Distributed Model Predictive Control of Nonlinear Process Systems Subject to Asynchronous Measurements

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Abstract: In this work, we address distributed model predictive control of nonlinear process systems subject to asynchronous measurements. Assuming that there exists an upper bound on the interval between two successive measurements of the process state, two separate Lyapunov-based model predictive controllers that coordinate their actions and take asynchronous measurements explicitly into account are designed. The proposed distributed control design only requires one directional communication between the two distributed controllers and provides the potential of maintaining stability and performance in the face of new or failing actuators. The results are illustrated through a chemical process example.

Keywords: Distributed model predictive control; Nonlinear systems; Networked control systems; Process control; Asynchronous measurements.

1. INTRODUCTION

We are currently witnessing an augmentation of the existing, dedicated local control networks, with additional networked (wired and/or wireless) actuator/sensor devices which have become cheap and easy-to-install the last few years. Such an augmentation in sensor information and networked-based availability of data has the potential (Ydstie (2002); Neumann (2007); Christofides et al. (2007)) to be transformative in the sense of dramatically improving the ability of the control systems to optimize process performance and prevent or deal with abnormal situations more quickly and effectively. However, augmenting dedicated, local control systems (LCS) with control systems that may utilize real-time sensor and actuator networks gives rise to the need to design/redesign and coordinate separate control systems that operate on a process. Model predictive control (MPC) is a natural control framework to deal with the design of coordinated, distributed control systems because of its ability to handle input and state constraints, and also because it can account for the actions of other actuators in computing the control action of a given set of control actuators in real-time. Motivated by the lack of available methods for the design of networked control systems (NCS) for chemical processes, in a previous work (Liu et al. (2008)), we introduced a decentralized control architecture for systems with continuous and asynchronous measurements. In this architecture, the local, pre-existing control system uses continuous sensing and actuation and an explicit control law. On the other hand, the NCS uses networked (wired or wireless) sensors and actuators and has access to heterogeneous, asynchronous measurements that are not available to the LCS. The NCS is designed via Lyapunov-based model predictive control (LMPC). Following up on this work, in another recent work (Liu et al. (in press)), we proposed a distributed model predictive control method for the design of networked control systems where both the pre-existing local control system and the networked control system are designed via Lyapunov-based model predictive control.

With respect to available results on distributed MPC design, several distributed MPC methods have been proposed in the literature that deal with the coordination of separate MPC controllers that communicate in order to obtain optimal input trajectories in a distributed manner (Rawlings and Stewart (2007); Dunbar (2007); Richards and How (2007); Keviczky et al. (2006); Magni and Scattolini (2006); Raimondo et al. (2007)). All of the above results on distributed MPC design are based on the assumption of continuous sampling and perfect communication between the sensor and the controller. However, one may encounter asynchronous measurement samplings because of measurement difficulties in process control applications.

In this work, we address distributed model predictive control of nonlinear process systems subject to asynchronous measurements. Assuming that there exists an upper bound on the interval between two successive measurements of the process state, two separate Lyapunov-based model predictive controllers that coordinate their actions and

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take asynchronous measurements explicitly into account are designed. Sufficient conditions are derived for the proposed distributed control design to guarantee that the state of the closed-loop system is ultimately bounded in a region that contains the origin. In addition, the proposed distributed control design only requires one directional communication between the two distributed controllers and provides the potential of maintaining stability and performance in the face of new or failing actuators. The results are illustrated through a chemical process example.

2. PRELIMINARIES

2.1 Control problem formulation

We consider nonlinear process systems described by the following state-space model

$$\dot{x}(t) = f(x(t), u_1(t), u_2(t), w(t)) \tag{1}$$

where $x(t) \in R^{n_x}$ denotes the vector of process state variables, $u_1(t) \in R^{n_{u_1}}$ and $u_2(t) \in R^{n_{u_2}}$ are two separate sets of manipulated inputs and $w(t) \in R^{n_w}$ denotes the vector of disturbance variables. The two manipulated inputs are restricted to be in two nonempty convex sets $U_1 \subseteq R^{n_{u_1}}$ and $U_2 \subseteq R^{n_{u_2}}$ and the disturbance vector is bounded, i.e., $w(t) \in W$ where

$$W := \{w \in R^{n_w} \text{ s.t. } |w| \leq \theta, \theta > 0\}\,^{2}.$$

We assume that f is a locally Lipschitz vector function and $f(0, 0, 0, 0) = 0$. This means that the origin is an equilibrium point for the nominal system (system (1) with $w(t) = 0$ for all t) with $u_1 = 0$ and $u_2 = 0$. System (1) is controlled with the two sets of manipulated inputs u_1 and u_2, which could be multiple inputs of a system or a single input divided artificially into two terms (e.g., $\dot{x}(t) = \hat{f}(x(t), u(t), w(t))$ with $u(t) = u_1(t) + u_2(t)$).

2.2 Lyapunov-based controller

We assume that there exists a Lyapunov-based controller $u_1(t) = h(x(t))$ which satisfies the input constraint on u_1 for all x inside a given stability region and renders the origin of the nominal closed-loop system asymptotically stable with $u_2(t) = 0$. Using converse Lyapunov theorems (Massera (1956); Lin et al. (1996)), this assumption implies that there exist functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ of class $\mathcal{K}$ [3] and a continuously differentiable Lyapunov function V for the nominal closed-loop system that satisfy the following inequalities

$$\begin{aligned} \alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \\ \frac{\partial V(x)}{\partial x} f(x, h(x), 0, 0) \leq -\alpha_3(|x|) \\ |\frac{\partial V(x)}{\partial x}| \leq \alpha_4(|x|) \\ h(x) \in U_1 \end{aligned} \tag{2}$$

for all $x \in D \subseteq R^{n_x}$ where D is an open neighborhood of the origin. We denote the region Ω_ρ [4] $\subseteq D$ as the stability region of the closed-loop system under the control $u_1 = h(x)$ and $u_2 = 0$.

[2] $|\cdot|$ denotes Euclidean norm of a vector.
[3] A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.
[4] We use Ω_r to denote the set $\Omega_r := \{x \in R^{n_x} | V(x) \leq r\}$.

By continuity and the local Lipschitz property assumed for the vector field $f(x, u_1, u_2, w)$ and the fact that the manipulated inputs u_1 and u_2 are bounded in convex sets, there exists a positive constant M such that

$$|f(x, u_1, u_2, w)| \leq M \tag{3}$$

for all $x \in \Omega_\rho$, $u_1 \in U_1$, $u_2 \in U_2$ and $w \in W$. In addition, by the continuous differentiable property of the Lyapunov function V and the Lipschitz property assumed for the vector field $f(x, u_1, u_2, w)$, there exist positive constants L_x, R_x, R_w such that

$$|\frac{\partial V}{\partial x} f(x, u_1, u_2, 0) - \frac{\partial V}{\partial x} f(x', u_1, u_2, 0)| \leq L_x |x - x'| \tag{4}$$

and

$$|f(x, u_1, u_2, w) - f(x', u_1, u_2, 0)| \leq R_x |x - x'| + R_w |w| \tag{5}$$

for all $x, x' \in \Omega_\rho$, $u_1 \in U_1$, $u_2 \in U_2$ and $w \in W$.

These constants will be used in section 4 in the proof of the main results of the present work.

2.3 Modeling of asynchronous measurements

Most control systems assume that measurements from sensors are obtained in a continuous periodic pattern. However, in many chemical processes, this assumption does not hold due to a host of measurement difficulties. In this case, the system is subject to asynchronous measurements. In the present work, we assume the state of system (1), $x(t)$, is sampled and available asynchronously at time instants t_k where $\{t_{k \geq 0}\}$ is a random increasing sequence of times. The distribution of $\{t_{k \geq 0}\}$ characterizes the time needed to obtain a new measurement in the case of asynchronous measurements. In general, there exists the possibility of arbitrarily large (but finite) periods of time in which a new measurement is not available. In such a case, it is not possible to provide guaranteed stability properties, because there exists a non-zero probability that the system operates in open loop for a period of time large enough for the state to leave the stability region. In order to study the stability properties in a deterministic framework, in the present work, we assume that there exists an upper bound T_m on the interval between two successive measurements, i.e., $\max_k \{t_{k+1} - t_k\} \leq T_m$. This assumption is reasonable from a process control perspective.

3. DISTRIBUTED LMPC

3.1 Distributed LMPC formulations

In our previous work (Liu et al. (in press)), we introduced a distributed model predictive control method where both the pre-existing LCS and the NCS are designed via Lyapunov-based model predictive control as shown in Fig. 1. The LMPCs computing the input trajectories of the LCS (i.e., u_1) and the NCS (i.e., u_2) are referred to as LMPC 1 and LMPC 2, respectively. Under the assumption of continuous and flawless measurements, in Liu et al. (in press), it was proved that this control scheme guarantees practical stability of the closed-loop system and has the potential to maintain the closed-loop stability and performance in the face of new or failing actuators (for example, the failure of the actuator of the NCS (zero input) does not affect the closed-loop stability) and to reduce

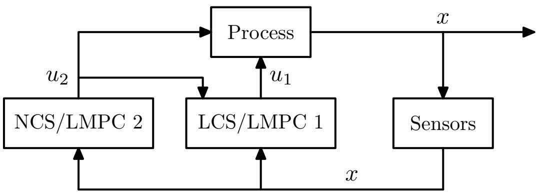

Fig. 1. Distributed LMPC design for networked control systems with continuous measurements (i.e., $x(t)$ is available to the controllers at $t_k = t_{k-1} + \Delta$ where Δ is a fixed sampling time for all k).

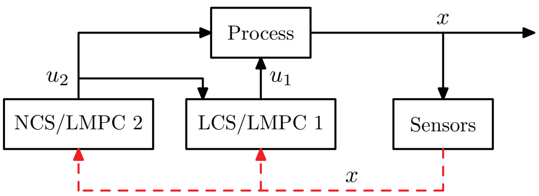

Fig. 2. Distributed LMPC design for networked control systems subject to asynchronous measurements.

computational burden in the evaluation of the optimal manipulated inputs compared with a centralized LMPC. However, when asynchronous measurements are present as shown in Fig. 2, these results do not hold. In this work, the distributed model predictive control method is extended to take into account asynchronous measurements explicitly, both in the constraints imposed on the LMPCs and in the implementation strategy.

In the presence of asynchronous measurements, the controllers need to operate in open-loop between successive new state measurements. We propose to take advantage of the model predictive control scheme to update the input based on a prediction obtained using the model. This is achieved by having the control actuators to store and implement the last computed optimal input trajectory. The proposed implementation strategy in the presence of asynchronous measurements is as follows:

(1) When a measurement $x(t_k)$ is available at t_k, LMPC 2 computes the optimal input trajectory of u_2;
(2) LMPC 2 sends the entire optimal input trajectory to its actuators and also sends the entire optimal input trajectory to LMPC 1.
(3) Once LMPC 1 receives the entire optimal input trajectory for u_2, it evaluates the future input trajectory of u_1;
(4) LMPC 1 sends the entire optimal input trajectory to its actuators.
(5) When a new measurement is received ($k = k+1$), go to step 1.

Note that in the proposed distributed scheme, only LMPC 2 is required to send its optimal input trajectory to LMPC 1 each time when a new measurement is available. This minimizes the communications required between the two controllers. Note also that the communication between LMPC 1 and LMPC 2 is in general done using a reliable link, and hence, it is not subject to data losses or delays.

We first design the optimization problem that charcterizes LMPC 2. This optimization problem depends on the latest state measurement $x(t_k)$, however, LMPC 2 does not have any information about the value that u_1 will take. In order to take a decision, LMPC 2 must assume a trajectory for u_1 along the prediction horizon. To this end, the Lyapunov-based controller $u_1 = h(x)$ is used. LMPC 2 is based on the following optimization problem:

$$\min_{u_{d2} \in S(\Delta)} \int_0^{N\Delta} L(\tilde{x}(\tau), u_{d1}(\tau), u_{d2}(\tau))d\tau \tag{6a}$$
$$\dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau), h(\tilde{x}(j\Delta)), u_{d2}(\tau), 0), \quad \forall \tau \in [j\Delta, (j+1)\Delta) \tag{6b}$$
$$\dot{\hat{x}}(\tau) = f(\hat{x}(\tau), h(\hat{x}(j\Delta)), 0, 0), \forall\, \tau \in [j\Delta, (j+1)\Delta) \tag{6c}$$
$$u_{d2}(\tau) \in U_2, \forall \tau \in [0, N\Delta) \tag{6d}$$
$$\tilde{x}(0) = \hat{x}(0) = x(t_k) \tag{6e}$$
$$V(\tilde{x}(\tau)) \leq V(\hat{x}(\tau)), \forall \tau \in [0, N_R\Delta) \tag{6f}$$

where $S(\Delta)$ is the family of piece-wise constant functions with sampling time Δ, N is the prediction horizon,

$$L(x, u_1, u_2) = x^T Q_c x + u_1^T R_{c1} u_1 + u_2^T R_{c2} u_2$$

is the performance index, Q_c, R_{c1} and R_{c2} are positive definite weight matrices that define the cost, $\tilde{x}$ is the predicted trajectory of the nominal system with u_2 being the input trajectory computed by the LMPC of Eq. 6 (i.e., LMPC 2) and u_1 being the Lyapunov-based controller h applied in a sample-and-hold fashion with $j = 0, ..., N-1$, $\hat{x}$ is the predicted trajectory of the nominal system with u_1 being h applied in a sample-and-hold fashion and $u_2 = 0$, $x(t_k)$ is the state measurement obtained at t_k and N_R is the smallest integer that satisfies the inequality $T_m \leq N_R\Delta$. To take full advantage of the nominal model in the computation of the control action, we take $N \geq N_R$.

The optimal solution to this optimization problem is denoted by $u_{d2}^*(\tau|t_k)$. Once this optimal input trajectory of u_2 is available, it is sent to LMPC 1 as well as the control actuators controlled by LMPC 1.

In order to inherit the stability properties of the Lyapunov based controller, u_2 must satisfy the constraint (6f) which guarantees that the predicted decrease of the Lyapunov function from t_k to $t_k + N_R\Delta$, if $u_1 = h(x)$ and $u_2 = u_{d2}^*$ are applied, is at least equal to the one obtained if the Lyapunov-based controller h is applied in a sample-and-hold fashion. Note that we have considered input constraints, see Eq. 6d.

The optimization problem of LMPC 1 depends on the latest state measurement $x(t_k)$ and the decision taken by LMPC 2 (i.e., u_{d2}^*). This allows LMPC 1 to compute an input u_1 such that the closed-loop performance is optimized, while guaranteeing that the stability properties of the Lyapunov-based controller are preserved. Specifically, LMPC 1 is based on the following optimization problem:

$$\min_{u_{d1} \in S(\Delta)} \int_0^{N\Delta} L(\check{x}(\tau), u_{d1}(\tau), u_{d2}(\tau))d\tau \tag{7a}$$
$$\dot{\check{x}}(\tau) = f(\check{x}(\tau), u_{d1}(\tau), u_{d2}(\tau), 0), \forall \tau \in [0, N\Delta) \tag{7b}$$
$$\dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau), h(\tilde{x}(j\Delta)), u_{d2}(\tau), 0), \quad \forall \tau \in [j\Delta, (j+1)\Delta) \tag{7c}$$
$$u_{d2}(\tau) = u_{d2}^*(\tau|t_k), \forall \tau \in [0, N\Delta) \tag{7d}$$
$$u_{d1}(\tau) \in U_1, \forall \tau \in [0, N\Delta) \tag{7e}$$
$$\check{x}(0) = \tilde{x}(0) = x(t_k) \tag{7f}$$
$$V(\check{x}(\tau)) \leq V(\tilde{x}(\tau)), \ \forall \tau \in [0, N_R\Delta) \tag{7g}$$

where $\check{x}$ is the predicted trajectory of the nominal system if $u_2 = u_{d2}^*$ and $u_1 = u_{d1}$ are applied, and $\tilde{x}$ is the predicted trajectory of the nominal system if $u_2 = u_{d2}^*$ and the Lyapunov-based controller h are applied in a sample-and-hold fashion.

The optimal solution to this optimization problem is denoted by $u_{d1}^*(\tau|t_k)$. The contractive constraint (7g) guarantees that the predicted decrease of the Lyapunov function from t_k to $t_k + N_R\Delta$, if $u_1 = u_{d1}^*$ and $u_2 = u_{d2}^*$ are applied, is at least equal to the one obtained when $u_1 = h(x)$ and $u_2 = u_{d2}^*$ are applied.

Note that the trajectory $\tilde{x}(\tau)$ predicted by constraint (7c) is the same optimal trajectory predicted by LMPC 2. This trajectory and the two contractive constraints (6f) and (7g) allow proving the closed-loop stability properties of the proposed controller.

The manipulated inputs of the proposed control scheme are defined as follows:

$$\begin{aligned} u_1(t) &= u_{d1}^*(t - t_k|t_k),\ \forall t \in [t_k, t_{k+1}) \\ u_2(t) &= u_{d2}^*(t - t_k|t_k),\ \forall t \in [t_k, t_{k+1}). \end{aligned} \quad (8)$$

Note that, as explained before, the controllers apply the last evaluated optimal input trajectory between two successive state measurements.

4. STABILITY PROPERTIES

In this section, we present the stability properties of the proposed distributed control scheme. We prove that the contractive constraints (6f) and (7g) guarantee that the proposed distributed control scheme inherits the stability properties of the Lyapunov-based controller (implemented in sample and hold and using the model to estimate the state of the system when a new measurement is not available). This property is presented in Theorem 1 below. To state this theorem, we need the following propositions.

Proposition 1. (c.f. Muñoz de la Peña and Christofides (2008)). *Consider the nominal sampled trajectory $\hat{x}$ of system (1) in closed-loop with the Lyapunov-based controller h applied in a sample-and-hold fashion and $u_2(t) = 0$. Let $\Delta, \epsilon_s > 0$ and $\rho > \rho_s > 0$ satisfy*

$$-\alpha_3(\alpha_2^{-1}(\rho_s)) + \alpha_4(\alpha_1^{-1}(\rho))L_x M\Delta \leq -\epsilon_s/\Delta. \quad (9)$$

Then, if $\rho_{\min} < \rho$ where

$$\rho_{min} = \max\{V(\hat{x}(t+\Delta)) : V(\hat{x}(t)) \leq \rho_s\} \quad (10)$$

and $\hat{x}(0) \in \Omega_\rho$, the following inequality holds

$$V(\hat{x}(k\Delta)) \leq \max\{V(\hat{x}(0)) - k\epsilon_s, \rho_{min}\}. \quad (11)$$

Proposition 1 ensures that if system (1) with $w(t) = 0$ for all t under the control law $u_1 = h(x)$ implemented in a sample-and-hold fashion and $u_2 = 0$ starts in Ω_ρ, then it is ultimately bounded in $\Omega_{\rho_{\min}}$. The following proposition provides an upper bound on the deviation of the state trajectory obtained using the nominal model, from the real-state trajectory when the same control input trajectories are applied.

Proposition 2. (c.f. Liu et al. (2008)). *Consider the following state trajectories*

$$\begin{aligned} \dot{x}_a(t) &= f(x_a(t), u_1(t), u_2(t), w(t)) \\ \dot{x}_b(t) &= f(x_b(t), u_1(t), u_2(t), 0) \end{aligned} \quad (12)$$

with initial states $x_a(t_0) = x_b(t_0) \in \Omega_\rho$. There exists a class $\mathcal{K}$ function $f_W(\cdot)$ such that

$$|x_a(t) - x_b(t)| \leq f_W(t - t_0), \quad (13)$$

for all $x_a(t), x_b(t) \in \Omega_\rho$ and all $w(t) \in W$ with

$$f_W(\tau) = \frac{R_w\theta}{R_x}(e^{R_x\tau} - 1).$$

The following proposition bounds the difference between the magnitudes of the Lyapunov function of two different states in Ω_ρ.

Proposition 3. (c.f. Liu et al. (2008)). *Consider the Lyapunov function $V(\cdot)$ of system (1). There exists a quadratic function $f_V(\cdot)$ such that*

$$V(x) \leq V(\hat{x}) + f_V(|x - \hat{x}|) \quad (14)$$

for all $x, \hat{x} \in \Omega_\rho$ with

$$f_V(s) = \alpha_4(\alpha_1^{-1}(\rho))s + Ms^2.$$

In Theorem 1 below, we provide sufficient conditions under which the proposed distributed LMPC design (8) guarantees the closed-loop stability of system (1) in the presence of asynchronous measurements.

Theorem 1. Consider system (1) in closed-loop with the distributed LMPC design (8) based on a controller $h(x)$ that satisfies (2). Let $\Delta, \epsilon_s > 0$, $\rho > \rho_{min} > 0$, $\rho > \rho_s > 0$ and $N \geq N_R \geq 1$ satisfy (9),(10) and the following inequality

$$-N_R\epsilon_s + f_V(f_W(N_R\Delta)) < 0. \quad (15)$$

If $x(t_0) \in \Omega_\rho$, then $x(t)$ is ultimately bounded in $\Omega_{\rho_c} \subseteq \Omega_\rho$ where

$$\rho_c = \rho_{min} + f_V(f_W(N_R\Delta)).$$

Proof: In order to prove that the closed-loop system is ultimately bounded in a region that contains the origin, we will prove that $V(x(t_k))$ is a decreasing sequence of values with a lower bound.

The proof consists of two parts. In the first part, we will prove that the stability results stated in Theorem 1 hold for the case where $t_{k+1} - t_k = T_m$ for all k and $T_m = N_R\Delta$. The proof of the stability results for the general case, that is $t_{k+1} - t_k \leq T_m$ for all k and $T_m \leq N_R\Delta$, will be shown in the second part.

Part 1: In this part, we prove that the stability results stated in Theorem 1 hold in the case that $t_{k+1} - t_k = T_m$ for all k and $T_m = N_R\Delta$. This case corresponds to the worst possible situation in the sense that LMPC 1 and LMPC 2 need to operate in open-loop for the maximum possible amount of time.

In order to simplify the notation, we will denote $\tilde{x}(t)$ the nominal closed-loop trajectory of system (1) with $u_1 = h$ implemented in a sample-and-hold fashion and $u_2 = u_{d2}^*$ from $x(t_k)$, $\hat{x}(t)$ the nominal closed-loop trajectory of system (1) under the Lyapunov-based controller $u_1 = h$ implemented in a sample-and-hold fashion and $u_2 = 0$ from $x(t_k)$, and denote $\check{x}(t)$ the nominal closed-loop trajectory of system (1) with $u_1 = u_{1d}^*$ and $u_2 = u_{2d}^*$ from $x(t_k)$.

By Proposition 1 and the fact that $t_{k+1} = t_k + N_R\Delta$, the following inequality can be obtained:

$$V(\hat{x}(t_{k+1})) \leq \max\{V(\hat{x}(t_k)) - N_R\epsilon_s, \rho_{\min}\}. \quad (16)$$

From the contractive constraints (6f) and (7g) in LMPC 2 and LMPC 1, the following inequality can be written:

$$V(\check{x}(t)) \leq V(\tilde{x}(t)) \leq V(\hat{x}(t)), \forall t \in [t_k, t_k + N_R\Delta). \quad (17)$$

From inequalities (16) (17) and taking into account that $\hat{x}(t_k) = \tilde{x}(t_k) = \check{x}(t_k) = x(t_k)$, the following inequality is obtained:

$$V(\check{x}(t_{k+1})) \leq \max\{V(x(t_k)) - N_R\epsilon_s, \rho_{\min}\}. \quad (18)$$

When $x(t) \in \Omega_\rho$ for all times (this point will be proved below), we can apply Proposition 3 to obtain the following inequalities:

$$\begin{aligned} V(x(t_{k+1})) &\leq V(\check{x}(t_{k+1})) \\ &\quad + f_V(|\check{x}(t_{k+1}) - x(t_{k+1})|). \end{aligned} \quad (19)$$

Applying Proposition 2 we obtain the following upper bound on the deviation of $\check{x}(t)$ from $x(t)$:

$$|x(t_{k+1}) - \check{x}(t_{k+1})| \leq f_W(N_R\Delta) \quad (20)$$

From inequalities (19) and (20), the following upper bound on $V(x(t_{k+1}))$ can be written:

$$V(x(t_{k+1})) \leq V(\check{x}(t_{k+1})) + f_V(f_W(N_R\Delta)). \quad (21)$$

Using inequality (18), we can re-write inequality (21) as follows:

$$\begin{aligned} V(x(t_{k+1})) \leq \; &\max\{V(x(t_k)) - N_R\epsilon_s, \rho_{\min}\} \\ &+ f_V(f_W(N_R\Delta)). \end{aligned} \quad (22)$$

If condition (15) is satisfied, from inequality (22), we know there exists $\epsilon_w > 0$ such that the following inequality holds:

$$V(x(t_{k+1})) \leq \max\{V(x(t_k)) - \epsilon_w, \rho_c\} \quad (23)$$

which implies that if $x(t_k) \in \Omega_\rho / \Omega_{\rho_c}$, then $V(x(t_{k+1})) < V(x(t_k))$, and if $x(t_k) \in \Omega_{\rho_c}$, then $V(x(t_{k+1})) < \rho_c$. Using inequality (23) recursively, it is proved that if $x(t_0) \in \Omega_\rho$, the closed-loop trajectories of system (1) under the proposed distributed LMPC design (8) satisfy

$$\limsup_{t \to \infty} V(x(t)) \leq \rho_c.$$

This proves that the closed-loop system is ultimately bounded in Ω_{ρ_c} for the case where $t_{k+1} - t_k = T_m$ for all k and $T_m = N_R\Delta$.

Part 2: In this part, we extend the results proved in Part 1 to the general case, that is, $t_{k+1} - t_k \leq T_m$ for all k and $T_m \leq N_R\Delta$ which implies that $t_{k+1} - t_k \leq N_R\Delta$. The proof is divided into two cases. The first case is that $t_{k+1} - t_k \leq \Delta$. In this case, the stability results hold as shown in Liu et al. (in press). The second case is that $\Delta < t_{k+1} - t_k \leq N_R\Delta$. Because f_V and f_W are convex and strictly increasing functions of their arguments (see Propositions 2 and 3 for the expressions of f_V and f_W) and following similar steps in Part 1, we can show that inequality (22) still holds. This proves that the stability results stated in Theorem 1 hold.

5. APPLICATION TO A CHEMICAL PROCESS

The process considered in this example is a three vessel, reactor-separator process consisting of two continuously stirred tank reactors (CSTRs) and a flash tank separator.A feed stream to the first CSTR F_{10} contains the reactant A which is converted into the desired product B. The desired product B can then further react into an undesired side-product C. The effluent of the first CSTR along with additional fresh feed F_{20} makes up the inlet to the second CSTR. The reactions $A \rightarrow B$ and $B \rightarrow C$ (referred to as 1 and 2, respectively) take place in the two CSTRs in series before the effluent from CSTR 2 is fed to a flash tank. The overhead vapor from the flash tank is condensed and recycled to the first CSTR and the bottom product stream is removed. A small portion of the overhead is purged before being recycled to the first CSTR. All the three vessels are assumed to have static holdup. The dynamic equations describing the behavior of the system, obtained through material and energy balances under standard modeling assumptions, can be found in Liu et al. (in press).

Each of the tanks has an external heat input. The manipulated inputs to the system are the heat inputs, Q_1, Q_2 and Q_3, and the feed stream flow rate to vessel 2, F_{20}.

The process was numerically simulated using a standard Euler integration method. Process noise was added to simulate disturbances/model uncertainty and it was generated as autocorrelated noise of the form $w_k = \phi w_{k-1} + \xi_k$ where $k = 0, 1, \ldots$ is the discrete time step of 0.001 hr, ξ_k is generated by a normally distributed random variable with standard deviation σ_p, and ϕ is the autocorrelation factor and w_k is bounded by θ_p, that is $|w_k| \leq \theta_p$.

We assume that the measurements of the temperatures T_1, T_2, T_3 and the measurements of mass fractions x_{A1}, x_{B1}, x_{A2}, x_{B2}, x_{A3}, x_{B3} are available asynchronously at time instants $\{t_{k\geq 0}\}$ with an upper bound $T_m = 3\Delta$ on the maximum interval between two successive measurements, where Δ is the controller sampling time and chosen to be $\Delta = 0.02\ hr = 1.2\ min$.

For each set of steady-state inputs Q_{1s}, Q_{2s}, Q_{3s} and F_{20s} corresponding to a different operating condition, the process has one steady-state x_s. The control objective is to steer the process to the steady state

$$x_s^T = [0.61, 0.39, 425.9, 0.61, 0.39, 422.6, 0.35, 0.63, 427.3].$$

The process belongs to the following class of nonlinear systems

$$\dot{x}(t) = f(x(t)) + g_1(x(t))u_1(t) + g_2(x(t))u_2(t) + w(t)$$

where $x^T = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9] = [x_{A1} - x_{A1s}\ x_{B1} - x_{B1s}\ T_1 - T_{1s}\ x_{A2} - x_{A2s}\ x_{B2} - x_{B2s}\ T_2 - T_{2s}\ x_{A3} - x_{A3s}\ x_{B3} - x_{B3s}\ T_3 - T_{3s}]$ is the state, $u_1^T = [u_{11}\ u_{12}\ u_{13}] = [Q_1 - Q_{1s}\ Q_2 - Q_{2s}\ Q_3 - Q_{3s}]$ and $u_2 = F_{20} - F_{20s}$ are the manipulated inputs which are subject to the constraints $|u_{1i}| \leq 10^6\ KJ/hr$ $(i = 1, 2, 3)$ and $|u_2| \leq 3\ m^3/hr$, and $w = w_k$ is a time varying noise.

To illustrate the theoretical results, we first design the Lyapunov-based controller $u_1 = h(x)$ which can stabilize the closed-loop system and the explicit expression of the controller can be found in Liu et al. (in press). We consider a Lyapunov function $V(x) = x^T P x$ with P being the following weight matrix

$$P = diag^{\,5}\left(5.2 \times 10^{12}\left[4\ 4\ 10^{-4}\ 4\ 4\ 10^{-4}\ 4\ 4\ 10^{-4}\right]\right).$$

The values of the weights in P have been chosen in a way such that the Lyapunov-based controller $h(x)$ satisfies the input constraints, stabilizes the closed-loop system and provides good closed-loop performance.

Based on the Lyapunov-based controller $h(x)$, we design LMPC 1 and LMPC 2. The prediction horizons of both LMPC 1 and LMPC 2 are chosen to be $N = 6$ and N_R is

[5] $diag(v)$ denotes a matrix with its diagonal elements being the elements of vector v and all the other elements being zeros.

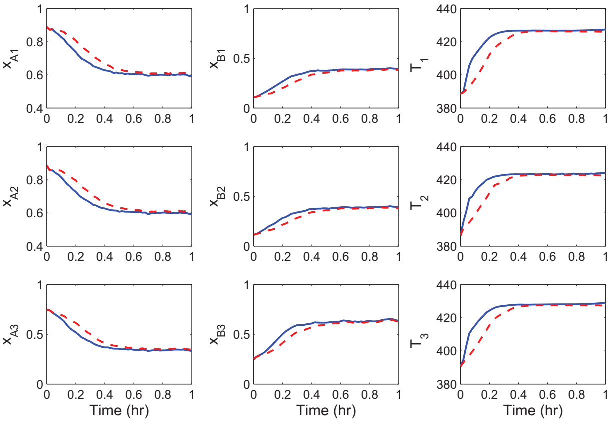

Fig. 3. State trajectories of the process under the proposed distributed LMPC design (8) (solid lines) and the original distributed LMPC design in Liu et al. (in press) (dashed lines) under continuous measurements.

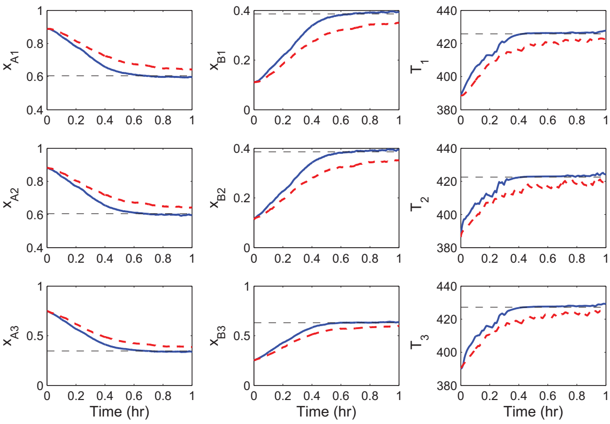

Fig. 4. State trajectories of the process under the proposed distributed LMPC design (8) (solid lines) and the original distributed LMPC design in Liu et al. (in press) (dashed lines) in the presence of asynchronous measurements.

chosen to be 4 so that $N_R\Delta \geq T_m$. The weight matrices for the LMPC designs are chosen as: $Q_c = diag(10^3 Q_v)$ with $Q_v = [2\ 2\ 0.0025\ 2\ 2\ 0.0025\ 2\ 2\ 0.0025]$, $R_{c1} = diag([5 \cdot 10^{-12}\ 5 \cdot 10^{-12}\ 5 \cdot 10^{-12}])$ and $R_{c2} = 100$.

We first carried out simulations to compare the proposed distributed LMPC design (8) with the original distributed LMPC design in Liu et al. (in press) in the case where no asynchronous measurements are present (i.e., state measurements $x(t_k)$ are available continuously with the interval between two successive measurements being Δ). The state trajectories under the two control designs are shown in Fig. 3. From Fig. 3, we can see that both the proposed and the original distributed LMPC designs stabilize the closed-loop system at the desired steady state.

We also carried out another set of simulations to compare both control laws in the presence of asynchronous measurements. To model the time sequence $\{t_{k\geq 0}\}$, we use an upper bounded random Poisson process. The Poisson process is defined by the number of events per unit time W. The interval between two successive concentration sampling times (events of the Poisson process) is given by $\Delta_a = \min\{-ln\chi/W, T_m\}$, where χ is a random variable with uniform probability distribution between 0 and 1. This generation ensures that $\max_k\{t_{k+1} - t_k\} \leq T_m$. In this example, W is chosen to be $W = 20$. The state trajectories of the system in closed-loop with both controllers are shown in Fig. 4. From Fig. 4, we can see that the proposed distributed LMPC design, which takes into account asynchronous measurements explicitly, can stabilize the closed-loop state at the desired steady state; however, the original distributed LMPC design failed to drive the closed-loop state to the desired steady state.

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