

**AUTO-TUNING OF PID CONTROLLERS FOR
MIMO PROCESSES BY RELAY FEEDBACK****Lucíola Campestrini** Péricles Rezende Barros*
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Abstract: In this paper, the auto-tuning of decentralized PID controllers for MIMO processes using different relay feedback experiments is studied. These methods consist in identifying the ultimate point of the process, using relay feedback and applying Ziegler-Nichols-like formulae in order to tune the controllers. A benchmark illustrates the benefits and drawbacks of each approach.

Keywords: Process control, PID control, Multivariable feedback control, Relay control, Sequential control, Decentralized control

1. INTRODUCTION

Relay feedback is widely applied as a tuning method for single-input-single-output (SISO) processes. The most usual applications are based on Ziegler-Nichols and related tuning formulae, in which the tuning is performed based on the identification of one point of the process' frequency response - the ultimate point (Astrom and Hagglund, 1995). More sophisticated tuning, providing better robustness and performance, can also be achieved by relay feedback, using different experiments which identify several points of the frequency response (Arruda, 2003), (Johansson *et al.*, 1998), (Arruda *et al.*, 2002), (Goncalves, 2001). Such methods have shown to be quite effective for many years, and many auto-tuning techniques and commercial products are based on them.

Attempts to generalize relay-feedback tuning methods to the tuning of multiple-input-multiple-output (MIMO) plants have been described recently. Two forms of generalization have been studied: Decentralized Relay Feedback (DRF) and

Sequential Relay Feedback (SRF). In this paper we review the theory on these different approaches, discuss some results, present and illustrate these ideas by means of a case study.

In this paper, the processes to be considered are MIMO square processes, described by a transfer matrix

$$Y(s) = G(s)U(s) \quad (1)$$

with $Y(s), U(s) \in \mathfrak{R}^m$. The processes are assumed to be BIBO-stable. The aim is to design decentralized PID controllers

$$U(s) = C(s)Y(s) \quad (2)$$
$$= \begin{bmatrix} p_1(s) & 0 & 0 & \dots & 0 \\ 0 & p_2(s) & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & & \\ 0 & 0 & 0 & \dots & p_m(s) \end{bmatrix} \begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_m(s) \end{bmatrix}$$

where $p_i(s) = k_{pi} + \frac{k_{ii}}{s} + k_{di}s$.

2. SISO RELAY FEEDBACK

The PID design based on the identification of one point of the frequency response is considered.

This design approach relies on the application of simple formulae to tune the parameters of the PID. These formulae are given in terms of the process' ultimate quantities. Hence, the method consists simply in identifying somehow these characteristics and then applying the formulae. This method is usually referred to in the literature as the closed-loop Ziegler-Nichols method. Different formulae have been proposed over the years, after the pioneer work by Ziegler and Nichols. Given these variations in the method, in which the same procedure is applied but with different formulae, this tuning method will be referred as the *ultimate quantities* method in order to avoid confusion.

The ultimate quantities are the ultimate frequency ω_c and the ultimate gain K_c and, for SISO process, are defined as follows.

Definition 1. Let a BIBO-stable SISO process be controlled by purely proportional controller $u = -Ky$, with gain $K \in [0, \infty)$. Since the process is stable, the feedback system is stable for sufficiently small K . Assume that there exists a value K_c such that the closed-loop system is stable $\forall 0 \leq K < K_c$ and unstable for $K = K_c + \epsilon$, for ϵ an arbitrarily small positive scalar; this value K_c is the *ultimate gain* of the process. On the other hand, for $K = K_c$ the closed-loop system is on the verge of stability and hence a sustained oscillation will be observed; the frequency of this oscillation is called the ultimate frequency ω_c .

Fact 1. The ultimate gain K_c exists if and only if there exists a frequency ω_π , such that $\arg G(j\omega_\pi) = -\pi$. Furthermore, in this case $\omega_c = \omega_\pi$. (Astrom and Hagglund, 1995).

Different formulae have been proposed, with different ranges of application. Clearly, none of these formulae can be guaranteed to provide specified performance or even closed-loop stability. Yet, for a wide range of processes, they provide successful tuning.

Ziegler-Nichols PID formulae, which are used in this paper, are $k_p = 0.6K_c$, $k_i = k_p/T_i = k_p/(T_c/2)$ and $k_d = k_pT_d = k_pT_c/8$, where $T_c = 2\pi/\omega_c$.

2.1 The relay experiment

A convenient procedure to determine the ultimate quantities is the relay experiment. This experiment consists in putting the process under *relay feedback*. The relay function $\eta(\cdot)$ is described as

$$\eta(e) = -\text{dsign}(e) + \text{bias} \quad (3)$$

where $\text{bias} = r/G(0)$. Then, if the process satisfies the assumptions in Fact 1, a symmetric oscillation is observed, and this oscillation satisfies the statements below.

Fact 2. Let ω_{osc} be the frequency of the oscillation observed in the relay experiment and A_{osc} be its amplitude at the relay input. Then

$$\omega_\pi = \omega_{osc} \quad (4)$$

$$K_c = \frac{4d}{\pi A_{osc}} \quad (5)$$

Since the ultimate frequency is such that $G(j\omega_c) = -\pi$, $\omega_{osc} = \omega_\pi$ gives an estimate for the ultimate frequency, so that both ultimate quantities can be obtained by means of the relay experiment.

More complex phenomena can occur in relay feedback systems, such as multiple limit cycles, multimodal limit cycles, and even chaotic behavior (Arruda, 2003), (Goncalves, 2001) and (Johansson *et al.*, 1998), which complicates and sometimes invalidates the relay experiment as a means to determine the ultimate quantities of the process. Safeguards against these phenomena and/or further signal processing to extract the ultimate quantities even in their presence must be provided in PID auto-tuning with relay feedback. Still, the relay experiment has been largely and successfully applied to SISO processes (Astrom and Hagglund, 1995).

3. SEQUENTIAL RELAY FEEDBACK

In Sequential Relay Feedback (SRF), the control loops are put under relay feedback one at a time, sequentially, and m relay experiments are performed. After the relay experiment is applied to a given loop, the loop is closed with the tuned PID before applying the relay experiment to the next loop.

At each step of the SRF procedure, one PID controller is applied to an input-output pair for which a SISO relay experiment has been performed. The input-output relationship "seen" by the controller is the same one that was identified by the relay experiment. So, from the point of view of closed-loop stability, the SRF procedure is as safe as a SISO tuning.

However, each controller is tuned taking into consideration only the dynamics of the previously tuned loops. Indeed, at the i -th step, all the loops indexed from $i + 1$ until m are open and therefore have no influence on the system behavior. To correct this drawback, an iterative procedure can be performed: after closing all the loops the procedure is repeated for all the loops. In this

second time the loops are closed, so that all the couplings are observed in the relay experiments. Although good results are usually obtained with this iterative procedure, convergence of the tuning parameters is not guaranteed, and - most important - the total number of experiments is large; if k iterations are to be performed, km relay experiments are necessary.

4. DECENTRALIZED RELAY FEEDBACK

In Decentralized Relay Feedback (DRF) only one experiment is performed, with all control loops in relay feedback, that is $u_i = \eta(e_i) \forall i$. Since all the input-output pairs are connected, the behavior of the whole multivariable system is observed at this one experiment.

However, the theoretical analysis necessary to the correct application of this procedure is by necessity of a MIMO nature. The SISO reasoning and theoretical analysis do not apply in this case, and can only be useful as intuitive guidelines. The current understanding of DRF is far from complete, so it is not clear how to make the best use of the information provided by it.

4.1 Theoretical Analysis

The relay experiment in the SISO case serves to identify the ultimate quantities of the process: the ultimate gain K_c and the ultimate frequency ω_c . Based solely on these ultimate quantities the PID tuning is determined. The definitions of ultimate quantities can be made *mutatis mutandis* for MIMO systems.

Definition 2. Let a BIBO-stable square process with m inputs be controlled by purely proportional controller $u = -Ky$, with gain $K = \text{diag}\{k_1 \ k_2 \ \dots \ k_m\}$, $k_i \in [0, \infty)$. Since the process is stable, the feedback system is stable for sufficiently small K . Assume that there exists a value K_c such that the closed-loop system is stable $\forall K = \alpha K_c$, $0 < \alpha < 1$ and unstable for $K = K_c(1 + \epsilon)$, with ϵ an arbitrarily small positive scalar; this value K_c is called an *ultimate gain* of the process. On the other hand, for $K = K_c$ the closed-loop system is on the verge of stability and hence a sustained oscillation will be observed; the frequency of this oscillation is called an *ultimate frequency* ω_c .

As discussed in Section 2, the ultimate quantities are usually unique in the SISO case; even when they are not unique, they are countable. The situation is quite different in the MIMO case, since the gain K can be increased from 0 in infinite

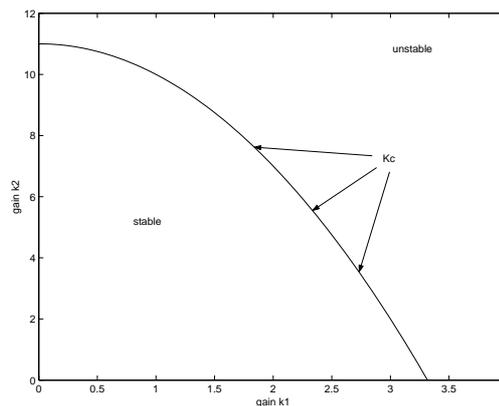


Fig. 1. MIMO case

different directions in the parameter space. It can be expected that a different K_c and ω_c will be found for each different direction, as depicted in Fig. 1. The ultimate gains form a curve in the parameter space. In the more general ($m > 2$) case these gains will form a surface of dimension $m - 1$; this surface will be called the *ultimate surface*.

If PID tuning is determined based on the ultimate quantities, using Ziegler-Nichols like formulae, then two things must be realized. First, all PID's will be tuned based on the same ultimate frequency. Second, the tuning will be dependent on which of the ultimate quantities has been identified.

For simplicity, let $m = 2$. Consider that the gain K is increased in the direction $K = \text{diag}\{k_1 \ 0\}$, that is, the second loop is kept open, and the proportional gain in loop 1 is increased. The ultimate gain that will be obtained in this experiment is the SISO ultimate gain k_{c1} of the first loop. Then, if the PID's are tuned according to these ultimate quantities, this tuning is the "correct" one for the first SISO loop. On the other hand, if the gain K as $K = \text{diag}\{0 \ k_2\}$ is increased then the "correct" SISO tuning for the second loop is obtained. If a different direction in the parameter space is picked to increase the gain K , then it is expected to find a tuning that will not be optimal for any of the two loops, but will represent some sort of "average" of the two. The closer this direction is to either one of the two SISO directions defined above, the closer the tuning will be to the corresponding "correct" SISO tuning for that loop (Halevi *et al.*, 1997).

Let r_i be the relative importance given by the designer to the i -th loop, so that $\sum_{i=1}^m r_i = 1$. Then the desired ultimate point is the one that is in the direction of the vector $K_d = \text{diag}\{g_{11}(0)r_1 \ g_{22}(0)r_2 \ \dots \ g_{mm}(0)r_m\}$, where g_{ii} 's are the diagonal terms of $G(s)$.

Once the ultimate quantities are identified, a gain sufficiently smaller than the ultimate one should guarantee stability and enough stability margins.

This is the reasoning in the SISO case, and also the reasoning justifying ZN-like formulae. However, this is not necessarily true in the MIMO case. If the ultimate curve in the parameter space is convex, then adequate stability margins are guaranteed by taking a gain $K = \alpha K_c$ for sufficiently small α . However, it is not possible to guarantee that the ultimate surface is convex, or even smooth. As a matter of fact, there are cases in which the ultimate surface is not convex, like in the benchmark studied in the sequel.

5. CASE STUDY - DISTILLATION COLUMN

Aiming to demonstrate the use of SRF and DRF methods, this section presents some results obtained for the control of the Wood and Berry distillation column, whose transfer matrix is presented in (6). This process has been widely used as a benchmark (Loh and Vasnani, 1994) and (Wang *et al.*, 1997). It is hard to control since it has significant transport delay and strong coupling.

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{10.9s + 1} & \frac{-18.9e^{-3s}}{14.4s + 1} \\ \frac{16.7s + 1}{6.6e^{-7s}} & \frac{21s + 1}{-19.4e^{-3s}} \end{bmatrix} \quad (6)$$

5.1 Real \times Estimated Ultimate Surface

The ultimate surface of a process can be obtained from the very definition of ultimate quantities, that is, by applying increasing gains to the inputs of the plant until a sustained oscillation is obtained. When a sustained oscillation is presented, the “gains” applied in the plant are one ultimate point of the surface. The ultimate gains are then determined within a certain precision, which is given by the size of the steps used to increase the gains. So, although this curve is still an approximation to the actual ultimate surface, we will refer to it as the “real ultimate surface”; the estimate obtained by DRF will be referred to as the “estimated ultimate surface”.

Fig. 2 shows two ultimate surfaces of the Wood and Berry distillation column: the estimated one, obtained by Decentralized Relay Feedback (dotted curve), and the real one (solid curve). It is clear from the figure that the DRF experiment does not always provide good estimates of the ultimate quantities.

As described in section 4.1, the closer the direction of the ultimate point is to either one of the two directions in the parameter space (k_1 or k_2), the closer the tuning will be to the corresponding “correct” SISO tuning for that loop. So, it is expected to find a point in the ultimate surface

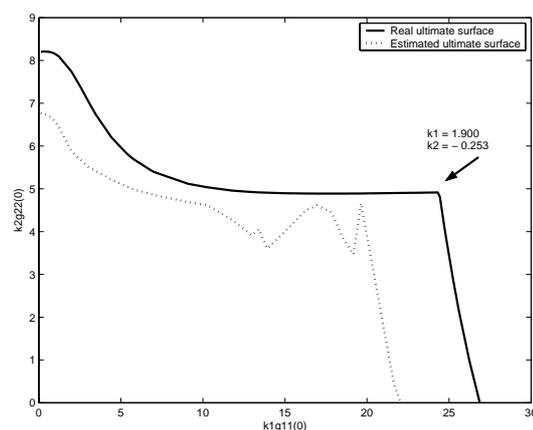


Fig. 2. Real and estimated ultimate surfaces of the Wood and Berry distillation column.

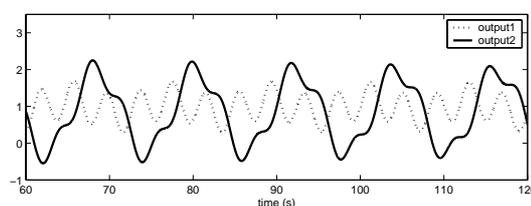


Fig. 3. Plant's outputs when a proportional controller $K = \text{diag}\{1.9 \ -0.2533\}$ is applied.

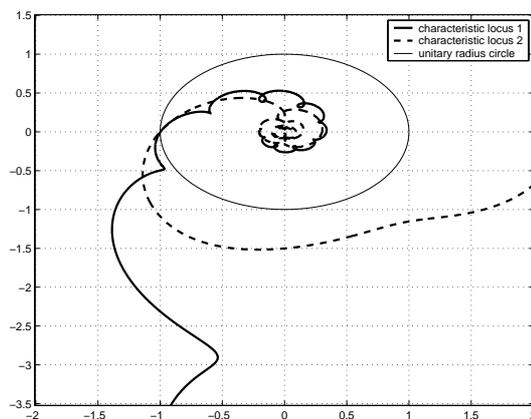


Fig. 4. Characteristic loci of the distillation column with a proportional controller $K = \text{diag}\{1.9 \ -0.2533\}$.

where information about both loops can be found. Hence, with this ultimate point, it is expected a better tuning of the PID controllers.

In fact, there is a point at the distillation column's ultimate surface for which the system presents multimodal oscillations, as presented in Fig. 3. This ultimate point is given by $k_1 = 1.9$ and $k_2 = -0.2533$. When this ultimate gain is applied on the plant, both characteristic loci cross the point -1 , as shown in Fig. 4.

Fig. 5 shows the response of the system controlled by PIs obtained with ultimate values $k_1 = 1.9$, $k_2 = -0.2533$ and $t = 12$ from the real ultimate surface and Ziegler-Nichols like formulae. It can be

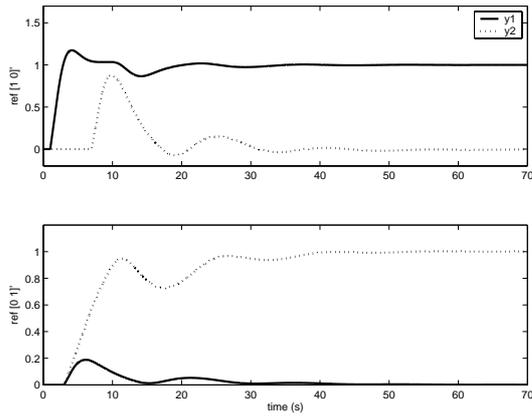


Fig. 5. Response of the system controlled by PI-Ziegler-Nichols like formulae - obtained with ultimate point $k_1 = 1.9$ and $k_2 = -0.2533$.

Table 1. Ultimate values and PID controller's gains obtained from SRF (1st iteration: detuning and 2nd iteration: Ziegler-Nichols).

	1 st iter.		2 nd iter.	
	loop 1	loop 2	loop 1	loop 2
K_c	1.72	-0.27	1.59	-0.19
T_c	3.90	12.37	3.80	12.27
k_p	0.342	-0.163	0.955	-0.112
k_i	0.058	-0.026	0.503	-0.018
k_d	0.500	-0.252	0.454	-0.172

seen that, choosing the “right point”, the relative controller provides a good performance to the system. Unfortunately, this point cannot be reached with relay experiment, because it presents multimodal oscillations and the ultimate quantities cannot be taken from it. Indeed, there is a region around this point in which the relay experiment does not work, as can be seeing in the estimated ultimate surface from Fig. 2.

5.2 PID Tuning

5.2.1. Sequential Relay Feedback A sequential relay feedback was performed in the distillation column. The results are shown in Table 1 and Figs. 6 and 7. Table 1 presents the ultimate quantities obtained for each loop and the related gains of the PID controllers. Besides, these values were obtained twice, for first and second iterations.

The results are satisfactory, suggesting that this method can be used in order to find the ultimate quantities for further PID tuning. The system presents a considerable overshoot for reference $r = [1 0]'$ because Ziegler-Nichols like formulae are used. On the other hand, the disturbance rejection is strongly enhanced from first to second iteration in the second loop.

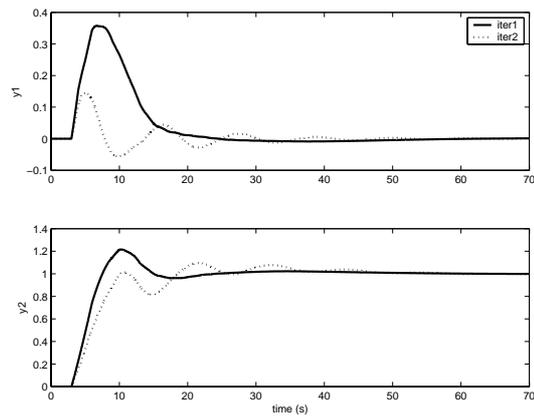


Fig. 6. Response of the system controlled by PID obtained from SRF and $r = [0 1]'$.

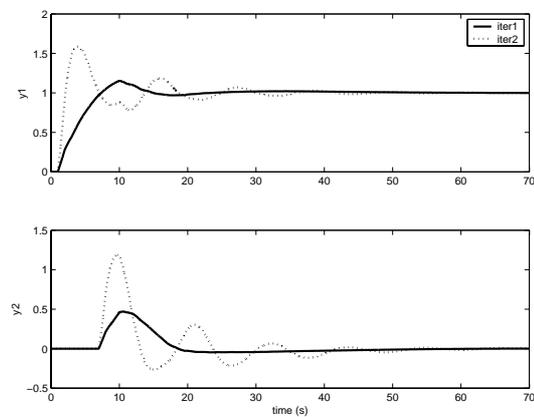


Fig. 7. Response of the system controlled by PID obtained from SRF and $r = [1 0]'$.

5.2.2. Decentralized Relay Feedback In order to identify the ultimate surface, several decentralized relay feedback experiments were performed in the system, in which different relay amplitude ratios d_2/d_1 were set. Hence, different ratios d_2/d_1 correspond to different weights $k_2g_{22}(0)/k_1g_{11}(0)$.

As can be seen in Fig. 2, the estimated surface with DRF is non-smooth for a wide range of gains. Within this range, the system presents multimodal limit cycles. So, it is only safe to tune PID controllers based on information taken from unimodal oscillations. Table 2 and Figs. 8 and 9 show the results obtained for different weights.

Fig. 8 shows the response of the system controlled by PID obtained from DRF and reference $r = [0 1]'$. It shows that, the higher the weight $k_2g_{22}(0)/k_1g_{11}(0)$, the larger is the overshoot of the system. However, for a reference equal to $r = [0 1]'$, the higher the weight $k_2g_{22}(0)/k_1g_{11}(0)$, the smaller is the overshoot of the system, as shown in Fig. 9. Thus, the hard task is to find a weight that will yield an ultimate point that provides good performance of both loops.

Table 2. Ultimate values and PID controller's gains obtained from DRF for different weights ($k_2g_{22}(0)/k_1g_{11}(0)$).

$k_2g_{22}(0)/k_1g_{11}(0)$		loop 1	loop 2
10.44	K_c	0.050	-0.345
	T_c	11.2	11.2
	k_p	0.029	-0.203
	k_i	0.005	-0.036
	k_d	0.041	-0.285
2.96	K_c	0.155	-0.303
	T_c	12.4	12.4
	k_p	0.091	-0.178
	k_i	0.015	-0.029
	k_d	0.142	-0.276
0.52	K_c	0.707	-0.241
	T_c	12.7	12.7
	k_p	0.416	-0.142
	k_i	0.065	-0.022
	k_d	0.660	-0.226

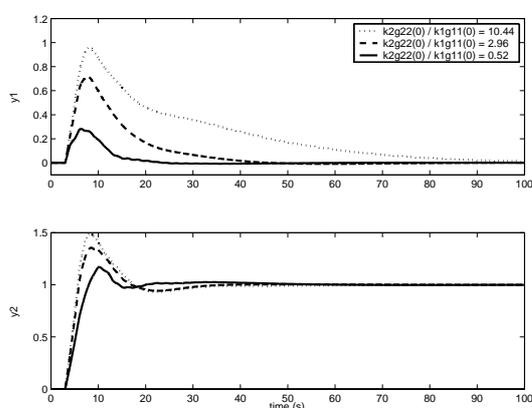


Fig. 8. Response of the process controlled by PID obtained from DRF, reference $r = [0 \ 1]'$ and different weights.

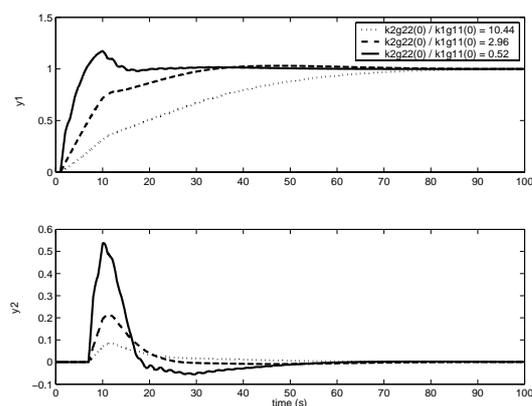


Fig. 9. Response of the process controlled by PID obtained from DRF, reference $r = [1 \ 0]'$ and different weights.

6. CONCLUDING REMARKS

In this paper, two forms of generalization of relay feedback tuning to multivariable processes are studied: Sequential and Decentralized Relay Feedback. These “multivariable” methods extend the SISO relay feedback to MIMO processes and

uses given formulae in order to tune the PID controllers.

The decentralized relay experiment seems to be the best way to obtain the ultimate quantities of the process, since it observes the behavior of the whole multivariable system and requires only one experiment. However, it can present multimodal oscillations, in which case the oscillations are not representative of the ultimate quantities. Hence, it is still a risky approach to tuning. Even if it is expected that this oscillation presents more information about the process, it is still not known how to get them from the experiment. Indeed, the point $k_1 = 1.9$, $k_2 = -0.2533$ of the real ultimate surface, seems to be the point that intersects both loops surface, and more characteristics of the process are expected to be obtained in this point.

On the other hand, the Sequential Relay Feedback requires more time to get the “right” controller parameters. However, as only one relay is applied at a time, it is a safer experiment because multimodal oscillations are not troublesome.

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