the derivation of explicit closed-loop stability conditions that depend in a transparent way on the observer design parameters.

3.2 Design of model-based networked control structure

To conserve battery power in the plant-wide WSN, we initially reduce the rate at which the information (i.e., $\bar{x}_j$) is transferred from the wireless sensor suite of each unit to the target control systems in the neighboring units as much as possible without sacrificing closed-loop stability. To this end, and following the idea presented in (Sun and El-Farra (2008b)), we embed in each unit (both in the local controller and in the wireless sensor suite) a set of dynamic models that provide estimates of the evolution of the states of the neighboring units when communication over the WSN is suspended. The model estimates are used to generate both the local state estimates and the local control action. The state of each model is then reset using the state estimate generated by the observer of the corresponding unit when the wireless sensor suite of the latter is allowed to transmit its data at discrete time instances. In mathematical terms, the local control and update laws for unit i are implemented as follows:

$$\begin{aligned} u_i(t) &= K_i\bar{x}_i(t) + \sum_{j=1,j\neq i}^{n} K_{ij}\widehat{x}_j^i(t),\ t\neq t_k^j,\ i=1,2,\cdots,n \\ \dot{\bar{x}}_i(t) &= (A_i - L_iC_i)\bar{x}_i(t) + \sum_{j=1,j\neq i}^{n} A_{ij}\widehat{x}_j^i(t) + B_iu_i(t) + L_iy_i(t) \\ \dot{\widehat{x}}_j^i(t) &= \widehat{A}_j\widehat{x}_j^i(t) + \widehat{B}_j\widehat{u}_j^i(t) + \widehat{A}_{ji}\bar{x}_i(t) + \sum_{l=1,l\neq i,l\neq j}^{n}\widehat{A}_{jl}\widehat{x}_l^i(t), t\neq t_k^j \\ \widehat{u}_j^i(t) &= K_j\widehat{x}_j^i(t) + K_{ji}\bar{x}_i(t) + \sum_{l=1,l\neq i,l\neq j}^{n} K_{jl}\widehat{x}_l^i(t),\ \ t\neq t_k^j \\ \widehat{x}_j^i(t_k^j) &= \bar{x}_j(t_k^j),\ \ j=1,\cdots,n, j\neq i,\ \ k=0,1,2,\cdots \end{aligned} \tag{3}$$

where $\widehat{x}_j^i$ is the estimate of x_j provided by a model of unit j embedded in unit i; $\widehat{A}_j$, $\widehat{B}_j$ and $\widehat{A}_{jl}$ are constant matrices; t_k^j indicates the k-th transmission time for the j-th sensor suite in the WSN. The fact that $\bar{x}_i$ appears directly in the model of the j-th unit follows from: (1) the structure of the plant and the way the i-th and j-th units are interconnected, and (2) the fact that the observer-generated estimates of x_i are assumed to be available continuously to the local control system of the i-th unit. Note that the models used by the i-th controller to recreate the behavior of the neighboring units do not necessarily match the actual dynamics of those processes, i.e., in general $\widehat{A}_j \neq A_j$, $\widehat{B}_j \neq B_j$, $\widehat{A}_{jl} \neq A_{jl}$.

3.3 Scheduling WSN transmissions and model updates

A key measure of the extent of WSN utilization is the update period for each sensor suite, $h^j := t_{k+1}^j - t_k^j$, which determines the frequency at which the j-th node sends observer estimates to the other units through the network to update the corresponding model states. A larger h implies larger savings in WSN resource utilization. To simplify the analysis, we consider in what follows only the case when the update period is constant and the same for all the units, so that $t_{k+1}^j - t_k^j := h,\ j = 1, 2, \cdots, n$. To further reduce network utilization, we perform sensor scheduling whereby only one wireless sensor suite is allowed to transmit its observer estimates to the appropriate units at any one time, while the other suites remain dormant until the next suite is allowed to transmit its data (the analysis can be generalized to cases where multiple suites transmit at the same time). The transmission schedule is defined by: (1) the sequence (or order) of transmitting nodes: $\{s_j,\ j = 1, 2, \cdots, n\},\quad s_j \in \mathcal{N} := \{1, 2, \cdots, n\}$, where s_j is a discrete variable that denotes the j-th transmitting entity in the sequence, and (2) the time at which each node in the sequence transmits observer estimates. To characterize the transmission times, we introduce the variable: $\Delta t_j := t_k^{s_{j+1}} - t_k^{s_j},\ \ j = 1, 2, \cdots, n-1$, which is the time interval between the transmissions of two consecutive nodes in the sequence. Fig.1 is a schematic representation of how

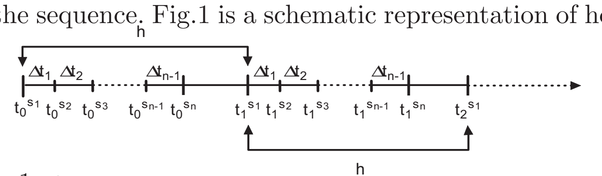

Fig. 1. A schematic showing the time-line for the transmission of each sensor suite in an h-periodic schedule.

sensor scheduling is performed. Note that the schedule is h-periodic in that the same sequence of transmitting nodes is executed repeatedly every h seconds (equivalently, each node transmits its data every h seconds). Note also from the definitions of both h and Δt_j that we always have the constraint $\sum_{j=1}^{n-1} \Delta t_j < h$. Since the update periods for all units are the same, the intervals between the transmission times of two specific units are constant, and within any single execution of the schedule (which lasts less than h seconds), each sensor suite can only transmit its observer estimates through the WSN and update its target models in the local control systems of its neighbors once. This can be represented mathematically by the condition: $s_i \neq s_j$ when $i \neq j$. By manipulating the time intervals Δt_j (i.e., the transmission times) and the order in which the nodes transmit, one can systematically search for the optimal sensor transmission schedule that leads to the largest update period (or smallest communication rate between each sensor suite and its target units).

4. NETWORKED CLOSED-LOOP STABILITY ANALYSIS

4.1 Characterizing the scheduled closed-loop response

In order to derive conditions for closed-loop stability, we need first to express the plant response as a function of the update period and the sensor transmission schedule. To this end, we define the model estimation errors by $e_j^i = \bar{x}_j - \widehat{x}_j^i$, for $j \neq i$, and $e_j^i = 0$, for $j = i$, where e_j^i represents the difference between the state of the observer of unit j (embedded in unit j) and the state of the model of unit j (embedded in unit i). Introducing the augmented vectors: $\mathbf{e}_j := [(e_j^1)^T\ (e_j^2)^T\ \cdots\ (e_j^n)^T]^T$, $\mathbf{e} := [\mathbf{e}_1^T\ \mathbf{e}_2^T\ \cdots\ \mathbf{e}_n^T]^T$, $\mathbf{x} := [x_1^T\ x_2^T\ \cdots\ x_n^T]^T$, $\bar{\mathbf{x}} := [\bar{x}_1^T\ \bar{x}_2^T\ \cdots\ \bar{x}_n^T]^T$, it can be shown that the overall closed-loop plant of Eq.1 and Eq.3 can be formulated as a combined discrete-continuous system of the form:

$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \Lambda_{11}\mathbf{x}(t) + \Lambda_{12}\bar{\mathbf{x}}(t) + \Lambda_{13}\mathbf{e}(t) \\
\dot{\bar{\mathbf{x}}}(t) &= \Lambda_{21}\mathbf{x}(t) + \Lambda_{22}\bar{\mathbf{x}}(t) + \Lambda_{23}\mathbf{e}(t) \\
\dot{\mathbf{e}}(t) &= \Lambda_{31}\mathbf{x}(t) + \Lambda_{32}\bar{\mathbf{x}}(t) + \Lambda_{33}\mathbf{e}(t),\ \ t \neq t_k^j \\
\mathbf{e}_j(t_k^j) &= \mathbf{0},\ \ j = 1, 2, \cdots, n,\ \ k = 0, 1, 2, \cdots,
\end{aligned} \tag{4}
$$

where Λ_{ij}'s are constant matrices whose explicit forms are omitted for brevity but can be obtained by substituting Eq.3 into Eq.1. Note that, unlike the case of simultaneous sensor transmissions (where no scheduling takes place) which was investigated in Sun and El-Farra (2008b), not all models within a given unit are updated (and hence not all estimation errors are re-set to zero) at each transmission time. Instead, only the model of the transmitting unit is updated using the observer-generated estimates provided by the wireless sensor suite of that particular unit. Defining the augmented state vector $\xi(t) := [\mathbf{x}^T(t)\ \bar{\mathbf{x}}^T(t)\ \mathbf{e}^T(t)]^T$, the dynamics of the overall closed-loop system can be cast in the following form:

$$
\begin{aligned}
\dot{\xi}(t) &= \Lambda_o \xi(t),\ \ t \neq t_k^j \\
\xi(t_k^j) &= \left[\mathbf{x}^T(t_k^j)\ \bar{\mathbf{x}}^T(t_k^j)\ \mathbf{e}^T(t_k^j)\right]^T,\ k = 0, 1, 2, \cdots \\
\mathbf{e}^T(t_k^j) &= \left[\mathbf{e}_1^T(t_k^j)\ \cdots\ \mathbf{e}_{j-1}^T(t_k^j)\ \mathbf{0}\ \mathbf{e}_{j+1}^T(t_k^j)\ \cdots\ \mathbf{e}_n^T(t_k^j)\right]^T \\
\Lambda_o &= \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{bmatrix}
\end{aligned} \tag{5}
$$

The following proposition provides an explicit characterization of the scheduled closed-loop response in terms of the update period and the transmission schedule. The proof can be obtained by solving the system of Eq.5 within each sub-interval in Fig.1, and is omitted for brevity.

Proposition 1. Consider the closed-loop system described by Eq.5 with a transmission schedule $\{s_1, s_2, \cdots, s_n\}$ and the initial condition $\xi(t_0^{s_1}) = [\mathbf{x}^T(t_0^{s_1})\ \bar{\mathbf{x}}^T(t_0^{s_1})\ \mathbf{e}^T(t_0^{s_1})]^T = \xi_0$, with $\mathbf{e}_{s_1}(t_0^{s_1}) = \mathbf{0}$. Then, for $k = 0, 1, 2, \cdots$,

$$
\xi(t) = \begin{cases} e^{\Lambda_o (t - t_k^{s_j})} \Gamma_j(\Delta t_j, I_o^{s_j}) M_o^k \xi_0,\ t \in [t_k^{s_j},\ t_k^{s_{j+1}}) \\ e^{\Lambda_o (t - t_k^{s_n})} \Gamma_n M_o^k \xi_0,\ t \in [t_k^{s_n},\ t_{k+1}^{s_1}) \end{cases} \tag{6}
$$

where $j = 1, 2, \cdots, n-1$, and

$$
\Gamma_j = \prod_{j-1-\mu=0}^{j-2} I_o^{s_{\mu+1}} e^{\Lambda_o \Delta t_\mu},\text{for } j \geq 2, \text{and } \Gamma_j = I,\ \text{for } j = 1 \tag{7}
$$

$$
M_o = I_o^{s_1} e^{\Lambda_o (h - \sum_{j=1}^{n-1} \Delta t_j)} \Gamma_n \tag{8}
$$

$$
I_o^{s_j} = \begin{bmatrix} I & O & \cdots & O \\ O & H_1 & \cdots & O \\ \vdots & \vdots & & \vdots \\ O & O & \cdots & H_n \end{bmatrix},\ H_i = \begin{cases} I,\ i \neq s_j \\ O,\ i = s_j \end{cases} \tag{9}
$$

for $j = 1, 2, \cdots, n$, $t_{k+1}^{s_j} - t_k^{s_j} = h$ and $\Delta t_j = t_k^{s_{j+1}} - t_k^{s_j}$, $j = 1, 2, \cdots, n-1$.

4.2 Characterizing the maximum allowable update period

Having expressed the overall closed-loop response in terms of the update period, the transmission times (which are determined by Δt_j) and the sequence of transmitting nodes (which determines the structure of $I_o^{s_j}$), we are in a position to state the main result of this section. The following theorem provides a necessary and sufficient condition for stability of the scheduled closed-loop plant under the quasi-decentralized networked output feedback control structure. The proof is omitted for brevity.

Theorem 2. Referring to the scheduled closed-loop system of Eq.5 whose solution is given by Eqs.6-9, the zero solution, $\xi = [\mathbf{x}^T \ \bar{\mathbf{x}}^T \ \mathbf{e}^T]^T = [\mathbf{0}\ \mathbf{0}\ \mathbf{0}]^T$, is globally exponentially stable if and only if the eigenvalues of the matrix in Eq.8 are strictly inside the unit circle.

By examining the structure of the test matrix M_o in Eq.8, it can be seen that its eigenvalues depend on the update period h, the closed-loop matrix Λ_o (which in turn depends on the plant-model mismatch as well as the controller and observer gains for all the units), the time intervals between sensor transmissions $\Delta t_1, \Delta t_2, \cdots, \Delta t_{n-1}$, as well as the sensor transmission sequence $\{s_1, s_2, \cdots, s_n\}$. The stability criteria in Theorem 2 can therefore be used to compare different schedules (by varying the transmission sequence as well as the transmission times) to determine the ones that require the least communication rate between the sensors and the target controllers and therefore produce the biggest savings in WSN battery power utilization. For a fixed schedule, the stability criteria can also be used to compare different models, as well as different controllers and state observers in terms of their robustness with respect to communication suspension (i.e., which ones require measurement updates less frequently than others). Note that choosing $\Delta t_1 = \Delta t_2 = \cdots = \Delta t_{n-1} = 0$ reduces the problem to one where all the nodes in the WSN transmit their observer estimates simultaneously. As expected, in this case stability of the networked closed-loop system depends only on Λ_o and h.

5. SIMULATION STUDY: APPLICATION TO CHEMICAL REACTORS WITH RECYCLE

We consider a plant composed of three non-isothermal continuous stirred-tank reactors (CSTRs) in a cascade. The reactant species A is consumed in each reactor by three parallel irreversible exothermic reactions. The output of the third CSTR is passed through a separator that removes the products and recycles unreacted A to the first CSTR. Under standard modeling assumptions, a plant model of the following form can be derived from conservation laws:

$$\begin{aligned}
\frac{dT_j}{dt} &= \frac{F_j^0}{V_j}(T_j^0 - T_j) + \frac{F_{j-1}}{V_j}(T_{j-1} - T_j) \\
&\quad + \sum_{i=1}^{3} \frac{(-\Delta H_i)}{\rho c_p} R_i(C_{Aj}, T_j) + \frac{Q_j}{\rho c_p V_j} \\
\frac{dC_{Aj}}{dt} &= \frac{F_j^0}{V_j}(C_{Aj}^0 - C_{Aj}) + \frac{F_{j-1}}{V_j}(C_{A(j-1)} - C_{Aj}) \\
&\quad - \sum_{i=1}^{3} R_i(C_{Aj}, T_j),\ j = 1, 2, 3
\end{aligned}$$

where T_j, C_{Aj}, Q_j, and V_j denote the temperature, the reactant concentration, the rate of heat input, and the volume of the j-th reactor, respectively, $R_i(C_{Aj}, T_j) = k_{i0} \exp\left(\frac{-E_i}{RT_j}\right) C_{Aj}$ is the rate of the i-th reaction, F_j^0 denotes the flow rate of a fresh feed stream associated with the j-th reactor, F_j is the flow rate of the outlet stream of the j-th reactor, with $F_0 = F_r, T_0 = T_3, C_{A0} = C_{A3}$ denoting the flow rate, temperature and reactant concentration of the recycle stream, ΔH_i, k_i, E_i, $i = 1, 2, 3$, denote the enthalpies, pre-exponential constants and activation energies of the three reactions, respectively, c_p and ρ denote the heat capacity and density of fluid in the reactor. Using typical values for the process parameters (see Sun and El-Farra (2008a)), the plant with $Q_j = 0$, $C_{Aj}^0 = C_{Aj}^{0s}$ and a recycle ratio of $r = 0.5$, has three steady-states (two locally asymptotically stable and one unstable). The control objective is to stabilize the plant at the (open-loop) unstable steady-state by manipulating Q_j and C_{Aj}^0, $j = 1, 2, 3$. Only the temperatures of the three reactors are assumed to be available as measurements. A plant-wide WSN composed of 3 wireless sensor suites is deployed. Each sensor suite collects estimates of the local process state variables provided by a state observer embedded within the unit and broadcasts it to the rest of the plant. It is desired to stabilize the plant with minimal data exchange over the WSN to conserve battery power for the wireless devices.

Linearizing the plant around the unstable steady-state yields a system of the form of Eq.1 to which the networked output feedback control and scheduling architecture described in the previous sections is applied. The synthesis details are omitted due to space limitations. In the remainder of this section, we will investigate the interplay between the communication rate and the sensor transmission schedule, and its impact on closed-loop stability. Since closed-loop stability requires all eigenvalues of M_o to lie within the unit circle, it is sufficient to consider only the maximum eigenvalue magnitude, denoted by $\lambda_{\max}(M_o)$.

Table 1. Sensor transmission schedules

Schedule	$s_1, s_2, s_3, s_1, s_2, s_3, \cdots$
1	$1, 2, 3, 1, 2, 3, \cdots$
2	$1, 3, 2, 1, 3, 2, \cdots$
3	$2, 1, 3, 2, 1, 3, \cdots$
4	$2, 3, 1, 2, 3, 1, \cdots$
5	$3, 1, 2, 3, 1, 2, \cdots$
6	$3, 2, 1, 3, 2, 1, \cdots$

We consider first the case when $\Delta t_1 = \Delta t_2 = \Delta t$. Fig.2(a) is a contour plot showing the dependence of $\lambda_{\max}(M_o)$ on both the interval between transmissions, Δt, and the update period, h, under the six possible sensor transmission schedules listed in Table 1 when imperfect models are embedded in the local control systems (each model has 10% parametric uncertainty in the heat of reaction). For each schedule, the area enclosed by the unit contour line is the stability region of the plant. It can be seen that, for sufficiently small Δt (below 0.03 hr), the maximum allowable update periods obtained under sequences 2 and 6 are larger than the one obtained when no scheduling takes place (i.e., with $\Delta t = 0$). As Δt is increased, however, the trend is reversed, indicating that the benefits of scheduling can be limited by a poor choice of the transmission times. For sequences 3 and 5, scheduling yields larger update periods (compared with the concurrent transmission configuration) only when the transmission times are chosen such that $\Delta t > 0.04$ hr. In general, allowing the different sensor suites to transmit their data and update their target models at different times (rather than simultaneously) can help provide a more targeted and timely (though only partial) correction to model estimation errors which in turn helps reduce the rate at which each node in the WSN must transmit its data. These predictions are further confirmed by the closed-loop state profile shown in Fig.2(b), which shows that the linearized plant is stable under sequence 6 but unstable under sequence 2, when $\Delta t = 0.02$ hr and

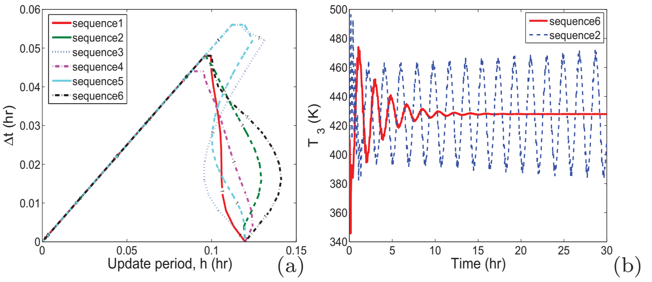

Fig. 2. (a) Dependence of $\lambda_{\max}(M_o)$ on Δt and h for different sensor transmission sequences under a model-based control scheme. (b) Closed-loop temperature profile for CSTR3 under the model-based quasi-decentralized output feedback control strategy using two different sensor transmission schedules with the same update period.

$h = 0.13$ hr (for brevity, only the temperature profile for CSTR 3 is shown; the state and input profiles for the other reactors exhibit similar behavior).

We consider next the more general case where $\Delta t_1 \neq \Delta t_2$. Fig.3(a) is a contour plot showing the dependence of $\lambda_{\max}(M_o)$ on Δt_1 and h for different values of Δt_2, when the WSN nodes transmit according to sequence 2 and an uncertain model is used (nominal value of the heat of reaction is 10% higher than the actual value). It can be seen that a larger update period (and hence larger reduction in WSN utilization) can be obtained by carefully choosing the transmission times for the sensor suites of different units than in the case when $\Delta t_1 = \Delta t_2$. For example, consider the case when $\Delta t_2 = 0.02$ hr and $h = 0.13$ hr. This point lies outside the stability region of schedule 2 when $\Delta t_2 = \Delta t_1 = 0.02$ hr (see Fig.2(a)). If we choose $\Delta t_1 = 0.08$ hr, however, the same update period becomes stabilizing under schedule 2 (the point now lies inside the stability region). These observations are further confirmed by the temperature profiles in Fig.3(b).

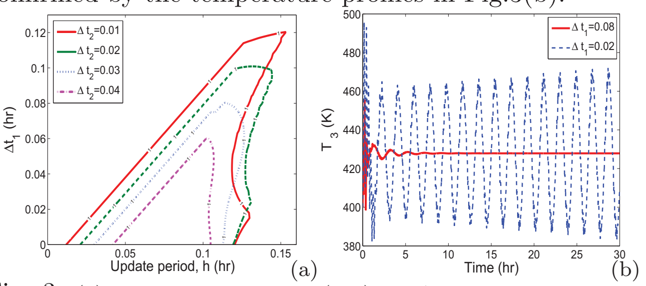

Fig. 3. (a) Dependence of $\lambda_{\max}(M_o)$ on Δt_1 and h for different values of Δt_2 under schedule 2 with a fixed model, and (b) Closed-loop temperature profile for CSTR 3 when $\Delta t_2 = 0.02$ hr and $h = 0.13$ hr for two different values of Δt_1.

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