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DYNAMIC PENALTY FORMULATION FOR SOLVING HIGHLY CONSTRAINED MIXED-INTEGER NONLINEAR PROGRAMMING PROBLEMS

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Abstract: This contribution presents a heuristic approach for solving nonconvex mixed-integer nonlinear programming (MINLP) problems with highly constrained discontinuous domains. A new fuzzy penalty strategy is proposed to make stochastic algorithms capable of solving optimization problems with a large number of difficult-to-satisfy constraints. The method consists of a dynamic penalty formulation based on the magnitude and frequency of the constraint violation, applied according to a hierarchical classification of the constraints. The new strategy is introduced to a multi-objective optimization algorithm based on evolutionary strategies. The performance of the proposed methodology is investigated on the basis of a multi-enterprise supply chain optimization problem.

Keywords: Nonlinear programming, multi-objective optimization, discrete-time system, heuristic search, integer programming, algorithms, hierarchical decision making, planning

1. INTRODUCTION

Mathematical programming techniques have been widely applied to solve process systems engineering problems. A variety of practical problems such as optimization for integrated process design and control, dynamic allocation and location-allocation problems, design of multi-product batch plants, etc, have been modeled. These problems often involve hybrid discrete-continuous systems and are therefore formulated as mixed-integer optimization problems. Continuous variables usually describe process states, while discrete ones are related to the structure of the process. Discrete variables may be restricted to binary values, when defining the assignments of equipments and sequencing of tasks.

The basic formulation of mixed-integer optimization problems, when represented in algebraic form is:

$$\text{Min } Z = f(\text{x, y}) \quad \text{s.t.} \begin{cases} h_i(x,y) = 0 & i \in I \\ g_j(x,y) \le 0 & j \in J \\ \text{x} \in \text{X, y} \in \text{Y} \end{cases}$$

where f (x, y) is the objective function, h(x, y) are the equality relationships that describe the performance of the system (material balances, production rates) and g (x, y) are inequalities that define specifications or constraints for feasible scheduling. I and J are the index sets of equalities and inequalities, and x and y are continuous and discrete variables. Optimization problems are classified according to the type of variables and important properties of the functions, like linearity, convexity and differentiability. Mixed-integer programming problems are commonly regarded as steady-state models. Dynamic models give rise to multi-period optimization problems, in case of discrete time models and optimal control problems, in case of continuous time. Powerful

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methods for solving large-scale mixed-integer linear programming (MILP) are well established and have been applied to practical problems for the last few decades. Methods for mixed-integer nonlinear programming (MINLP) problems, on the other hand, have become available recently. Some reviews on optimization methods have been published (Biegler and Grossmann, 2004, Grossmann, 2002). Most common optimization algorithms are based on branch and bound and on decomposition methods. Such algorithms, however, are not guaranteed to locate the global optimum in case of nonconvexity of objective functions or constraints, as it may give rise to multiple local optima (Stein *et al.*, 2004). Relaxation of integer variables as continuous ones and subsequent rounding of the solutions may lead to inaccuracy and infeasible solutions. Decomposition of the original problem to a set of sub-problems may require the objective functions and constraints to be differentiable, which restricts its applicability for a large number of real-life problems (Cheung *et al*., 1997). Moreover, algorithms based on classical nonlinear optimization theory may not be capable of solving large-scale applications, due to their high computational effort requirement (Stein *et al.*, 2004).

Evolutionary algorithms (EAs) have received considerable attention over the last decade, as they have shown to be robust for solving highly nonlinear, nondifferentiable and multimodal optimization problems. Some studies have confirmed the capability of EA-based methods to solve MINLP problems involving local optima and nonconvexities (Ryoo and Sahinidis, 1995, Ostermark, 1999, Cheung *et al*., 1997, Lin *et al*., 2004). Ostermark (1999) has successfully tested EA on a set of complex problems that could not be solved by the GAMS/MINOS package. Hybrid stochastic algorithms have been also employed to solve MINLP problems. Cheung *et al.* (1997) employed a modified grid search method with a genetic algorithm. Lin *et al.* (2004) proposed a migration operation and a population diversity measure to avoid clustering. Ko and Evans (2005) applied a genetic algorithm-based heuristic to solve a set of NP-hard problems. Stochastic methodologies have been also used to treat multi-objective optimization problems. Guillén *et al*. (2005) solve a supply chain design problem as a multi-objective stochastic MILP model. Chan *et al*. (2005) develop a hybrid genetic algorithm based on analytic hierarchy process to solve multi-factory supply chain models. Zhou and Hua (2000) use goal programming and analytic hierarchy process to address sustainable supply chain optimization and scheduling of continuous process industries. Azapagic and Clift (1999) use life cycle assessment in environmental management to solve a multi-objective optimization system.

Besides the inherent complexity of MINLP, the problem of finding any feasible solution may be itself NP-hard. Different approaches are employed to deal with constrained optimization problems. Some methods reject the infeasible solutions while others adopt repair operations. Modifying nearly-feasible solutions, however, may disrupt the schema excessively or incur undue computational overhead. The most promising methods make use of penalty functions (Ostermark, 1999). By penalizing infeasible individuals, these methods turn such individuals into mediocre ones. This procedure prevents the propagation of the infeasible solutions to future generations, since mediocre individuals have little chance to survive. Such strategy transforms constrained problems into unconstrained ones.

In this contribution, a new penalty function method based on fuzzy logic theory has been specially developed to treat problems in which feasible regions are very difficult to reach. The approach was first developed for multi-objective optimization, but it can be extended to any stochastic optimization algorithm. It comprises a dynamic penalty function based on the constraint classification and the intensity and frequency of violation. The optimization is encouraged to solve the constraints according to pre-established priority, until the feasible region is reached. The proposed formulation is illustrated on a numerical example of a multi-enterprise supply chain network. A multi-product, multistage and multi-period production and distribution-planning model is addressed. A multi-objective optimization algorithm based on evolutionary strategies is applied to determine the best configuration of the supply chain network. The proposed method has successfully attained a compromise solution among all participant enterprises, providing a balanced satisfaction for all objectives. The results of a hypothetical case study confirmed the ability of the proposed method in solving complex MINLP problems.

2. EVOLUTIONARY ALGORITHMS

Evolutionary algorithms are robust stochastic methods for global and parallel optimization. These methods are founded on the principles of natural genetics, in which the fittest species survive and propagate while the less successful tend to disappear. The evolution process consists of performing a population of individuals with operators to generate the next generation. The basic operators simulate the processes of selection, crossover and mutation, which happen according to pre-established probabilities. Selection is based on the survival potential, expressed by the fitness function. Crossover involves random exchange of characters between pairs of individuals, in order to produce new ones. Mutation is an occasional change in individual's characters randomly chosen. It introduces diversity to a model population. Evolutionary methods are able to deal with ill-behaved problem domains, such as the ones presenting multimodality, discontinuity, time-variance, randomness and noise.

Evolutionary algorithms are regarded to be suitable to solve multi-objective optimization problems as they work on a population of individuals. Multi-objective optimization is a special extension of the optimization theory in which multiple opposing targets must be accomplished simultaneously. The search process aims to find solutions that are the best on all objectives. The optimal solution constitutes a family of points, called Pareto optimal front, that equally satisfy the set of objective functions. An important characteristic of the Pareto set is that no improvement can be obtained in any objective without deteriorating at least one of the other objectives. As all objective functions are optimized at the same time, the solution constitutes a compromise between the conflicting aims.

3. THE PROPOSED STRATEGY

This work focuses on the solution of nonconvex MINLP optimization problems that involve discontinuous domains and a large number of constraints. Such problems are difficult to solve as they present multiple local optima dispersed in a discontinuous search space. Even stochastic algorithms can be easily trapped in a local optimum surrounded by an infeasible region. Additional difficulty emerges in case of discrete problems with binary variables. Any change in these variables may interrupt the search progress. Also, constraints involving these variables are easily violated, which makes feasible regions hard to be found.

In order to face these drawbacks, a heuristic strategy is proposed to provide multi-objective stochastic algorithms with an efficient tool to handle these difficulties. The strategy consists of a penalization procedure that incorporates the constraints into the objective function by means of a penalty function. This function associates a certain value with the extent each constraint is violated by each individual. The procedure is formulated as follows:

Minimize $F(\boldsymbol{x}, \boldsymbol{y}) = f(\boldsymbol{x}, \boldsymbol{y}) + P(\alpha, \boldsymbol{x}, \boldsymbol{y})$

where $P(\alpha, \boldsymbol{x}, \boldsymbol{y}) = \alpha_k \, \mathrm{SVC}(\boldsymbol{x}, \boldsymbol{y})$

$\boldsymbol{x} \in \boldsymbol{R}^n$, $\boldsymbol{y} \in \boldsymbol{Z}^m$, α is a predefined constant related to the k-th rank and $P(\alpha, \boldsymbol{x}, \boldsymbol{y})$ is the dynamic penalty function. SVC $(\boldsymbol{x}, \boldsymbol{y})$ is the sum of violated constraints, which incorporates the distance from the feasible set and the frequency of constraint violation:

$$\mathrm{SVC}(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=1}^{p} D_i(\boldsymbol{x}, \boldsymbol{y}) + \sum_{j=1}^{q} D_j(\boldsymbol{x}, \boldsymbol{y})$$

Inequality constraints:

$$D_i(\boldsymbol{x}, \boldsymbol{y}) = \begin{cases} 0, & \text{if } g_i(\boldsymbol{x}, \boldsymbol{y}) \geq -\varepsilon, \quad i = 1, \ldots, p \\ |g_i(\boldsymbol{x}, \boldsymbol{y})| & \text{otherwise} \end{cases}$$

Equality constraints:

$$D_j(\boldsymbol{x}, \boldsymbol{y}) = \begin{cases} 0, & \text{if } -\varepsilon \leq h_j(\boldsymbol{x}, \boldsymbol{y}) \leq \varepsilon, \; j = 1, \ldots, q \\ |h_j(\boldsymbol{x}, \boldsymbol{y})| & \text{otherwise} \end{cases}$$

A procedure based on the fuzzy logic theory is adopted to generate a hierarchical sequencing on the optimization process. The constraints are first arranged into classes, according to the level of difficulty offered to search evolution. Three classes are suggested: rank 1 - binary variables; rank 2 - discrete or integer variables; rank 3 - continuous variables. A finite value is associated with each rank, differing by at least one order of magnitude. The aim is to encourage the hardest constraints to be satisfied first. Constraints involving binary variables are usually the most demanding and must be strongly penalized. This strategy is essential to the process to succeed. As the degree of freedom continuously reduces during optimization, it will probably fail if the hardest constraints are left to the end.

The proposed strategy also introduces a modified mutation operator, which is applied when the search is trapped in an infeasible region. This operator consists of a Monte Carlo-based mutation, which simulates different procedures for early and final iterations, due to the distinct degrees of freedom. A wide mutation, called extensive Monte Carlo-based mutation, is first performed, alternating discrete and continuous variables. In final stages, a local Monte Carlo-based mutation is applied. In this case only variables directly associated to the violated constraints are mutated within a certain range. An EA-based algorithm developed in a previous work (Silva and Biscaia, 2003) is employed to solve MINLP problems. Some adaptations are required to introduce the proposed strategy into a general genetic algorithm, as suggested as follows:

1) create a random initial population;
2) evaluate the individuals and apply the penalty function method;
3) rank the individuals and calculate their fitness;
4) apply selection, crossover, mutation operators;
5) if progress fails for N iterations, apply extensive Monte Carlo-based mutation in a best individual;
6) repeat steps (2)-(5);
7) if iterative process stagnates for M attempts of step (5), apply a local Monte Carlo-based mutation in a best individual;
8) repeat steps (2)-(7) until no constraint is violated or a limit number of attempts is reached;
9) if all constraints are satisfied, register non-dominated individuals in the Pareto set filter;
10) if a limit number of attempts is reached repeat (1)-(9).

Other modifications of the original algorithm include a rounding procedure to operate in discrete variable space, reformulation of the mutation operator to perform changes in a random number of the characters and grouping of decision variables associated to each constraint. High mutation probabilities are also used to increase the algorithm's exploitation ability and improve the convergence.

4. PROBLEM STATEMENT

An example problem dealing with a multi-enterprise supply chain is considered in this work. It consists of a centralized three-echelon structure including manufacturing, storage and market. The structure comprises two retailers, two warehouses and one plant. The distribution channels consist of a smaller-scale distributor with fast delivery service and a larger-scale distributor with a slower delivery service. The larger-scale service implies lower operating costs, but has a transportation lead-time of one week. Delayed shipment problem is considered in the distribution system. The plant batch manufactures two different products. The production has a fixed cost associated and can be conducted in regular time or overtime, to satisfy customer demand. If the production line is idle, a fixed idle cost is added to the total manufacturing cost. The raw material purchasing cost is included in the manufacturing cost.

The overall problem aims to determine: a) production schedule, including production rates for all time intervals; b) transportation of products; c) sale quantity; d) costs and revenue of each enterprise and e) inventory level of each enterprise. Given: a) product sale prices; b) costs of unit manufacturing, transport, handling and inventory; c) manufacturing data in regular time and overtime; d) transportation data - capacity level and lead time; e) inventory capacity and safe inventory quantity and f) forecasted customer demand over a time horizon.

The objective is to determine the configuration of the supply chain that maximizes the profit of each enterprise, the customer service level and safe inventory level, taking into account a fair distribution of these targets among all the participants.

4.1 Model Formulation

The optimization problem is formulated as a multi-objective mixed-integer nonlinear programming (MOMINLP) problem. The mathematical formulation for the supply chain model was originally proposed by Chen *et al*. (2003). All parameters and system information are presented in the above-mentioned reference.

Objective functions:

Overall profit:

$$\max \sum_t Z_{rt} = \sum_i USR_{ir} S_{irt} - \sum_d \sum_i USR_{idr} S_{idrt} - \sum_i UIC_{ir} I_{irt} - \sum_i UHC_{ir} (\sum_d S_{idr,t-TLTdr} + S_{idrt}) \quad (1)$$

$$\max \sum_t Z_{dt} = \sum_r \sum_i USR_{idr} S_{idrt} - \sum_p \sum_i USR_{ipd} S_{ipdt} - \sum_i UIC_{id} I_{idt} - \sum_i UHC_{id} (\sum_d S_{ipd,t-TLTpd} + S_{idrt}) - \sum_k \sum_r (FTC_{kdr} Y_{kdrt} + UTC_{kdr} Q_{kdrt}) + \sum_{k'} \sum_p (FTC_{k'pd} Y_{k'pdt} + UTC_{k'pd} Q_{k'pdt}) \quad (2)$$

$$\max \sum_t Z_{pt} = \sum_d \sum_i USR_{ipd} S_{ipdt} - \sum_i [FMC_{ip} \gamma_{ipt} + FIC_{ip} (\beta_{ipt} - \alpha_{ipt}) + UMC_{ip} FMQ_{ip} \alpha_{ipt} + OMC_{ip} OMQ_{ip} o_{ipt}] - \sum_i UIC_{ip} I_{ipt} - \sum_i UHC_{ip} (FMQ_{ip} \alpha_{ip,t-1} + OMQ_{ip} o_{ip,t-1} + \sum_d S_{ipdt}) \quad (3)$$

Average customer service level:

$$\max \frac{1}{T} \sum_t \frac{100}{I} \sum_i \frac{S_{irt}}{FCD_{irt} + B_{ir,t-1}} \quad (4)$$

Average safe inventory level:

$$\max \frac{1}{T} \sum_t \frac{100}{I} \sum_i \left(1 - \frac{D_{irt}}{SIQ_{ir}}\right) \quad (5)$$

$$\max \frac{1}{T} \sum_t \frac{100}{I} \sum_i \left(1 - \frac{D_{idt}}{SIQ_{id}}\right) \quad (6)$$

$$\max \frac{1}{T} \sum_t \frac{100}{I} \sum_i \left(1 - \frac{D_{ipt}}{SIQ_{ip}}\right) \quad (7)$$

Constraints:

Inventory balance - Retailer:

$$I_{irt} = I_{irt,t-1} + \sum_d S_{idr,t-TLTdr} - S_{irt}$$

$$I_{irT} \geq SIQ_{ir}$$

Backlog level - Retailer:

$$B_{irt} = B_{irt,t-1} + FCD_{irt} - S_{irt}$$

$$B_{irT} = 0 \;\; I_{irt} \geq 0 \;\; B_{irt} \geq 0 \;\; S_{irt} \geq 0$$

Maximum inventory capacity - Retailer:

$$\sum_i I_{irt} \leq MIC_r$$

Safe inventory - Retailer:

$$SIQ_{ir} - I_{irt} \leq D_{irt} \leq SIQ_{ir}$$

$$D_{irT} = 0 \;\; D_{irt} \geq 0$$

Inventory balance - Distribution center:

$$I_{idt} = I_{idt,t-1} + \sum_p S_{ipd,t-TLTpd} - \sum_r S_{idrt}$$

$$I_{idT} \geq SIQ_{id} \;\; I_{idt} \geq 0 \;\; S_{idrt} \geq 0$$

Maximum inventories- Distribution center:

$$\sum_i I_{idt} \leq MIC_d$$

Shortage in safe inventories - Distribution center:

$$SIQ_{id} - I_{idt} \leq D_{idt} \leq SIQ_{id}$$

$$D_{idT} = 0 \;\; D_{idt} \geq 0$$

Output transportation - Distribution center:

$$\sum_k Q_{kdrt} = \sum_i S_{idrt}$$

$$TCL_{k-1,dr} Y_{kdrt} \leq Q_{kdrt} \leq TCL_{kdr} Y_{kdrt}$$

$$\sum_k Y_{kdrt} \leq 1$$

$$\sum_r \sum_k TCL_{kdr} Y_{kdrt} \leq MOTC_d$$

Input transportation - Distribution center:

$$\sum_{k'} Q_{k'pdt} = \sum_i S_{ipdt}$$

$$TCL_{k'-1,pd} Y_{k'pdt} \leq Q_{k'pdt} \leq TCL_{k'pd} Y_{k'pdt}$$

$$\sum_{k'} Y_{k'pdt} \leq 1$$

$$\sum_p \sum_{k'} TCL_{k'pd} Y_{k'pdt} \leq MITC_d$$

Inventory balance - Plant:

$$I_{ipt} = I_{ipt,t-1} + FMQ_{ip} \alpha_{ip,t-1} + OMQ_{ip} o_{ip,t-1} - \sum_d S_{ipdt}$$

$$I_{ipT} \geq SIQ_{ip} \;\; I_{ipt} \geq 0 \;\; S_{ipdt} \geq 0$$

Maximum inventory - Plant:

$$\sum_i I_{ipt} \leq MIC_p$$

Shortage in safe inventory constraints - Plant:

$$SIQ_{ip} - I_{ipt} \le D_{ipt} \le SIQ_{ip}$$

$$D_{ipT} = 0 \;\; D_{ipt} \ge 0$$

Manufacturing - Plant:

$$\sum_i \beta_{ipt} = 1 \;\; \alpha_{ipt} \le \beta_{ipt}$$

$$\gamma_{ipt} \ge \beta_{ipt} - \beta_{ip,t-1} \qquad o_{ipt} \le \alpha_{ipt}$$

$$\sum_i \sum_t o_{ipt} \le MTO_p$$

$$\sum_i \sum_n o_{ip,t-n+1} \le N - 1$$

5. RESULTS AND DISCUSSION

According to the problem description, three-levels of enterprises are integrated in a multi-objective optimization problem. Planning horizons varying from 3 to 8 weeks are tested. The multi-objective optimization problem consists of 12 objective functions. A population size of 25 individuals, crossover probability of 90% and mutation probability of 30% are used to solve the problem. Some of the results obtained for a three-week planning horizon are shown in Figure I. Each line on the graphics represents an optimal result. Table 1 presents the best results obtained for each objective function in some of the optimization cases. For the sake of space, the complete Pareto set is omitted.

Table 1. Best results

i	t = 3	t = 5	t = 6	t = 8
1	5.84×10^5	1.22×10^6	9.69×10^5	9.25×10^5
2	1.0	0.83	0.91	0.86
3	0.97	0.61	0.74	0.78
4	5.68×10^5	1.09×10^6	1.06×10^6	1.45×10^6
5	0.97	0.84	0.97	0.68
6	0.94	0.73	0.71	0.76
7	1.68×10^5	4.61×10^5	3.05×10^5	6.26×10^3
8	1.0	0.99	0.67	0.73
9	7.63×10^5	2.14×10^6	1.99×10^6	1.61×10^6
10	1.0	0.96	0. 94	0.69
11	1.05×10^6	1.52×10^6	1.09×10^6	2.54×10^6
12	1.0	0.76	0.77	0.72

High values for all objective functions are obtained, which indicates that the proposed strategy leads to an unbiased search process. A balanced exploration process is mandatory to obtain a good compromise solution for all objectives and satisfy a fair distribution. The results obtained for the most relevant objective functions, which represent each enterprise profit, do not differ in order of magnitude for each case study. Hence, any of the solutions in the Pareto set would be satisfactory to all participant enterprises.

Table 2 presents the number of variables and constraints of each optimization case, as well as the number of generations required. The three-week period problem was solved in 112 seconds on a Pentium IV 2.4 GHz. The eight-week problem, on the hand, took around 12 hours to perform 8,790 iterations. It should be highlighted that this computational effort is required to find the feasible region. In a previous work, the original version of the algorithm was used to solve the same problem (Silva and Biscaia Jr., 2005). The maximum planning horizon the algorithm was able to solve was three weeks.

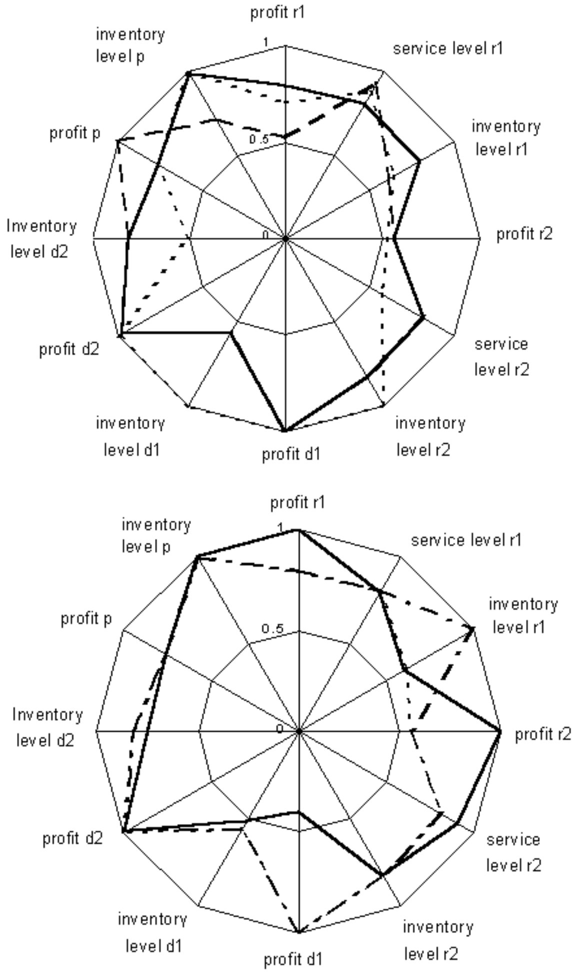

Fig. I. Optimal solutions

Table 2. Number of variables, constraints and iterations

	t = 3	t = 5	t = 6	t = 8
# var	122	254	320	452
# constr	172	344	430	602
# iter	1,066	3,190	4,090	8,790

6. CONCLUSIONS

In this contribution, a dynamic penalty formulation based on the fuzzy logic theory is proposed to solve highly constrained MINLP problems in which feasible regions are very difficult to be achieved. The strategy includes a constraint classification, which induces a hierarchy search progress; a penalty function, which incorporates different levels of penalization into the fitness function according to the constraint classification, the intensity and frequency of constraint violation; and a mutation operator, to prevent the search to stagnate. A problem involving a large number of difficult-to-satisfy constraints is presented to evaluate the performance of the algorithm. A multi-product, multistage and multi-period production and distribution-planning model, formulated as a multi-objective mixed-integer nonlinear programming (MOMINLP) problem, was selected. A compromise solution among all participant enterprises of the supply chain is achieved, ensuring a fair distribution profit. The results confirm the efficiency of the proposed approach to solve nonconvex MINLP problems involving large search spaces, number of constraints and objective functions.

NOTATION

Indices

i	products	r	retailers
d	distribution centers	p	plants
t	periods		
k	transportation capacity level from d to r		
k'	transportation capacity level from p to d		

Parameters

USR{i, pd, dr, r}	unit sale revenue of i
UICi{i, p, d, r}	unit inventory cost of i
UHC{i, p, d, r}	unit handling cost of i for p, d, r
UTC{k, dr}	kth-level unit transportation cost
FTC{k, dr}	kth-level fixed transportation cost
FTC{k', pd}	k'th-level fixed transportation cost
UMC{i, p}	unit manufacturing cost of i
OMC{i, p}	overtime unit manufacturing cost
FMC{i, p}	fixed manufacturing cost for changing plant to make i
FIC{i, p}	fixed idle cost to keep plant idle
FCD{i, r, t}	forecasted customer demand for i
TLT{pd, dr}	transportation lead time
SIQ{i, p, d, r}	safe inventory quantity
MIC{i, p, d, r}	maximum inventory capacity
TCL{k, dr}	kth transportation capacity level
MITC{d}	max. input transportation capacity
MOTC{d}	max. output transportation capacity
FMQ{i, p}	fixed manufacturing quantity of i
OMQ{i, p}	overtime fixed production quantity
MTO{p}	maximum total overtime in manufacturing period

Binary Variables

Y{k, dr, t}	kth transportation capacity
α {i, p, t}	manufacture in regular-time
β {i, p, t}	set up plant to manufacture i
γ {i, p, t}	change plant over to manufacture i
o{i, p, t}	manufacture with overtime workforce

Integer variables

S{pd, dr, r, t}	sales quantity of i
Q{k, dr, t}	kth-level transportation quantity
Q{pd, dr, t}	total transportation quantity
I{i, p, d, r, t}	inventory level of i in p, d, r
B{i, r, t}	backlog level of i in r at end of t
D{i, p, d, r, t}	shortage in safe inventory level
TMC{p, t}	total manufacturing cost of p
TPC{d, r, t}	total purchase cost of d, r
TIC{p, d, r, t}	total inventory cost of p, d, r
THC{p, d, r, t}	total handling cost of p, d, r
TTC{d, t}	total transportation cost of d
PSR{p, d, r, t}	product sales revenue of p, d, r
Z{p, d, r, t}	net profit of p, d, r

Continuous variables

SIL{ p, d, r,, t}	safe inventory level of p, d, r
CSL{r, t}	customer service level of r

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