# ACTIVE CONTROL STRATEGY FOR DENSITY-WAVE IN GAS-LIFTED WELLS

**Laure Sinègre** [*] **Nicolas Petit** [*]
**Thierry Saint-Pierre** [**] **Pierre Lemétayer** [**]

[*] *CAS, École des Mines de Paris, France*
[**] *CSTJF, TOTAL Exploration-Production, Pau, France*

Abstract: We focus on the control of gas-lifted wells in the context of instable flows. Two cases are considered: casing-heading and density-wave. While it is known that active control can stabilize the casing-heading phenomenon, (passive) hardware upgrading solutions are sometimes preferred. In this paper, we advocate active control solutions in contrast to these strategies. Our aim is to stress that density-wave, which is a complicated issue not addressed by hardware solutions yet, can also be stabilized by the same simple control strategies that proved successful against casing-headings.

Keywords: Process Control, Gas-Lifted Well, Density-wave, Stabilization.

## 1. INTRODUCTION

Producing oil from deep reservoirs and lifting it through wells to surface facilities often requires activation to maintain oil output at a commercial level. In the gas-lift activation technique (Brown, 1973), gas is injected at the bottom of the well through the injection valve (point C in Figure 1) to lighten up the fluid column and to lower the gravity pressure losses. High pressure gas is injected at well head through the gas valve (point A), then goes down into the annular space between the drilling pipe (casing, point B) and the production pipe (tubing, point D) where it enters. Oil produced from the reservoir (point F) and injected gas mix in the tubing. They flow through the production valve (point E) located at the surface.

As wells and reservoirs get older, liquid rates begin to decrease letting wells be more sensitive to flow instabilities commonly called headings. These induce important oil production losses (see (Hu and Golan, 2003)) along with possible facilities damages. The best identified instability is the “casing-heading”. It consists of a succession of pressure build-up phases in the casing without production and high flow rate phases due to intermittent gas injection rate from the casing to the tubing (see (Jansen *et al.*, 1999) or (Torre *et al.*, 1987) for a complete description). Yet, keeping the gas injection constant in the tubing does not always prevent the instability. It has been pointed out in (Hu and Golan, 2003) that headings still occur on wells equipped with NOVA valves, i.e. valves maintaining the flow critical. In such a case one refers to the density-wave instability. In details, even though the gas injection rate through valve C is kept constant, self-sustained oscillations, confined in the tubing D can occur. Out-of-phase effects between the well influx and the total pressure drop along the tubing are usually reported at the birth of this phenomenon. More details about modelling under the form of a distributed delay system can be found in (Sinègre *et al.*, 2005).

Interestingly, almost all casing-heading control strategies aim at maintaining the gas flow rate injected in the tubing at a given set-point. In practice, under the assumption of a constant well head gas (in-)flow rate, stabilizing the casing head pressure achieves this goal. One can find details in (der Kinderen *et al.*, 1998) and also in (Eikrem and Golan, 2002) where the more advanced case of two interconnected wells is addressed.

Hardware upgrades to the NOVA valves are sometimes preferred to such active feedback control strategies. Technically, the valves track a critical flow point. This implies that flow does not depend on downstream pressure. Decoupling is thus achieved, and casing-heading stabilization is guaranteed.

Yet, further feedback control strategies have emerged. Another idea is to stabilize the pressure at the bottom of the well. As measurements at such depths are often not reliable and sometimes even not available at all, the need for estimators is critical. In (Eikrem *et al.*, 2004) example of stabilization relying on downhole pressure estimation is given. The controller relies on downhole pressure measurement and can handle sensor failures. Up to the authors' knowledge, when the well head pressures are the only measured variables, controlling the casing head pressure is the only proposed strategy.

We believe that even though very effective for casing-heading phenomenon, hardware upgrading solutions do not address all the instabilities of gas-lifted wells yet. To illustrate our point, we focus on the density-wave phenomenon. While it is known since (Hu and Golan, 2003) that density-wave on NOVA valve equipped wells can occur, we demonstrate that the original simple feedback control strategy of casing head pressure setpoint tracking does stabilize the well.

Controlling the density-wave phenomenon is studied in (Hu and Golan, 2003) and implicitly in (Dalsmo *et al.*, 2002). In both cases manipulating the production choke is used to stabilize the downhole pressure. The promising results at Brage field are reported in (Dalsmo *et al.*, 2002). Although the density-wave is not explicitly mentioned, they state that the slugging is not caused by casing-heading. They also stress that the strategy is efficient as long as the downhole pressure sensor works properly. Unfortunately, technical issues and high cost premiums usually prevent the use of the sensors required for real-time control purposes. In this paper we aim at showing that it is possible to control the density-wave using only well head measurements. We show that the control strategy described for casing-heading, i.e. stabilization of the casing head pressure through production choke actuation, is also efficient in the density-wave case. This is the contribution of our paper.

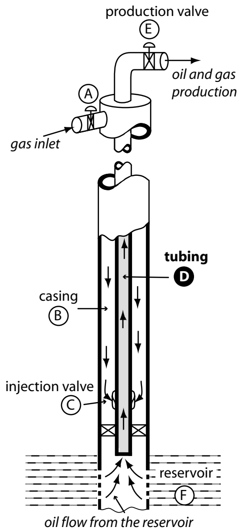


Fig. 1. Gas-lift activated well. Casing-heading involves both tubing D and casing B while density-wave takes place in the tubing D.

The article is organized as follows. In Section 2, we detail the model of controlled gas-lifted wells. This model implies an ordinary differential equation coupled to a distributed parameters system with boundary control. In Section 3, we propose a control strategy and prove local convergence. In Section 4, we give OLGA®2000 simulation results that illustrate the relevance of the approach. Conclusions and future directions are given in Section 5.

## 2. MODELLING

In this section, we present a gas-lifted well model. First, we detail the casing and tubing subsystems and their interconnection by feedback loops. Then, we explain through OLGA®2000 simulations why we choose the well head pressure as control variable.

### *2.1 Gas-lifted well modelling*

*Casing model* The well is divided in two parts. Nomenclature is given in Table 1. The annular part, called casing, can be considered as a tank filled with gas. The dynamics is simply represented by a mass balance equation

$$\dot{x} = w_{\bullet\bullet} - w_{\bullet\bullet} \tag{1}$$

where $w_{\bullet\bullet}$ is the gas inlet and $w_{\bullet\bullet}$ the gas outlet. The expression of $w_{\bullet\bullet}$ with respect to upstream and downstream pressures, respectively $P_{\bullet\bullet}$ and $P_{\bullet\bullet}$, is given by

$$w_{\bullet\bullet} \triangleq C_{\bullet\bullet}\sqrt{\max(0, \rho_{\bullet\bullet}(P_{\bullet\bullet} - P_{\bullet\bullet}))}$$

Assuming that the gas is ideal and that the column is at equilibrium state, we get

$$\rho_{\bullet\bullet} \triangleq \alpha x \text{ and } P_{\bullet\bullet} \triangleq \beta x$$

where $\alpha$ and $\beta$ are defined by

$$\beta = \alpha RT \triangleq \frac{g}{S_\bullet} \frac{1}{1 - \exp\left(-\frac{\bullet\bullet_a}{\bullet\bullet}\right)}$$

The casing is considered as a one-dimensional system of length $L$, which state is the gas mass, $x$, with two inputs $P_{\bullet\bullet}$ and $w_{\bullet\bullet}$

$$\dot{x} = w_{\bullet\bullet} - C_{\bullet\bullet}\sqrt{\max(0, \alpha x(\beta x - P_{\bullet\bullet}))} \qquad (2)$$

*Tubing model* Following (Imsland, 2002), we could use the gas and the oil masses as states and then model the tubing dynamics by two balance equations. The system resulting from the coupling of this model and the casing model accurately reports the casing-heading instability. Yet, it can not represent the density-wave phenomenon, which originates in the propagation of the gas mass fraction. For that purpose, we use the model presented in (Sinègre *et al.*, 2005).

Mass conservation laws along with proper choice of slip velocity law (see (Cholet, 2000) and (Duret, 2005)) yield the existence of a Riemann invariant (as defined in (Chorin and Marsden, 1990)) being the gas mass fraction. We assume that the gas is ideal and that no phase change occurs. Following (Asheim, 1988), we neglect transient inflow from the reservoir as well as acceleration and friction terms in Bernoulli's law. In other words we assume the flow to be dominated by gravitational effects. Furthermore, for sake of simplicity, we approximate the gas mass fraction by the gas volume fraction.

Under these assumptions, the tubing model writes under the integral form

$$P_{\bullet\bullet} = P_0 + \rho_\bullet g L + \int_0^\bullet k(\zeta)\left(1 - \frac{P_\bullet - P_{\bullet\bullet}(t-\zeta)}{\lambda w_{\bullet\bullet}(x(t-\zeta), P_{\bullet\bullet}(t-\zeta))}\right) d\zeta \qquad (3)$$

where $\tau \triangleq L/V_\bullet$ is the propagation delay. The right hand side is the sum of $P_0 + \rho_\bullet g L$ which corresponds to the weight of the column full of oil, and an integral which corresponds to the lightening effect of the gas. This (convolution) integral consists of the product of the propagating gas mass fraction by a negative function $k$ with finite support, which is proportional to the difference of density between gas and oil. The expression of $k$ over $[0, \tau]$ is given by

$$k(t) \triangleq V_\bullet g\left(\frac{tP_0 + (\tau - t)P_\bullet}{\tau RT} - \rho_\bullet\right) < 0$$

Notice that $k$ is a strictly decreasing affine function. For sake of simplicity, we shall write from now on

$$k(t) = (k_1 t + k_2)\mathbf{1}_{[0\bullet\bullet]} \qquad (4)$$

where $\mathbf{1}_{[0\bullet\bullet]}$ is zero over the entire real line except for the interval $[0, \tau]$ where it is equal to 1.

*Gas-lifted well model* Coupling equations (2) and (3) gives

$$\begin{cases} \dot{x} = w_{\bullet\bullet} - w_{\bullet\bullet}(x, P_{\bullet\bullet}) \\ P_{\bullet\bullet} = P^{\bullet}_{\bullet\bullet} + k * \left(1 - \frac{P_\bullet - P_{\bullet\bullet}}{\lambda w_{\bullet\bullet}(x, P_{\bullet\bullet})}\right) \end{cases} \qquad (5)$$

The state is $(x, P_{\bullet\bullet})$, where $P_{\bullet\bullet}$ is a function mapping $[0, \tau]$ onto $\mathbb{R}$. The considered output is $x$. In practice, $x$ is proportional to the well head casing pressure, which is actually measured. So far, the input corresponding to the production choke does not appear in model (5). Since manipulating this choke has a direct impact on the well head tubing pressure, one can assume that the input is $P^{\bullet}_{\bullet\bullet} \triangleq P_0 + \rho_\bullet g L$. We stress the relevance of this approach in section 2.2.

A gas-lifted well consists of two coupled subsystems. On one hand is the casing with inputs $w_{\bullet\bullet}$ and $P_{\bullet\bullet}$ and output $w_{\bullet\bullet}$. On the other hand is the tubing with inputs $w_{\bullet\bullet}$ and $P^{\bullet}_{\bullet\bullet}$ and output $P_{\bullet\bullet}$. This structure is reported in Figure 2. The two possibly positive feedback loops are at the birth of instabilities. The first loop appears in the tubing, it corresponds to the self-correlation of $P_{\bullet\bullet}$ detailed in (3). This internal loop creates the density-wave. On the other hand, the casing-heading arises from the coupling of these two subsystems via the explicit feedback loop in (5).

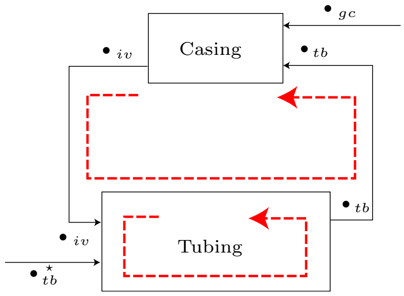


Fig. 2. Block scheme of the gas-lifted well model. The system consists of two coupled subsystem. The two arrows stand for possibly positive feedback loops, yielding instabilities.

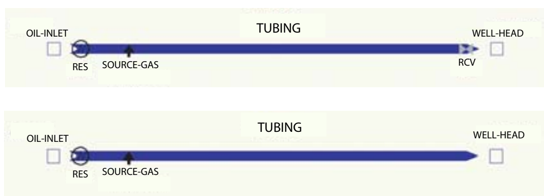


Fig. 3. Block scheme of the OLGA®2000 simulation setup. First case (top) with a production choke, second case (bottom).

### 2.2 Manipulated variable definition

We now investigate the role of the tubing well head pressure as input variable. With OLGA®2000 we consider two setups simulating the flow in a single vertical pipe (see Figure 3). Oil is supplied by a reservoir and gas is injected at the bottom of the pipe. In the first setup, the pipe is equipped with a production choke that we progressively open. In the second setup, there is no production choke. Instead, the tubing is modelled as a pipe with a downstream pressure boundary condition. Gradually, we decrease this boundary pressure, simulating a reduction of the well head pressure.

Figure 4 shows the steady state well head pressure values as a function of the production choke opening. Classically, our focus is on comparing the oil and gas velocities histories obtained from the two simulation setups. Figure 5 reports the static values of the oil and gas velocities as a function of the well head pressure. Over almost the whole well head pressure operating range (from 23 to 29 bar, i.e. from 0.2 to 1 choke opening), the curves coincide. It is only when the choke is almost closed that differences appear. Figure 6 shows the comparison of the step responses to an increase of the well head pressure and to a consistent decrease of the production choke opening, respectively. We notice similar undershoots of approximately 0.02 m/s. It takes between four and five noticeable oscillations for both systems to settle. This experiment suggest it is valid to consider $P_0 + \rho_\bullet g L$ as our input variable. From now on, we denote $u \triangleq P_{\bullet\bullet}^{\bullet}$.

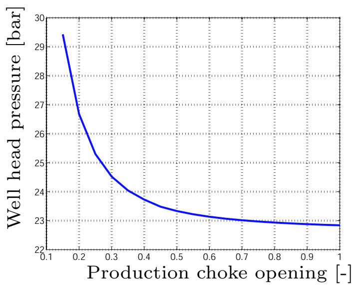


Fig. 4. Well head pressure as a function of the production choke opening.

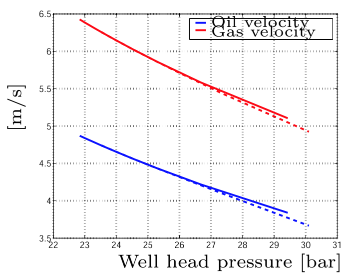


Fig. 5. Comparison of the oil and gas velocities between the fist and second cases (continuous and dashed line respectively).

Table 1. Nomenclature

| Symb. | | Values | Units |
|---|---|---|---|
| $R$ | Ideal gas constant | 523 | S.I. |
| $T$ | Temperature of the well | 323 | K |
| $C_{iv}$ | Injection valve constant | | |
| $S_a$ | Casing section | 0.081 | m$^2$ |
| $\alpha$ | Constant | | 1/m$^3$ |
| $\beta$ | Constant | | 1/m/s$^2$ |
| $P_r$ | Reservoir pressure | 170$e$5 | Pa |
| $P_{tb}^\star$ | Pres. of the column of oil | $P_0 + \rho_l g L$ | Pa |
| $P_0$ | Separator pressure | 22$e$5 | Pa |
| $g$ | Gravity constant | 9.81 | m/s$^2$ |
| $\rho_l$ | Density of oil | 781 | kg/m$^3$ |
| $V_g$ | Gas velocity | | m/s |
| $L$ | Pipe length | 3000 | m |
| $\lambda$ | Constant | | 1/(ms) |
| $k_p, k_i$ | Controller gains | | |
| $x(t)$ | Mass of gas in the casing | | kg |
| $P_a(t)$ | Casing head pressure | | Pa |
| $P_{ab}(t)$ | Casing head pressure | | Pa |
| $\rho_{ab}(t)$ | Casing gas density | | kg/m$^3$ |
| $w_{gc}(t)$ | Gas mass flow rate | | kg/s |
| $w_{iv}(t)$ | Gas mass flow rate in the tubing | | kg/s |
| $P_{tb}(t)$ | Bottom-hole pressure | | Pa |
| $u(t)$ | Production choke opening | | - |

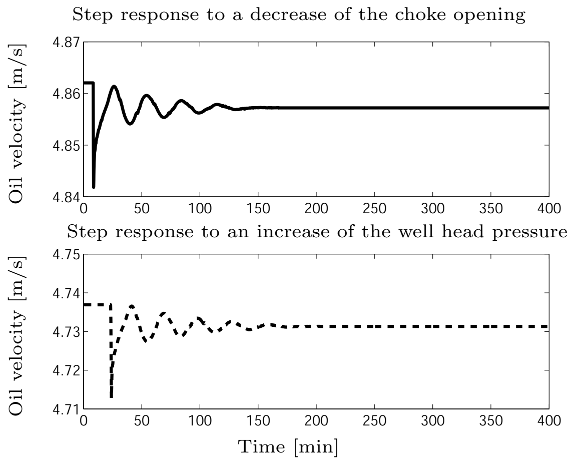


Fig. 6. Comparison of the step responses to an increase of the well head pressure and to a decrease of the production choke opening.

## 3. CLOSED-LOOP STABILITY ANALYSIS

We now aim at showing that it is theoretically possible to stabilize the well using a simple PI controller with the well head pressure as input and the mass of gas in the casing as output.

Linearization of equation (5) gives

$$\delta\dot{x} = -\partial_\bullet w_{\bullet\bullet}\delta x - \partial_{\bullet_{tb}} w_{\bullet\bullet}\delta P_{\bullet\bullet}$$

$$\delta P_{\bullet\bullet} = \delta u +$$

$$\int_0^\bullet k(\zeta)\left(\frac{1}{\lambda w_{\bullet\bullet}} + \frac{P_\bullet - P_{\bullet\bullet}}{\lambda w_{\bullet\bullet}^2}\partial_{\bullet_{tb}} w_{\bullet\bullet}\right)\delta P_{\bullet\bullet}(t-\zeta)d\zeta$$

$$+ \int_0^\bullet k(\zeta)\frac{P_\bullet - P_{\bullet\bullet}}{\lambda w_{\bullet\bullet^2}}\partial_\bullet w_{\bullet\bullet}\delta x(t-\zeta)d\zeta$$

Therefore, one can rewrite

$$\delta \dot{x} = a_1 \delta x + a_2 \delta P_{\bullet\bullet}$$
$$\delta P_{\bullet\bullet} = \delta u + \int_0^{\bullet} a_3 k(\zeta) \delta P_{\bullet\bullet}(t-\zeta) d\zeta + \int_0^{\bullet} a_4 k(\zeta) \delta x(t-\zeta) d\zeta$$

which, in Laplace coordinates, leads to

$$\begin{cases} \delta \tilde{x} = \dfrac{a_2}{s-a_1} \delta \tilde{P}_{\bullet\bullet} \\ \delta \tilde{P}_{\bullet\bullet} = \delta \tilde{u} + a_3 \tilde{k}(s) \delta \tilde{P}_{\bullet\bullet} + a_4 \tilde{k}(s) \delta x \end{cases}$$

Finally, the transfer function is

$$\delta \tilde{x} = \frac{a_2}{s - a_1 - a_3 s \tilde{k}(s) + a_5 \tilde{k}(s)} \delta \tilde{u}$$

with $a_5 \triangleq a_1 a_3 - a_2 a_4$. We now study the stability of this SISO system when closing the loop with

$$\delta u = k_{\bullet} \left(1 + \frac{k_{\bullet}}{s}\right) (\delta x_{\bullet\bullet} - \delta x),$$

where $k_{\bullet} > 0$. For that purpose, one can investigate the location of the roots of

$$s - a_1 - a_3 s \tilde{k}(s) + a_5 \tilde{k}(s) + a_2 k_{\bullet} \left(1 + \frac{k_{\bullet}}{s}\right) = 0 \tag{6}$$

The following result holds

*Lemma 1.* There exists $k_{\bullet}^* > 0$ such that for all $k_{\bullet} \geq k_{\bullet}^*$ the closed loop system, which characteristic equation is (6), is stable.

*Proof 1.* Consider $k_{\bullet} > 0$, and assume that one can find a root $s$ of the characteristic equation (6) such that $Re(s) \geq 0$. Then, $|e^{-\bullet\bullet}| < 1$. Using the mean-value inequality, $\left|\frac{1-\bullet^{-s\tau}}{\bullet\bullet}\right| < 1$ and $\left|\frac{1-\bullet^{-s\tau} - \bullet\bullet\bullet^{-s\tau}}{(\bullet\bullet)^2}\right| < 1$. Therefore,

$$|\tilde{k}(s)| = \left| k_2 \frac{1 - e^{-\bullet\bullet}}{s} + k_1 \frac{1 - e^{-\bullet\bullet} - s\tau e^{-\bullet\bullet}}{s^2} \right| < |k_1 \tau^2| + |k_2 \tau|$$

Furthermore,

$$|s\tilde{k}(s)| < \left| k_2 (1 - e^{-\bullet\bullet}) - k_1 \tau e^{-\bullet\bullet} + k_1 \frac{1 - e^{-\bullet\bullet}}{s} \right| < 2(|k_1 \tau| + |k_2|)$$

Thus,

$$|a_3 s \tilde{k}(s) - a_5 \tilde{k}(s)| < 2|a_3|(|k_1 \tau| + |k_2|) + |a_5|(|k_1 \tau^2| + |k_2 \tau|)$$

On the other hand, since $Re(s)$, $k_{\bullet}$, $k_{\bullet}$, $a_2$ and $-a_1$ are all positive, then

$$\left| s - a_1 + a_2 k_{\bullet} \left(1 + \frac{k_{\bullet}}{s}\right) \right| \geq Re(s) - a_1 + a_2 k_{\bullet} \left(1 + k_{\bullet} \frac{Re(s)}{|s|}\right) \geq -a_1 + a_2 k_{\bullet} \geq 0$$

In summary, if $s$ is a solution of the characteristic equation (6) with positive real part then

$$| - a_1 + a_2 k_{\bullet}| < 2|a_3|(|k_1 \tau| + |k_2|) + |a_5|(|k_1 \tau^2| + |k_2 \tau|) \tag{7}$$

Let

$$k_{\bullet}^* \triangleq \frac{2|a_3|(|k_1 \tau| + |k_2|) + |a_5|(|k_1 \tau^2| + |k_2 \tau|)}{a_2}$$

For $k_{\bullet} \geq k_{\bullet}^*$, equation (7) does not hold. This proves that, for such values, one cannot find a solution of equation (6) with positive real part. Necessarily, the closed loop system is stable which concludes the proof. Finally, notice that this lower bound does not depend on $k_{\bullet}$ (which is positive by assumption). ∎

## 4. OLGA SIMULATIONS

### *4.1 Control structure*

Based on the theoretical analysis of section 3 and Lemma 1 in particular, we propose the following control scheme. We use a simple P-controller on the casing head pressure, $P_{\bullet}$, using the well head pressure $P_{\bullet}$. Then, we derive the production choke values through the static map in Figure 4

$$P_{\bullet} = P_{\bullet}^{\bullet\bullet} + k(P_{\bullet}^{\bullet\bullet} - P_{\bullet})$$
$$u = \frac{a}{P_{\bullet} - b} + c$$

where $a$, $b$ and $c$ are fit parameters.

### *4.2 Simulation setup*

Tests of our control structure are conducted on a well simulated in OLGA®2000. We use the compositional tracking and the Matlab-OLGA link toolboxes. We consider that the gas mass flow rate injected at the casing head can be arbitrarily chosen. The reservoir has constant $PI$, pressure and temperature. Along the well, temperatures are kept constant as well as the separator pressure, i.e. the boundary pressure at the well head.

The following scenario is considered. In the beginning, the controller is switched on. The gas injection rate is 0.4 kg/s. This corresponds to a stable equilibrium. Then, at $t$ = 1h the gas injection rate is decreased to 0.3 kg/s. About the corresponding steady-state, the open-loop system is unstable. When eventually the well is almost stabilized (at $t$ = 2h50), the proportional gain is discontinuously lowered to provide a soft landing and avoid unnecessary damped oscillations. Finally at $t$ = 13h50, the controller is switched off. As expected, the system diverges toward a self-sustained oscillatory regime. The gas injection rate in the tubing is almost constant. The observed behavior is indeed a density-wave as shown in the fourth graph of Figure 8.

## 5. CONCLUSION

Our point is to demonstrate the relevance of feedback control to address the various instabilities of gas-lifted wells. Among these are the casing-heading and density-wave. The first case was already addressed in (Eikrem and Golan, 2002). The results reported here stress that, theoretically and in simulations, the density-wave phenomenon can be handled by a similar strategy. For that purpose, we use a straightforward controller. Clearly, results could be improved upon using, at least, gain-scheduling and feed-forward terms. In our approach, no extra sensors are required. It is debatable whether such performance can be achieved in actual wells, given the actuation limitations and sensor noises. This point is currently under investigation.

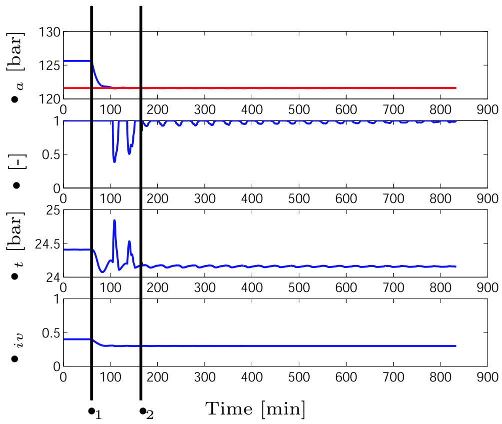

Fig. 7. Stabilization of the casing head pressure through production choke manipulations (first 840 min). At time $t_1$ the injection rate is switched from 0.4 to 0.3 kg/s. At time $t_2$ the proportional gain is reduced from 12 to 2.

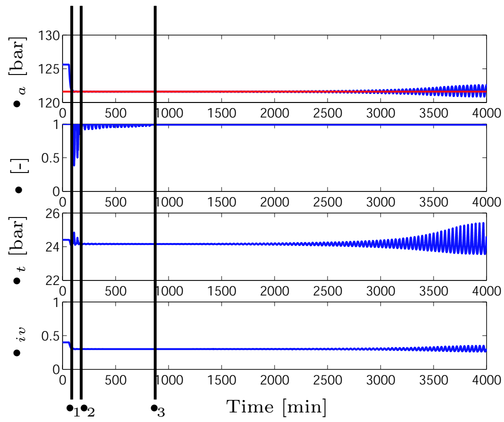

Fig. 8. Stabilization of the casing head pressure through production choke manipulations. At time $t_3$ the controller is switched off. The well gradually diverges towards its self-sustained oscillatory regime.

## REFERENCES


Asheim, H. (1988). Criteria for gas-lift stability. *Journal of Petroleum Technology* pp. 1452–1456.

Brown, K. E. (1973). *Gas lift theory and practice.* Petroleum publishing CO., Tulsa, Oklahoma.

Cholet, H. (2000). *Well production. Practical handbook.* Editions TECHNIP.

Chorin, A. J. and J. E. Marsden (1990). *A mathematical introduction to fluid mechanics.* Springer-Verlag.

Dalsmo, M., E. Halvorsen and O. Slupphaug (2002). Active feedback of unstable wells at the Brage field. In: *SPE Annual technical Confererence and Exhibition.* number SPE 77650.

der Kinderen, W. J. G. J., C. L. Dunham and H. N. J. Poulisse (1998). Real-time aritificial lift optimization. In: *8th Abu Dhabi International Petroleum Exhibition and Conference.* number SPE 49463.

Duret, E. (2005). Dynamique et contrôle des écoulements polyphasiques. PhD thesis. École des Mines de Paris.

Eikrem, G. O., B. Foss L. Imsland B. Hu and M. Golan (2002). Stabilization of gas-lifted wells. In: *Proc. of the 15th IFAC World Congress.*

Eikrem, G. O., L. Imsland and B. Foss (2004). Stabilization of gas-lifted wells based on state estimation. In: *International Symposium on Advanced Control of Chemical Processes.*

Hu, B. and M. Golan (2003). Gas-lift instability resulted production loss and its remedy by feedback control: dynamical simulation results. In: *SPE International Improved Oil Recovery Conference in Asia Pacific.* number SPE 84917. Kuala Lumpur, Malaysia.

Imsland, L. S. (2002). Topics in Nonlinear Control - Ouput Feedback Stabilization and Control of Positive Systems. PhD thesis. Norwegian University of Science and Technology, Department of Engineering and Cybernetics.

Jansen, B., M. Dalsmo, K. Havre, L. Nøkleberg, V. Kritiansen and P. Lemétayer (1999). Automatic control of unstable gas-lifted wells. *SPE Annual technical Confererence and Exhibition.*

Sinègre, L., N. Petit and P. Ménégatti (2005). Distributed delay model for density wave dynamics in gas lifted wells. In: *Proc. of the 44th IEEE Conf. on Decision and Control, to appear.*

Torre, A. J., Z. Schmidt, R. N. Blais, D. R. Doty and J. P. Brill (1987). Casing heading in flowing wells. *SPE Production Engineering.*