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OPTIMAL CONTROL OF MULTIVARIABLE PROCESSES USING BLOCK STRUCTURED MODELS

G. Harnischmacher, **W. Marquardt** [1]

Lehrstuhl für Prozesstechnik, RWTH Aachen University
D–52056 Aachen, Germany

Abstract: Block structured models have been used in nonlinear model predictive control to reduce computational cost. The solution of the nonlinear dynamic optimization problem has been evaded by inverting the nonlinear element and solving the resulting linear problem in the past. However, by exploiting the block structure for sensitivity calculation, the original nonlinear problem can also be solved at low computational cost, and at the same time this o ers much greater modeling exibility. This paper deals with dynamic optimization and, in particular, the e cient calculation of rst order sensitivity information for the case of multivariable Hammerstein and Uryson systems. In a simulation example the method is shown to combine low computational cost with the possibility to signi cantly reduce the losses of optimality compared to the previous methods.

Keywords: Hammerstein model, sensitivity system, nonlinear model predictive control, dynamic optimization, multivariable block structured model

1. INTRODUCTION

Nonlinear model predictive control (NMPC) poses challenging problems both in modeling and computation. Obtaining nonlinear, dynamic process models either requires large amounts of identi -cation data or deep physical insight for rigorous modeling. Afterwards, the optimization problem has to be solved within short sampling times required in closed loop NMPC. Numerous model reduction techniques have been explored to reduce the original process model (Marquardt, 2002), or to totally avoid online optimization (Kadam *et al.*, 2005).

Block structured models consisting of nonlinear static and linear dynamic elements have been used to reduce both the modeling and computation e orts. Structuring the model in this way leads to an approximate model, which is inferior in prediction quality to a rigorous nonlinear model, but provides a viable compromise between the low predictive capabilities of a linear model and the costly development of a non-structured nonlinear dynamic model. Applications range from such di erent elds as neuroprothesis, where a rigorous nonlinear model could not be obtained (Hunt *et al.*, 1998), to the control of an industrial C2-splitter (Norquay *et al.*, 1999). For Wiener (Norquay *et al.*, 1999) and Hammerstein (Zhu and Seborg, 1994) models tailored solution algorithms have been developed. They are based on the inversion of the nonlinear element to reduce the original nonlinear dynamic optimization problem to a linear one. We will refer to this method as the "inversion based method" in the sequel. To obtain a unique solution with the inversion based method, the nonlinearity of the model needs to be bijective, which is generally not the case. Especially for the multi-input multi-output (MIMO) case, this poses restrictions on the model structures. In particular, the MIMO model structure suggested

[1] Corresponding author: marquardt@lpt.rwth-aachen.de

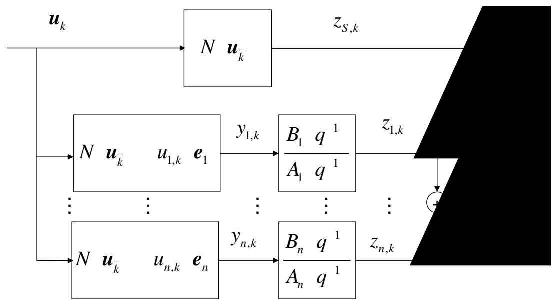

Fig. 1. Block diagram of the HM model.

by Kortmann and Unbehauen (Kortmann and Unbehauen, 1987) has been used previously and we will refer to it as the KU model in the sequel.

In contrast to the inversion based method, we are directly solving the nonlinear dynamic optimization problem constrained by block structured models. Therefore, rst order derivatives of the objective and constraints with respect to the degrees of freedom of the dynamic optimization problem are required. For rigorous dynamic models the calculation of this sensitivity information oftentimes dominates the computational cost of the solution process. We aim at reducing the computational cost by exploiting the block structure for e cient calculation of sensitivity information. Our method covers all MIMO Hammerstein as well as Uryson (Gallman, 1975) models. It allows the solution of the offline optimal control problem. State estimation for such models, required for closed loop control implementation, is the focus of current research.

2. PROBLEM STATEMENT

The constrained, discrete time optimal control problem

$$\min_{\{\boldsymbol{u}_k\}} \ \Phi(\{\boldsymbol{x}_k\}, \{\boldsymbol{u}_k\}) \tag{1a}$$
$$s.t. \ \ \boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{(k\ \ 1)}, \boldsymbol{u}_{(k\ \ 1)}) \tag{1b}$$
$$\boldsymbol{0} \geq \boldsymbol{g}(\boldsymbol{x}_k, \boldsymbol{u}_k, t_k) \tag{1c}$$
$$\boldsymbol{x}_0, \boldsymbol{u}_0 \tag{1d}$$
$$k = 1 \ldots K \tag{1e}$$

is given with the objective function $\Phi(\ )$, the manipulated variables $\{\boldsymbol{u}_k\}$, partly measurable state variables $\{\boldsymbol{x}_k\}$, inequality constraints $\boldsymbol{g}(\ )$, process model $\boldsymbol{f}(\ )$, and initial conditions $\boldsymbol{x}_0$, $\boldsymbol{u}_0$. By $\{\ \}$ we denote discrete time sequences of variables, while bold symbols denote vector variables. A function $h(\{\boldsymbol{x}_k\})$ denotes $h(\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_K)$. Given the limited computation time available for NMPC, some form of model reduction is required for large process models $\boldsymbol{f}(\ )$. In this paper we assume, that $\boldsymbol{f}(\ )$ can be approximated by a discrete time Hammerstein or Uryson model (Pearson, 1999). Gradient based solution methods require at least rst order derivatives of the objective and constraints with respect to the degrees of freedom, for which sensitivity equations are developed in this paper.

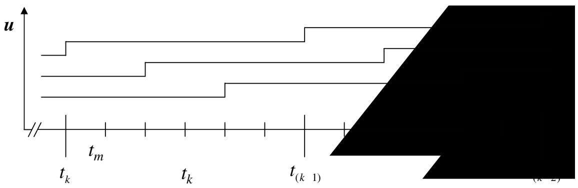

Fig. 2. Oversampling example.

To derive the sensitivity equations we rst treat the single-input single-output (SISO) case for the sake of simplicity. For this case, we approximate problem (1) by

$$\min_{\{u_k\}} \ \Phi(\{z_k\}, \{u_k\}) \tag{2a}$$
$$s.t. \ \sum_{i=0}^{dim(\boldsymbol{a})} a_i \ \ z_{(k\ \ i)} = \sum_{i=0}^{dim(\boldsymbol{b})} b_i \ \ y_{(k\ \ i)} \tag{2b}$$
$$y_k = N(u_k) \tag{2c}$$
$$0 \geq \boldsymbol{g}(z_k, u_k, t_k) \tag{2d}$$
$$\{z_0\} = \boldsymbol{z}_0, \ \{u_0\} = \boldsymbol{u}_0 \tag{2e}$$
$$k = 1 \ldots K\,. \tag{2f}$$

In problem (2) the reduced process model is de ned by the nonlinear static map $N(\ ) : \mathbb{R}^1 \rightarrow \mathbb{R}^1$, and a linear dynamic process model with gain normalized to one, which is de ned by $\boldsymbol{a}$ and $\boldsymbol{b}$. $\{u_k\}$ and $\{z_k\}$ are the measurable input and output variables and $\{y_k\}$ is the nonmeasurable intermediate variable. $\Phi(\ )$ is the objective function, $\boldsymbol{g}(\ )$ are inequality constraints, and $\{u_0\}$ and $\{z_0\}$ are sequences of delayed inputs and outputs at t_0 de ning the initial condition of the system. Note that the di erence between problems (1) and (2) is a replacement of the original process model by a reduced model. The objective and inequality constraints in (2) only contain the measurable variable $\{z_k\}$ instead of the full state vector $\{\boldsymbol{x}_k\}$.

We extend the method to the more relevant MIMO case in Section 3.2. For the MIMO case several Hammerstein structures have been developed. In this paper we will use the Hammerstein model based on deviation dynamics, which is discussed in detail and compared to the other structures by Harnischmacher and Marquardt (2005). This model is the only one to consistently extend the concept of the Hammerstein model comprising a nonlinear static map followed by an independent linear process model to the multi-input single-output (MISO) case. We will term it HM model in the sequel. The model consists of a static channel and $n = dim(\boldsymbol{u})$ dynamic channels j as depicted in Figure 1. As this model is similar to Uryson models (Gallman, 1975), the results for the MISO case straightforwardly extend to this model class as well. For the MIMO case, problem (1) is approximated using the HM model by

$$\min_{\{\boldsymbol{u}_k\}} \Phi(\{\boldsymbol{z}_k\},\{\boldsymbol{u}_k\}) \quad (3a)$$

$$s.t.\ z_{l,k} = N_l(\boldsymbol{u}_k) + \sum_{j=1}^{dim(\boldsymbol{u})} \quad z_{l,j,k} \quad (3b)$$

$$\sum_{i=0}^{dim(\boldsymbol{a}_{l,j})} a_{l,j,i} \quad z_{l,j,(k \quad i)} = \sum_{i=0}^{dim(\boldsymbol{b}_{l,j})} b_{l,j,i} \quad y_{l,j,(k \quad i)} \quad (3c)$$

$$y_{l,j,k} = N_l(\boldsymbol{u}_{\overline{k}} + \quad u_{j,k}\ \boldsymbol{e}_j) \quad (3d)$$

$$0 \geq \boldsymbol{g}(\boldsymbol{z}_k, \boldsymbol{u}_k, t_k) \quad (3e)$$

$$\{\boldsymbol{z}_{l,0}\} = Z_{l,0}, \{\boldsymbol{u}_0\} = U_0 \quad (3f)$$

$$k = 1...K,\ l = 1...dim(\boldsymbol{z}),\ j = 1...dim(\boldsymbol{u}). \quad (3g)$$

In this case, each element of the input and output sequences $\{\boldsymbol{u}_k\}$ and $\{\boldsymbol{z}_k\}$ is of dimension $dim(\boldsymbol{u})$ and $dim(\boldsymbol{z})$ respectively. $\boldsymbol{N}(\ ) : \mathbb{R}^{dim(\boldsymbol{u})} \rightarrow \mathbb{R}^{dim(\boldsymbol{z})}$ is a nonlinear static map of the process and $N_l(\ ) : \mathbb{R}^{dim(\boldsymbol{u})} \rightarrow \mathbb{R}^1$ denotes the l^{th} component of $\boldsymbol{N}(\ )$. $\boldsymbol{u}_{\overline{k}}$ is a reference value for $\boldsymbol{u}$, which is updated at every t_k, and $\ u_{j,k} = u_{j,k} \quad u_{j,\overline{k}}$ is the deviation thereof in the direction of the unit vector $\boldsymbol{e}_j$. In this structure the nonlinear element in each channel j represents the local gain of the nonlinear map $N_l(\ )$ in the direction of $\boldsymbol{e}_j$ at $\boldsymbol{u}_k$. To derive the linear elements, linear SISO systems $G_{l,j} : \mathbb{R}^1 \rightarrow \mathbb{R}^1$ are identi ed for all $l = 1...dim(\boldsymbol{u})$ and $j = 1...dim(\boldsymbol{z})$. The parameters $\boldsymbol{a}_{l,j}$ and $\boldsymbol{b}_{l,j}$ are then derived analytically after normalizing the gain to one just as in the SISO case. $\boldsymbol{g}(\ )$ are inequality constraints, $\Phi(\ )$ the objective, and U_0, $Z_{l,0}$ the initial conditions as before.

Model (3b-d) decouples the static response of the system with respect to its inputs to maintain the independence of the nonlinear and linear elements. This decoupling is based on the decomposition of the Taylor expansion of $N_l(\ )$. It is exact, i the second and higher order terms of the Taylor expansion, e.g. $\ u_{j_1,k} \quad u_{j_2,k} \frac{\partial^2 N_l(\cdot)}{\partial u_{j_1} \partial u_{j_2}}\Big|_{\boldsymbol{u}=\boldsymbol{u}_{\overline{k}}}$ are equal to zero, which is generally not the case. To meet this condition, we ensure that the input $\ u_{j,k}$ is di erent from zero for at most one j for $dim(\boldsymbol{b}_{j^*}) \quad 1$ intervals by oversampling the model. The model is sampled at an internal sampling interval of $\ t_m$, such that $\ t_k = \ t_m \sum_{j=1}^{dim(\boldsymbol{u})} (dim(\boldsymbol{b}_j) \quad 1)$. The response of the system to the input $\boldsymbol{u}_k$ is then calculated by sequentially processing the inputs u_j. We de ne

$$\boldsymbol{u}_{k_n} = [u_{1,k}, ..., u_{n,k}, u_{(n+1),(k \ 1)}, ..., u_{dim(\boldsymbol{u}),(k \ 1)}]^T \quad (4)$$

for $n = 1...dim(\boldsymbol{u})$. The sequential processing is depicted in Fig. 2 for an example with $dim(\boldsymbol{u}) = 3$, $dim(\boldsymbol{z}) = 1$, and $dim(\boldsymbol{b}_j) = 3\ \forall\ j$. At time t_k the input $u_{1,k}$ is processed and the input is held constant for the following interval $\ t_m$. Hence, for the oversampled model, the input is $\boldsymbol{u}_{k_1}$ for $dim(\boldsymbol{b}_1) \quad 1$ intervals. $u_{2,k}$ is processed at $t_k + 2 \quad t_m$ and again the input unchanged in the following interval $\ t_m$ ensuring a constant input $\boldsymbol{u}_{k_2}$ for $dim(\boldsymbol{b}_2) \quad 1$ intervals and so on. By oversampling, the input $\boldsymbol{u}_k$ is turned into a sequence of inputs $\boldsymbol{u}_m$ for the oversampled model, which will be of importance for the sensitivity calculation. The input $\boldsymbol{u}_m$ to the oversampled model is given by

$$u_{j,m} = \begin{cases} u_{j,k \ \ 1}\ \forall\ t_k \leq t_m < t_k + \sum_{i=1}^{j} (dim(\boldsymbol{b}_i) \quad 1) \quad t_m \\ u_{j,k} \quad \forall\ t_k + \sum_{i=1}^{j} (dim(\boldsymbol{b}_i) \quad 1) \quad t_m \leq t_m < t_{k+1}. \end{cases} \quad (5)$$

3. SENSITIVITY EQUATIONS FOR HAMMERSTEIN SYSTEMS

3.1 SISO Case

For the SISO case the sensitivity of z_k with respect to an input u_{k^*} is straightforwardly calculated using the chain rule of di erentiation from

$$\frac{\partial z_k}{\partial u_{k^*}} = \frac{\partial z_k}{\partial y_{k^*}} \quad \frac{\partial y_{k^*}}{\partial u_{k^*}}. \quad (6)$$

As Eq. (2b) is linear in y_k, solving the recursion for z_k yields

$$z_k = \xi_{k,k^*}(\boldsymbol{a},\boldsymbol{b})\ y_{k^*} + \quad (\boldsymbol{a},\boldsymbol{b},\{y_{k \neq k^*}\},\boldsymbol{u}_0,\boldsymbol{z}_0), \quad (7)$$

where $\ (\ )$ is a polynomial containing all elements of $\{y_k\}$ but y_{k^*} and ξ_{k,k^*} is a constant polynomial of $\boldsymbol{a}$ and $\boldsymbol{b}$. The rst term of Eq. (6) is therefore

$$\frac{\partial z_k}{\partial y_{k^*}} = \xi_{k,k^*}(\boldsymbol{a},\boldsymbol{b}) := const. \quad (8)$$

The second term of Eq. (6)

$$\frac{\partial y_{k^*}}{\partial u_{k^*}} = \frac{\partial N(u)}{\partial u}\Big|_{u=u_{k^*}} \quad (9)$$

is just the rst order derivative of the nonlinear static element $N(u)$ at $u = u_{k^*}$.

Due to the structure of the Hammerstein model, the sensitivity calculation can thus be reduced to the calculation of one rst order derivative of $N(\ )$ and one vector multiplication

$$\frac{\partial \{z_k\}}{\partial u_{k^*}} = \boldsymbol{\xi}_{k^*} \quad \frac{\partial N(u)}{\partial u}\Big|_{u=u_{k^*}} \quad (10)$$

with $\boldsymbol{\xi}_{k^*} = [\xi_{1,k^*}, \ldots, \xi_{K,k^*}]$.

3.2 MIMO Case

MIMO Hammerstein and Uryson structures generally consist of parallel branches of MISO or SISO Hammerstein models. Hence, the sensitivity calculation is a straight forward extension of the SISO case. The computational e ort varies with the respective Hammerstein structure. For the KU model (Kortmann and Unbehauen, 1987) only the derivatives of $dim(\boldsymbol{u})$ scalar functions are required, while the model based on combined nonlinearities (Eskinat *et al.*, 1991) requires $dim(\boldsymbol{u})$

gradients of the respective nonlinear models. However, to our knowledge no control application based on the solution of the nonlinear dynamic optimization problem has been reported.

Because of the oversampling the sensitivity calculation for the HM model is a little more complex, but since it also consists of parallel Hammerstein channels, the structure of the solution remains the same. As Eq. (3) contains $dim(\boldsymbol{z})$ parallel MISO models, we will only treat the MISO case in this section and therefore drop the index l of Eq. (3) for the remainder of this section to ease the notation. Since Eq. (3) consists of parallel branches of Hammerstein systems, the sensitivity equations developed in this section are structurally equivalent to the SISO case. In particular Eq. (8) holds for each of the dynamic channels of Eq. (3c). We therefore use the following notation for the remainder of this section:

$$\xi_{j,k,k^*} := \frac{\partial z_{j,k}}{\partial y_{j,k^*}}. \tag{11}$$

The sensitivity of z_k with respect to $\boldsymbol{u}_{k^*}$ is given by

$$\frac{\partial z_k}{\partial \boldsymbol{u}_{k^*}} = \frac{\partial z_{S,k}}{\partial \boldsymbol{u}_{k^*}} + \sum_{j=1}^{dim(\boldsymbol{u})} \frac{\partial z_{j,k}}{\partial \boldsymbol{u}_{k^*}}. \tag{12}$$

The first term in Eq. (12) contains the sensitivity of the static channel S of the model, which is simply

$$\frac{\partial z_{S,k}}{\partial \boldsymbol{u}_{k^*}} = \left.\frac{\partial N(\boldsymbol{u})}{\partial \boldsymbol{u}}\right|_{\boldsymbol{u}=\boldsymbol{u}_{k^*}} \tag{13}$$

and zero for all $k \neq k$

The sensitivity calculation for the dynamic channels follows the same concept and the same simplification as in the SISO case. However as depicted in Fig. 2 the input $\boldsymbol{u}_{k^*}$ is in fact an input sequence to the oversampled model. $\left.\frac{\partial N(\boldsymbol{u})}{\partial \boldsymbol{u}_{k^*}}\right|_{\boldsymbol{u}=\boldsymbol{u}_{k_n}}$ is nonzero for the sequence $\{\boldsymbol{u}_{k_n^*}, ..., \boldsymbol{u}_{(k^*+1)_{n-1}}\}$. $\frac{\partial z_{j,k}}{\partial u_{j,k^*}}$ for the dynamic channels is then given by

$$\frac{\partial z_{n,k}}{\partial u_{j,k^*}} = \xi_{n,k,k^*} \left.\frac{\partial N(\boldsymbol{u})}{\partial u_j}\right|_{\boldsymbol{u}_{k_n^*}} \ \xi_{n,k,(k^*+1)} \left.\frac{\partial N(\boldsymbol{u})}{\partial u_j}\right|_{\boldsymbol{u}_{(k^*+1)_n}} \tag{14}$$

for channel $n = j$, by

$$\frac{\partial z_{n,k}}{\partial u_{j,k^*}} = \xi_{n,k,(k^*+1)} \left(\left.\frac{\partial N(\boldsymbol{u})}{\partial u_j}\right|_{\boldsymbol{u}_{(k^*+1)_n}} \ \left.\frac{\partial N(\boldsymbol{u})}{\partial u_j}\right|_{\boldsymbol{u}_{(k^*+1)_{(n-1)}}} \right) \tag{15}$$

for all channels $n = 1 \dots j-1$, and analogously

$$\frac{\partial z_{n,k}}{\partial u_{j,k^*}} = \xi_{n,k,k^*} \left(\left.\frac{\partial N(\boldsymbol{u})}{\partial u_j}\right|_{\boldsymbol{u}_{k_n^*}} \ \left.\frac{\partial N(\boldsymbol{u})}{\partial u_j}\right|_{\boldsymbol{u}_{k^*_{(n-1)}}} \right) \tag{16}$$

for all channels $n = j+1 \dots dim(\boldsymbol{u})$.

As in the SISO case, the integration of the sensitivity system for the MISO case can therefore be reduced to calculation of the 2 $dim(\boldsymbol{u})$ gradients of $N(\boldsymbol{u})$ at $\boldsymbol{u}_{k_1^*} \dots \boldsymbol{u}_{(k^*+1)_{dim(\boldsymbol{u})}}$ and a set of matrix multiplications

$$\begin{aligned} \frac{\partial \{z_k\}}{\partial \boldsymbol{u}_{k^*}} = & \sum_{j=1}^{dim(\boldsymbol{u})} \left.\frac{\partial N(\boldsymbol{u})}{\partial \boldsymbol{u}}\right|_{u_{k_j^*}} \Xi_{k^*,j} \ + \\ & \sum_{j=1}^{dim(\boldsymbol{u})} \left.\frac{\partial N(\boldsymbol{u})}{\partial \boldsymbol{u}}\right|_{u_{(k^*+1)_j}} \Xi_{(k^*+1),j} \ , \end{aligned} \tag{17}$$

where $\Xi_{k^*,j}$ and $\Xi_{(k^*+1),j}$ contain the respective vectors $\boldsymbol{\xi}_{j,k^*}$ and $\boldsymbol{\xi}_{j,(k^*+1)}$ analogously to Eq. (10).

4. COMPARISON WITH COMPETING METHODS

Directly competing are the inversion based methods using Wiener or Hammerstein models (Zhu and Seborg (1994), Norquay *et al.* (1999)). They offer slight advantages in computational cost, but are known to possibly suffer from non-uniqueness, when the nonlinear map is not bijective over the input space. This severely limits the nonlinear maps as well as the multivariable structures that can be used. Further, the objective function of the linear optimization problem contains the intermediate variable of the model as a proxy variable for either the output or the input to the system. As these are nonlinearly linked, the solution of the linear problem generally does not minimize the original objective. Finally, the inversion based solution of nonlinear dynamic optimization problems constrained by Uryson models is not possible, because intermediate variables y_κ of the different channels κ of the Uryson model, which are independent variables in the linear optimization problem, are in fact nonlinearly coupled.

The efficiency of the sensitivity calculation for Hammerstein systems is greatly increased by making use of Eq. (10), which does not hold for Wiener systems. The sensitivity of z_k with respect to u_{k^*} for a SISO Wiener system can be calculated from

$$\frac{\partial z_k}{\partial u_{k^*}} = \frac{\partial z_k}{\partial y_k} \ \frac{\partial y_k}{\partial u_{k^*}}. \tag{18}$$

In this case $\frac{\partial z_k}{\partial y_k} = \left.\frac{\partial N(\cdot)}{\partial y}\right|_{y=y_k}$ needs to be evaluated at every t_k. Thus, for Wiener systems the solution of the nonlinear dynamic optimization problem is computationally much more demanding, because the derivative of the nonlinear map has to be evaluated on the discretization of the output instead of the discretization of the input. When nonlinear maps other than polynomials are used, the evaluation of the nonlinear map dominates the computational cost (Harnischmacher *et al.*, 2006).

5. SIMULATION EXAMPLE

As a simulation example we choose the industrially relevant uid catalytic cracking (FCC) unit, for which several models exist in the open literature. We use the model originally developed by Kurihara and comprehensively discussed by Denn (1986). This model has been validated and used for control by Ansari and Tadé (2000). We will not restate the equations here due to space limitations. The nomenclature and units used in the sequel are the same as those of Denn (1986), where the complete model may be found. Ansari and Tadé (2000) also state the complete model, but with some typographical error and a slightly di erent notation. Detailed process descriptions can be found in both references. The example shows, that the solution of the nonlinear dynamic optimization problem can be performed in very short time and the increased modeling exibility leads to signi cant improvements in performance.

5.1 Simulated FCC Unit

The main manipulated variables of the process are the air owrate R_{ai} and the catalyst circulation rate R_{rc}, while the feed rate R_{tf} and feed temperature T_{fp} are treated as disturbances. To control the main quality variable, the cracking severity, several controlled variables have been explored due to the complex dynamics of the system. However the riser outlet temperature T_{ra} is directly related to the cracking severity and has recently been used for control (Jia *et al.*, 2003). The control problem is therefore non-square with manipulated variables R_{ai} and R_{rc} and controlled variable T_{ra}.

5.2 Identi cation

The simulated FCC unit is identi ed using two di erent Hammerstein model structures. For the inversion based method we use the KU model (Kortmann and Unbehauen, 1987). Quadratic functions are used in each of the two channels of the model. For the proposed method, the HM model (Harnischmacher and Marquardt, 2005) is used. Here, the nonlinear map is an arti cial neural network (ANN) identi ed from steady state data. For both models fourth order linear elements are identi ed from step response data.

The FCC process is known to exhibit a two timescale behavior (Christo des and Daoutidis, 1997). The models identi ed above give a poor description of the short time scale behavior of the process and a Uryson model, containing two dynamic channels for each input, is much more suitable (Gallman, 1975). As the response on the fast time scale is close to linear, constant gains are used in these two channels, while the same ANN as in the HM model is used in the two long time scale channels. The long time scale dynamic behavior of the system is described by rst order models, while models of third order are identi ed for the fast time scale channels.

5.3 Open-Loop Optimal Control

The control objective

$$\Phi = (\boldsymbol{T}_{ra} \quad \boldsymbol{T}_{set})^T(\boldsymbol{T}_{ra} \quad \boldsymbol{T}_{set}) + \quad \sum R_{rc,i} \quad (19)$$

is to be minimized. The time horizon is 1000 intervals t_k corresponding to two hours simulation time. The inputs $R_{ai} \in [390; 420]\frac{Mlb}{hr}$ and $R_{rc} \in [40; 42]\frac{ton}{min}$ are piecewise constant for 100 intervals t_k. $\boldsymbol{T}_{ra} = [z_{50}, z_{100}, ..., z_{1000}]^T$ contains the model output sampled every 50 intervals. The set point $\boldsymbol{T}_{set}$ changes from 950°F to 960°F at $k = 201$. $\boldsymbol{R}_{rc}$ contains the absolute values of R_{rc} as a proxy for process cost. $= 10 \quad 4$ is a weighting parameter.

For the inversion based method $\{\boldsymbol{u}_k\}$ is given by the roots of two independent quadratic functions, i.e. the nonlinear maps of the model. This leads to four possible solutions. In our case, however, the nonlinear functions are monotonous on the respective input spaces. While this leads to a poor description of the process nonlinearity in a certain section of the input space with steady state errors of up to 13°F, it follows that only one of the four solutions lies in the input space and the solution of the optimization problem is therefore unique. Such behavior of the nonlinear map cannot be expected in general and would pose severe restrictions on the nonlinear map.

5.4 Discussion

The nonlinear optimization problems with both the Hammerstein and Uryson models are solved in less than 1 second using MATLAB on a 1.5 GHz PC. Such computation times are well acceptable for NMPC applications in the process industry.

Simulation results for the manipulated variable trajectories obtained by using the di erent models are depicted in Fig. 3, which as a reference also contains the result obtained by solving the original dynamic optimization problem with the original model. This solution clearly outperforms all approximate solutions. It should be noted though, that for this simulation example, there is absolutely no plant model mismatch when the original model is used. The inversion based method, in contrast, performs worst. We compare the performance by the objective values obtained by simulating the original model with the inputs $\{\boldsymbol{u}_k\}$ calculated with the four di erent models. Using the HM model leads to a slight improvement of 15% in the original objective compared to the inversion based method. The weak performance of

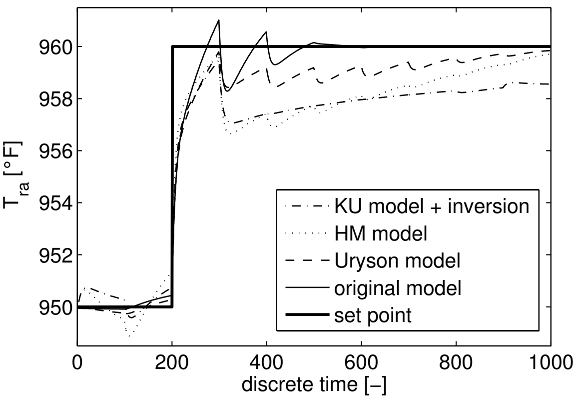

Fig. 3. Trajectory optimization results.

both methods is mainly due to insu cient modeling of the process dynamics.

Solving the nonlinear dynamic optimization problem constrained by a Uryson model leads to a reduction of over 80% in the original objective compared to the inversion based method. Further performance increases can be achieved by using a rigorous steady state model instead of the ANN. This leads to a reduction of 85% in the original objective. However this slight additional improvement comes at a cost of 160 seconds of computation time making this model computationally unattractive. For comparison the improvement in objective for the original model is 94% after 270 seconds of computation time.

6. CONCLUSIONS

Block structured models are well suited for nonlinear model predictive control because of the simple identi cation and low computational cost. Previous approaches aimed at reducing the computational cost by the inversion of the nonlinear element. This requires the nonlinear map to be bijective, excludes the use of Uryson models, and leads to a loss in optimality because of the nonlinear coupling between the proxy variable used in the objective of the linear optimization problem and its counterpart in the original objective. Sensitivity equations have been derived for multivariable Hammerstein and Uryson models to allow the solution of nonlinear optimization problems constrained by these models at low computational cost. An example problem with a non-square controller with two inputs parameterized on 10 intervals each was solved in less than 1 second and at the same time reduced the optimality loss by over 80% compared to previous methods, because of the increased modeling exibility. Future research will be directed at developing a tailored state estimation method for multivariable Hammerstein models to solve the closed loop NMPC problem. Further, online updating methods for the linear elements will be investigated to increase model accuracy.

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