Dynamic Operability for the Calculation of Transient Output Constraints for Non-Square Linear Model Predictive Controllers

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Abstract: This paper introduces a dynamic operability-based approach for the determination of feasible output constraints during transient operation. This approach is based on previously published steady-state operability developments and the concept of output funnels. In this study, high-dimensional non-square systems with more outputs than inputs are of particular interest. Such systems are challenging because it is impossible to control all the outputs at specific set-points when there are fewer degrees of freedom available than the controlled variables. Thus, interval, instead of set-point, control is needed for at least some of the output variables. In order to motivate the new concepts, two non-square case studies are addressed, one illustrative and one industrial - obtained from the control system of a Steam Methane Reformer process. The calculated constraints are validated by running DMCplus^{TM} (AspenTech) closed-loop simulations for the extreme values of the disturbances. These constraints are intended for use online in model-based controllers (e.g., Model Predictive Controllers) to ensure that each of the outputs will remain inside a feasibility envelope during transient operation.

Keywords: Output Variables, Constraints, Model-based Control, Operability, Dynamic Systems.

1. INTRODUCTION

Model Predictive Control (MPC) is a long standing multivariable constrained control methodology that utilizes an explicit process model to predict the future behavior of a chemical plant. At each control interval, the MPC algorithm attempts to optimize the future plant behavior by computing a sequence of future manipulated variable adjustments. The first of the optimal sequence of calculated input moves is implemented into the plant and the entire calculation is repeated at subsequent control intervals using updated process measurements. MPC has been extensively studied in academia and widely accepted in the chemical industry for its ability to handle complex multivariable and highly interactive process control problems (Qin and Badgwell, 2003). MPC-type controllers in industrial practice aim to control non-square systems in which there are more controlled outputs than manipulated inputs. In such systems it is impossible to control all the outputs at specific set-points because there are fewer degrees of freedom available than the controlled variables.

Based on the input constraints, generally specified *a priori* due to the physical limitations of the process, an important design task is to determine the output ranges within which one wants to control the process. The improper selection of these constraints can make the controller infeasible when a disturbance moves the process far away from its usual operating region. Past practice requires that output constraints are enforced whenever feasible and softened whenever they become infeasible (Rawlings, 2000). The steady-state operability methodology originally introduced for square systems (Vinson and Georgakis, 2000) and extended for non-square systems (Lima and Georgakis, 2006; Lima and Georgakis, 2008a) provides a method for selecting such output constraints systematically, so that they are as tight as possible but also do not render the controller infeasible. Specifically for non-square systems, the interval operability framework was introduced (Lima and Georgakis, 2006) to assess the input-output open-loop operability of multivariable non-square systems at the steady-state, a necessary condition for the overall process operability. The application of this framework to high-dimensional square and non-square systems is discussed in another set of publications (Lima and Georgakis, 2008b; Lima, Georgakis, Smith and Schnelle, 2008), where a Linear Programming (LP) based approach is introduced to calculate the tightest feasible set of steady-state output constraints when interval operability is necessary.

This paper extends this interval operability framework to enable the determination of feasible output constraints during transient for high-dimensional non-square systems. Although the previously developed steady-state operability approaches are necessary to quantify the overall operability of a process and to determine the steady-state output constraints for MPC, the development of a dynamic operability methodology for non-square systems will have great impact on MPC controller design. Specifically, dynamic operability analysis can be used to systematically calculate the amount of constraint relaxation necessary in order to prevent the occurrence of transient infeasibilities, when disturbances affect the process (see Dimitriadis and Pistikopoulos (1995) for dynamic flexibility analysis). This extension is accomplished here by designing a funnel for each of the output variables, which provides output constraints that guarantee feasible process operation in closed-loop. Previously, output funnels have been used to define MPC controllers' output trajectories in commercial packages (Qin and Badgwell, 2003). Specifically, Honeywell's RMPCT (Robust Multivariable Predictive Control Technology) controller defines a funnel for the outputs or Controlled Variables (CV) constraints. When a predicted CV trajectory leaves its funnel, the controller algorithm penalizes this trajectory to bring the CV back within its range (Qin and Badgwell, 2003; Maciejowski, 2002). Here such funnels are used to design output constraints during transient operation. This design is especially important for underdamped systems in general, where overshoots may occur during process operation, and overdamped or critically damped systems when disturbance dynamics are faster than input dynamics. For the opposite case when input dynamics are faster than disturbance ones, the output constraints calculated using one of the steady-state operability methodologies are also applicable during transient.

2. PROCESS OPERABILITY

Before introducing the dynamic operability approach, it is necessary to briefly define the sets of variables used for steady-state interval operability calculations (Lima and Georgakis, 2008a). The Available Input Set (AIS) is the set of values that the process input, or manipulated, variables (**u**) can take, based on the constraints of the process. For an n x m x q (*n outputs*, *m inputs* and *q disturbances*) linear system:

$$\mathbf{AIS} = \left\{ \mathbf{u} \mid \; u_i^{\min} \le u_i \le u_i^{\max};\; 1 \le i \le m \right\} \tag{1}$$

The Desired Output Set (DOS) is given by the ranges of the outputs (**y**) that are desired to be achieved and is represented by:

$$\mathbf{DOS} = \left\{ \mathbf{y} \mid \; y_i^{\min} \le y_i \le y_i^{\max};\; 1 \le i \le n \right\} \tag{2}$$

The Expected Disturbance Set (EDS) represents the expected steady-state values of the disturbances (**d**):

$$\mathbf{EDS} = \left\{ \mathbf{d} \mid \; d_i^{\min} \le d_i \le d_i^{\max};\; 1 \le i \le q \right\} \tag{3}$$

Based on the steady-state linear model of the process, expressed by the process gain matrix (**G**) and the disturbance gain matrix ($\mathbf{G}_d$), the Achievable Output Set for a specific disturbance vector, AOS(**d**), is defined by the ranges of the outputs that can be achieved using the inputs inside the AIS:

$$\mathbf{AOS}(\mathbf{d}) = \left\{ \mathbf{y} \mid \; \mathbf{y} = \mathbf{G}\mathbf{u} + \mathbf{G}_d \mathbf{d};\; \mathbf{u} \in \mathbf{AIS},\; \mathbf{d} \text{ is fixed} \right\} \tag{4}$$

The Achievable Output Interval Set (AOIS) is defined as the tightest feasible set of output constraints that can be achieved, with the available range of the manipulated variables and when the disturbances remain within their expected values (see references (Lima and Georgakis, 2008a; Lima and Georgakis, 2008b; Lima, Georgakis, Smith and Schnelle, 2008) for the algorithms developed for the calculation of this important set). Using these defined sets and some of the previously published interval operability concepts and calculations, a dynamic operability approach, based on the design of output funnels, for the determination of output constraints during transients is introduced next through a 2-D illustrative example. This is followed by the analysis of the Steam Methane Reformer (SMR) process example, which is 9-D and underdamped.

3. ILLUSTRATIVE EXAMPLE

In order to introduce the dynamic operability approach, consider a 2-D example from Lima and Georgakis (2006) with 2 outputs, 1 input and 1 disturbance (2 x 1 x 1). This example has the following steady-state gain model and constraining sets (see information on process dynamics below):

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1.41 \\ 0.66 \end{pmatrix} u_1 + \begin{pmatrix} -0.6 \\ 0.4 \end{pmatrix} d_1$$
$$\mathbf{AIS} = \{ u_1 \mid -1 \le u_1 \le 1 \}$$
$$\mathbf{EDS} = \{ d_1 \mid -1 \le d_1 \le 1 \}$$
$$\mathbf{DOS} = \left\{ \mathbf{y} \in \Re^2 \mid \|y\|_\infty \le 1 \right\} \tag{5}$$

Two funnels, one for each output, with specific amplitudes and decay characteristics will be designed for the two output variables of this system. Each of these funnels is designed from the moment that a disturbance is inserted into the system and provides an envelope where the control problem is always feasible if the output constraints remain inside of this envelope. This envelope starts at the funnel amplitude value (defined below), decays at a specific rate and ends at a designed steady-state constraint calculated using one of the interval operability approaches cited above. Cases where the disturbance variable takes its extreme values are of particular interest because they represent the worst cases, which if satisfied, ensure feasible operation for all the other cases.

For the system above, the dynamics of each of the input-output and disturbance-output pairs are plotted in Figs. 1, 2, 3, and 4 for pairs (y_1, u_1), (y_1, d_1), (y_2, d_1), and (y_2, u_1), respectively. In these figures, these dynamics are represented by step response coefficients, which would be obtained in practice by plant testing.

The funnel amplitude associated with output i (a_i) is defined as follows:

$$a_i = k_{d,j-i} s_j \tag{6}$$

where $k_{d,j-i}$ corresponds to the value of the steady-state disturbance gain associated with the disturbance-output pair j-i and s_j is the step disturbance value. For this example, s_j will be assumed at the extreme values of the disturbance within the EDS, i.e. d_1 is moved from 0 to ± 1. If d_1 = 1, then a_1 = -0.6 and a_2 = 0.4. When d_1 = -1, a_1 = 0.6 and a_2 = -0.4. It is assumed that using steady-state disturbance gains to calculate the starting point of the funnel decay, as opposed to the maximum absolute value of the dynamic gains, will be enough to provide an envelope that contains the entire closed-loop response. This is based on the assumption that the inputs are able to compensate for the presence of overshoots, caused by these dynamic gains, in most practical cases during closed-loop operation, especially if a model-based controller, such as MPC, is implemented. The decay for each output funnel (λ_i) is determined by the slowest dynamics among all the input-output and disturbance-output pairs for the corresponding output. Each output is analyzed separately because the disturbance dynamics might be slower for one of the outputs, while the input dynamics may be slower for the other. These dynamics are estimated from the step response curves using two approaches, depending on the characteristics of the analyzed curve:

1) Exponential fit (typically for oscillatory responses): an exponential is fitted to two selected points of the step response curve. These points are selected such that most of the curve is below (or above, depending on the sign of the dynamic gains) the fitted exponential. The dynamics of the analyzed pair are estimated by the following exponential decay:

$$y_{\exp} = a_{\exp} \exp\left(-\lambda_{\exp} t\right) + y_{\infty} \tag{7}$$

where y_{∞} corresponds to the steady-state gain of the analyzed step response curve. Using the two selected points and eq. (7), a system of 2 equations and 2 unknowns can be solved for the two parameters of the exponential, $a_{\exp}$ and $\lambda_{\exp}$. For the example above, this approach is used for pairs (y_1, u_1), (y_1, d_1), and (y_2, d_1), whose exponential fits are shown in Figs. 1, 2, and 3, respectively, along with the fitted points selected for each case. For such pairs, the following exponential fits are obtained:

$$\begin{aligned} \left(y_1,\ u_1\right) &= 0.47 \exp\left(-1.89\times10^{-2} t\right) + 1.41 \Rightarrow \lambda_{\exp} = 1.89\times10^{-2} \\ \left(y_1,\ d_1\right) &= -1.70 \exp\left(-2.12\times10^{-2} t\right) - 0.60 \Rightarrow \lambda_{\exp} = 2.12\times10^{-2} \\ \left(y_2,\ d_1\right) &= 0.18 \exp\left(-4.22\times10^{-2} t\right) + 0.40 \Rightarrow \lambda_{\exp} = 4.22\times10^{-2} \end{aligned} \tag{8}$$

2) First-order models estimated using ARX (Auto-Regressive model with eXogenous inputs, subroutine ARX in Matlab) (Ljung, 1999): the following first-order ARX model with a zero-order holder in the z domain is fitted to the step response coefficients of a specific pair:

$$y(z) = \frac{bz^{-1}}{1+az^{-1}} u(z) \text{ with } \lambda_{\text{arx}} = -\ln\left(p_z\right) \tag{9}$$

where λ_{arx} represents the model dynamics and is calculated by taking the negative natural log of the transfer function pole in the z domain, $p_z = -a$. This approach is used whenever a pair dynamics can be well approximated by a first-order model. This approach is applied here to pair (y_2, u_1), which is shown in Fig. 4, and the following model and λ_{arx} are obtained:

$$y_2(z) = \frac{0.0283}{z-0.9601} u_1(z) \Rightarrow \lambda_{\text{arx}} = -\ln\left(0.9601\right) = 4.07\times10^{-2} \tag{10}$$

After calculating all λs for all possible pairs (4 in this case), using the two approaches above, their values are compared and the one with smallest absolute value for each output (representing the slowest dynamics) is retained and used in the funnel design for the corresponding output. For example, for output 1, $\lambda_1 = 1.89 \times 10^{-2}$ is chosen, which is the smallest between 1.89×10^{-2} and 2.12×10^{-2}. Therefore, for this example, the following values of λ_i are selected:

$$\lambda_1 = 1.89\times10^{-2},\ \lambda_2 = 4.07\times10^{-2} \tag{11}$$

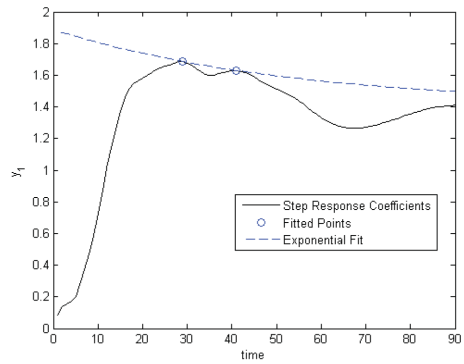

Fig. 1. Step Response Coefficients, Exponential Fit and Fitted Points for (y_1, u_1) pair.

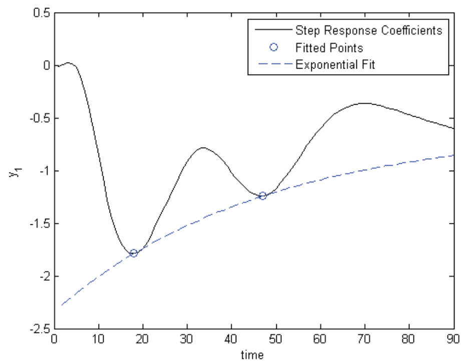

Fig. 2. Step Response Coefficients, Exponential Fit and Fitted Points for (y_1, d_1) pair.

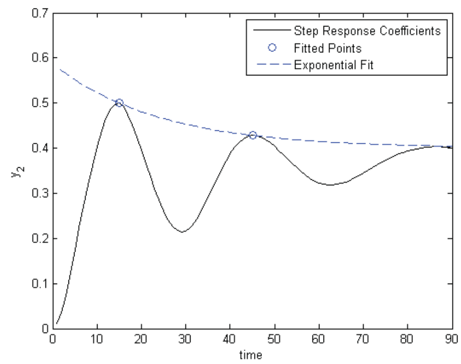

Fig. 3. Step Response Coefficients, Exponential Fit and Fitted Points for (y_2, d_1) pair.

Using the calculated amplitudes and decays, the following equation represents the funnel for each of the output variables:

$$f_i = \left(1+\alpha_f\right) a_i \exp\left[-\left(1+\beta_f\right)\lambda_i t\right] + y_{ss,i} \tag{12}$$

where $y_{ss,i}$ is one of the steady-state output constraints (upper or lower limit) for output i, which is calculated using the previously published interval operability approaches (Lima and Georgakis, 2008a; Lima and Georgakis, 2008b). Also, α_f

and β_f are adjustable tuning parameters, associated with amplitude and decay, respectively, and are independent of the output selected. As explained above for a_i, depending on the magnitude of the disturbance inserted, which in this case takes either its maximum or minimum value, $y_{ss,i}$ will have different values. For the case study here, if d_1 = 1, then $y_{ss,1}$ = -0.464 and $y_{ss,2}$ = 0.464. When d_1 = -1, $y_{ss,1}$ = 0.464 and $y_{ss,2}$ = -0.464. These values were extracted from the steady-state operability results presented in the ADCHEM 2006 paper by Lima and Georgakis (see case 2, section 3). Thus, each output envelope actually has upper and lower limits that start at different points and end at the upper and lower calculated steady-state constraints, respectively. The same decay holds for both cases. Therefore, for the 2-D case study above, selecting α_f = 0 and β_f = -0.74, the following funnels are obtained for each output for the two extreme values of the disturbance:

For $d_1 = 1$: $y_{ss,1} = -0.464$, $y_{ss,2} = 0.464$

$$f_1 = -0.6\exp\left[-(1-0.74)1.89\times10^{-2}t\right]-0.464 \quad (13)$$

$$f_2 = 0.4\exp\left[-(1-0.74)4.07\times10^{-2}t\right]+0.464 \quad (14)$$

For $d_1 = -1$: $y_{ss,1} = 0.464$, $y_{ss,2} = -0.464$

$$f_1 = 0.6\exp\left[-(1-0.74)1.89\times10^{-2}t\right]+0.464 \quad (15)$$

$$f_2 = -0.4\exp\left[-(1-0.74)4.07\times10^{-2}t\right]-0.464 \quad (16)$$

The funnels for outputs 1 (eqs. 13 and 15) and 2 (eqs. 14 and 16) are plotted in Figs. 5 and 6, respectively, along with the DMCplus™ (Dynamic Matrix Control - AspenTech, a multivariable constrained controller) trend obtained for each case. For all cases, the controller is operating in closed-loop mode and the disturbance was inserted at time = 0.

4. HIGH-DIMENSIONAL INDUSTRIAL SYSTEM

The design of output constraints during transient for the Steam Methane Reformer (SMR; Vinson, 2000) process example will now be performed using the output funnels defined above. This process has 9 outputs, 4 inputs and 1 disturbance variable and it is defined by the following set of steady-state equations and constraining sets (see information on process dynamics below):

$$\begin{pmatrix}\delta y_1\\ \delta y_2\\ \delta y_3\\ \delta y_4\\ \delta y_5\\ \delta y_6\\ \delta y_7\\ \delta y_8\\ \delta y_9\end{pmatrix} = \begin{pmatrix} 1.00 & 0 & 3.99 & 2.22\\ 2.50 & 0.25 & 16.51 & -11.80\\ -0.14 & 0 & -1.53 & -1.45\\ 0.55 & 0 & 2.76 & 0.69\\ -0.04 & 0.02 & 0 & 0\\ 2.34 & -0.66 & 36.71 & 5.09\\ 3.96 & -0.19 & 44.40 & 5.79\\ -0.04 & 0 & -3.11 & -1.71\\ 0.36 & 0.05 & 1.10 & -16.54\end{pmatrix}\begin{pmatrix}\delta u_1\\ \delta u_2\\ \delta u_3\\ \delta u_4\end{pmatrix} + \begin{pmatrix}0.40\\ 1.65\\ -0.15\\ 0\\ 0\\ 3.67\\ 4.44\\ -0.31\\ 0.11\end{pmatrix}\delta d_1 \quad (17)$$

$$\mathbf{AIS} = \left\{\begin{aligned}&\mathbf{u}\in\Re^4 \mid 10\le u_1\le 48;\ \ 60\le u_2\le 140;\\ &0.2\le u_3\le 2.0;\ \ -2.4\le u_4\le -0.7\end{aligned}\right\} \quad (18)$$

$$\mathbf{EDS} = \{d_1 \mid -4\le d_1\le 4\}$$

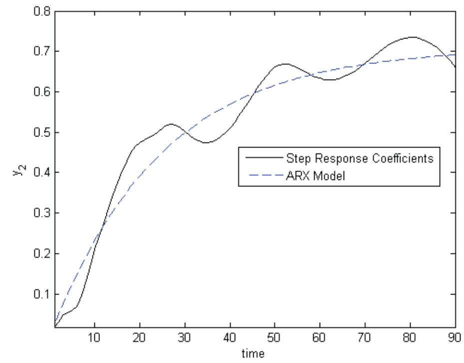

Fig. 4. Step Response Coefficients, Exponential Fit and First-order ARX Model for (y_2, u_1) pair.

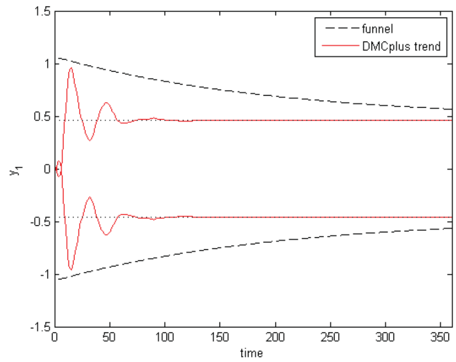

Fig. 5. Funnel Design and DMCplus trend for output y_1 with (α_f, β_f) = (0, -0.74).

where $\delta\mathbf{y}$, $\delta\mathbf{u}$ and δd_1 are deviation variables from the steady-state values for the outputs ($\mathbf{y}_{ss}$), the inputs ($\mathbf{u}_{ss}$), and the disturbance ($d_{1,ss}$), respectively. These steady-state values are given by:

$$\mathbf{y}_{ss} = (44.35,\ 94.10,\ 1.50,\ 21.5,\ 1.80,\ 431.45,\ 510.75,\ 5.35,\ 37.1)^T \quad (19)$$

$$\mathbf{u}_{ss} = (29.00,\ 100.00,\ 1.10,\ \text{-}1.55)^T;\ d_{1,ss} = 0$$

Also, the original output constraints (**DOS**), lower and upper limits, are given in Table 1. The SMR process has underdamped dynamics for several input-output/disturbance-output pairs, which are represented by the step response coefficients obtained by plant testing that are shown in Fig. 7. For this case, the disturbance gains, given in eq. (17), and the designed output constraints at the steady-state in Table 1 (from Lima, Georgakis, Smith, Vinson and Schnelle, 2009) will be used here to define each output funnel.

Following the same procedure as in the illustrative example above, exponential and ARX fits were obtained and λs calculated for all the input-output and disturbance-output pairs. The calculated λs for all these pairs are presented in Table 2, where the smallest absolute values of λ for each output, which will be used in the funnel design, are highlighted. Thus, using eq. (12), the funnel equations (20) and (21) are calculated for all outputs when d_1 moves from 0 to ± 4 (extreme cases) and (α_f, β_f) = (1.00, 0.36).

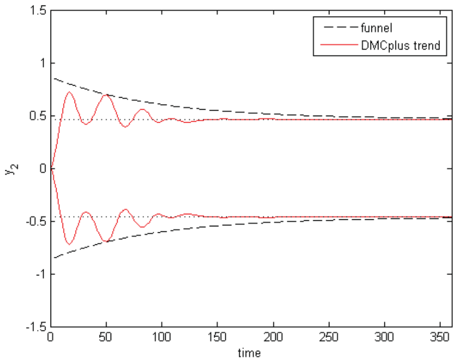

Fig. 6. Funnel Design and DMCplus trend for output y_2 with $(\alpha_f, \beta_f) = (0, -0.74)$.

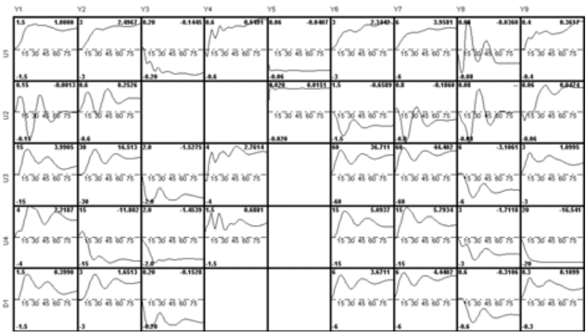

Fig. 7. Step Response Model for the SMR Problem. Responses for outputs y_1- y_9 to a step in inputs u_1 - u_4 and disturbance d_1. Empty boxes represent that there is no interaction between the input-output or disturbance-output pair.

Table 1. SMR Example: original and designed set of output constraints at the steady-state (Lima, Georgakis, Smith, Vinson and Schnelle, 2009).

Process Outputs	Original Lower Bound	Original Upper Bound	Designed Lower Bound	Designed Upper Bound
y_1	43.00	45.70	43.98	44.72
y_2	26.90	161.30	84.89	103.31
y_3	0.80	2.20	1.02	1.98
y_4	0	43.00	20.91	22.09
y_5	1.70	1.90	1.73	1.87
y_6	424.70	438.20	426.82	436.08
y_7	430.10	591.40	455.48	566.02
y_8	3.20	7.50	3.87	6.83
y_9	21.50	52.70	26.41	47.79

For $d_1 = 4$:

$$
\begin{aligned}
f_1 &= (1+1)1.60\exp\left[-(1+0.36)1.51\times10^{-2}t\right]+44.72\\
f_2 &= (1+1)6.60\exp\left[-(1+0.36)2.86\times10^{-2}t\right]+103.31\\
f_3 &= (1+1)(\text{-}0.60)\exp\left[-(1+0.36)1.40\times10^{-2}t\right]+1.02\\
f_4 &= 22.09\\
f_5 &= 1.87\\
f_6 &= (1+1)14.68\exp\left[-(1+0.36)1.95\times10^{-2}t\right]+436.08\\
f_7 &= (1+1)17.76\exp\left[-(1+0.36)2.21\times10^{-2}t\right]+566.02\\
f_8 &= (1+1)(\text{-}1.24)\exp\left[-(1+0.36)2.48\times10^{-2}t\right]+3.87\\
f_9 &= (1+1)0.44\exp\left[-(1+0.36)1.10\times10^{-2}t\right]+47.79
\end{aligned}
\tag{20}
$$

For $d_1 = -4$:

$$
\begin{aligned}
f_1 &= (1+1)(\text{-}1.60)\exp\left[-(1+0.36)1.51\times10^{-2}t\right]+43.98\\
f_2 &= (1+1)(\text{-}6.60)\exp\left[-(1+0.36)2.86\times10^{-2}t\right]+84.89\\
f_3 &= (1+1)0.60\exp\left[-(1+0.36)1.40\times10^{-2}t\right]+1.98\\
f_4 &= 20.91\\
f_5 &= 1.73\\
f_6 &= (1+1)(\text{-}14.68)\exp\left[-(1+0.36)1.95\times10^{-2}t\right]+426.82\\
f_7 &= (1+1)(\text{-}17.76)\exp\left[-(1+0.36)2.21\times10^{-2}t\right]+455.48\\
f_8 &= (1+1)1.24\exp\left[-(1+0.36)2.48\times10^{-2}t\right]+6.83\\
f_9 &= (1+1)(\text{-}0.44)\exp\left[-(1+0.36)1.10\times10^{-2}t\right]+26.41
\end{aligned}
\tag{21}
$$

Note that, for outputs y_4 and y_5, the steady-state disturbance gains are 0, and thus, their funnel's upper and lower bounds are constants at their upper and lower designed steady-state limits, respectively. Figs. 8, 9, and 10 show the DMCplus trends for y_1, y_2, and y_3, respectively, as well as the funnels for each case, where the controller is operating in closed-loop mode and the disturbance was inserted at time = 0. The funnels for the other outputs are not shown here due to space limitations of the manuscript.

Table 2. Calculated λs for each input-output and disturbance-output pairs for the SMR Example (smallest λ for each of the outputs in bold; dash (-) for pairs with no model)

y/ u, d	u_1	u_2	u_3	u_4	d_1
y_1	0.0189	0.0323	0.0212	**0.0151**	0.0212
y_2	0.0335	0.0699	0.0288	**0.0286**	0.0288
y_3	0.0794	-	**0.0140**	0.0847	**0.0140**
y_4	0.0537	-	0.0939	**0.0364**	-
y_5	1.4437	**0.5177**	-	-	-
y_6	0.0407	0.0453	0.0422	**0.0195**	0.0422
y_7	0.0530	0.0391	0.0681	**0.0221**	0.0681
y_8	**0.0248**	0.0280	0.0316	0.0706	0.0316
y_9	0.0243	0.0290	**0.0110**	0.1213	**0.0110**

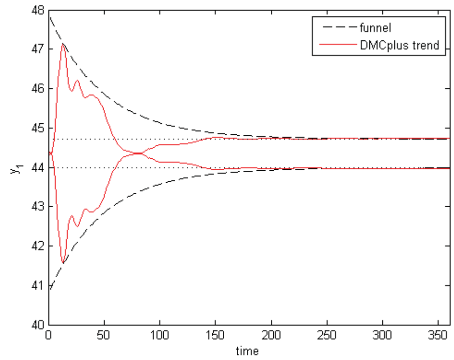

Figure 8: SMR Example: funnel design and DMCplus trend for output y_1 with $(\alpha_f, \beta_f) = (1.00, 0.36)$.

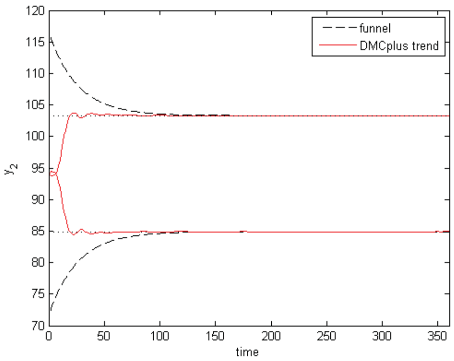

Figure 9: SMR Example: funnel design and DMCplus trend for output y_2 with $(\alpha_f, \beta_f) = (1.00, 0.36)$.

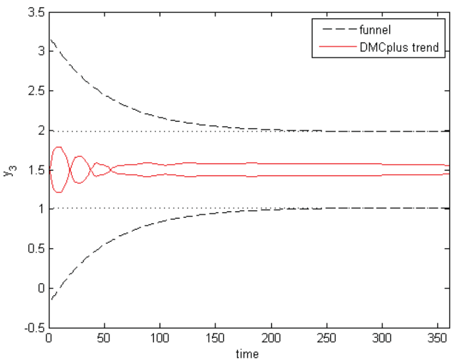

Figure 10: SMR Example: funnel design and DMCplus trend for output y_3 with $(\alpha_f, \beta_f) = (1.00, 0.36)$.

6. CONCLUSIONS

In this paper we have presented an extension of the previously developed steady-state interval operability approach to dynamical systems. Through the detailed examination of an illustrative case study we have motivated the calculation of output funnels for the design of output constraints during transient operation. The developed methodology was then applied to determine feasible output constraints for the Steam Methane Reformer industrial process. The analysis presented here provides a starting point for the verification of the achievability of control objectives in the entire control horizon. As potential future directions, an extension of this framework to address systems with multiple disturbances is necessary. Moreover, a moving horizon operability approach could be developed, where operability calculations would be performed online at each time instant, as the control horizon advances.

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