ON INPUT-OUTPUT SELECTION FOR MULTILOOP CONTROL: FROM RGA TO ROMA

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Abstract: A new tool for selecting the right pairing between inputs and outputs in a multiloop system is introduced. Similar in structure to the classical RGA the new array, called a Relative Omega Array, is based on the characteristic frequencies in open and closed loop under perfect control, as eventually detected by a classical relay test. This way the dynamic properties of the system are simply taken into account. Some examples show that the new tool is effective, giving the correct pairing also when the RGA approach fails.

Keywords: Multivariable control, RGA, interaction, auto-tuning, integrity.

1. SELECTION OF PAIRING INPUT AND OUTPUT VARIABLES IN DECENTRALIZED CONTROL SCHEMES

In multi-loop industrial control systems decentralised multi-loop SISO controllers are widely used, despite the interactions between the input/output variables (Luyben, 1997, Ogunnaike, 1994, Mayne, 1973). A decentralised structure is usually preferred for large scale industrial processes, since its simplicity, especially in case of sensor or actuator failures, where a process engineer can easily modify the controller parameters in order to counteract the abnormal operating condition. Experience show that usually performance of decentralised control structures meets satisfactory process design requirements. Performance improvements obtained adopting more refined (and complex) controllers are not usually so relevant to justify additional costs for their implementation and maintenance. Before developing a control structure design for a multivariable process, some basic question must be answered (Havre, 1996):

- What outputs must be controlled?
- How to select the control variables?
- How to pair input and output variables?
- How to tune the controllers?

The selection of controlled outputs essentially depends on the decision of the expert and on the physical insight of the process. In the selection it will be necessary to keep into account costs of production, safety in terms of protecting plant personnel and plant investments, physical limitations, availability and reliability of the sensors.
It can be required to control simultaneously levels of liquids, pressures, temperatures, positions, speed, product quality, production rates. In real plants often the choice of the variable to be controlled is clear, but not always such variables are directly measurable and they must be estimated from other measures. The problem of loop pairing between controlled and manipulated variables was usually solved by the relative gain array (RGA) method, introduced in 1966 by Bristol, and its several extensions. Suppose for simplicity to consider a multi-variable system with an equal number of controlled and manipulated variables, described by the matrix of transfer functions G(s). In the classic formulation due to Bristol, it is considered the matrix of the steady-state process gains of the system A = G(0) and it is defined the Relative Gain Array (RGA) as:

$$\Lambda = A \otimes (A^T)^{-1} = A \otimes A^{-T} \qquad (1)$$

where $\otimes$ denotes element-by-element product.

Definition: the RGA matrix $\Lambda = \{ \lambda_{ij} \}$ is formed by the generic λ_{ij} element which corresponds to the ratio of the open loop and closed loop gains between input j and output i, representing first the process gain in an isolated loop and, second, the apparent process gain in that same loop when all other control loops

are closed. Two main hypotheses are posed: matrix Λ is evaluated at steady-state and the control is perfect, i.e., the closed loop gains are evaluated when all other outputs $h \neq i$ are ideally regulated to zero.
RGA matrix (1) has several properties: the more relevant for following developments (Ogunnaike, 1994, Skogestad, 1997) are:

1) Any row or column sums to one.
2) The relative gain is invariant under scaling, i.e., $\Lambda(M) = \Lambda(PMQ)$, where P and Q are arbitrary diagonal matrices.
3) The only effect of altering the order of rows or columns in K is to introduce the same alteration of order in Λ.

The meaning of the relative gains λ_{ij} is that for ideal decentralised control the pairing should have a value of $\lambda_{ij} = 1$.
If $\lambda_{ij} = 0$, it implies that the steady-state gain of a single loop is zero, or that the interaction is so high that the behaviour of the loop is totally affected by the other loops. Such interaction has opposite effects if λ_{ij} is negative: in this case the open and closed loop gains have opposite signs, that is the closed loop multiloop process is unstable, or the single loops with negative λ_{ij}'s are unstable if the remaining loops are turned off, or the multiloop process is unstable if the loops with negative λ_{ij}'s are turned off (e.g., in case of failure in the i-j loop (Grosdidier, 1986)).
Based on the previous properties a suitable use of RGA matrix leads to an easy and practical rule for selecting the less interacting pairings: the variable pairings corresponding to positive relative gains λ_{ij} as close to unity as possible are preferred.

To illustrate the use of the RGA method and its limitations, consider the following examples:

Example 1.

Wood and Berry process (Ogunnaike, 1994):

$$P_1(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}$$

The steady-state RGA is:

$$\Lambda_1 = \begin{pmatrix} 2.0094 & -1.0094 \\ -1.0094 & 2.0094 \end{pmatrix}$$

It suggests the use of a diagonal pairing (y_1-u_1, y_2-u_2) in a good agreement the physical behaviour of the process.

Example 2.

Process described by the following matrix transfer function (Meeuse, 2002):

$$P_2(s) = \begin{pmatrix} \dfrac{e^{-s}}{1+s} & \dfrac{1}{1+s} \\ \dfrac{-1}{1+s} & \dfrac{e^{-2s}}{1+s} \end{pmatrix}$$

The steady-state RGA is:

$$\Lambda_2 = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

In this example the steady-state RGA does not suggest any preferential pairing.

Example 3.

Process described by the following matrix transfer function (Seider, 1999):

$$P_3(s) = \begin{pmatrix} \dfrac{2.5e^{-5s}}{(1+15s)(1+2s)} & \dfrac{1}{1+4s} \\ \dfrac{1}{1+3s} & \dfrac{-4e^{-5s}}{1+20s} \end{pmatrix}$$

The steady-state RGA is:

$$\Lambda_3 = \begin{pmatrix} 0.9091 & 0.0909 \\ 0.0909 & 0.9091 \end{pmatrix}$$

It suggests the use of a diagonal pairing (y_1-u_1, y_2-u_2), but a practical implementation based on dynamical considerations leads to off-diagonal pairing (y_1-u_2, y_2-u_1). In this example the static RGA fails.

2. A CRITICAL REVIEW

As suggested from the previous examples a simple application of the steady-state RGA can lead to wrong pairings or may not help the designer. The most important limitations of the static RGA can be summarized as:

- it doesn't include dynamics and a correct pairing should be frequency-dependent
- an optimal pairings may vary with the structure of the SISO controllers adopted
- RGA cannot discriminate diagonal processes from processes with triangular structure
- it does not consider disturbances.

Anyway, a primary advantage of RGA is that it requires only minimal process information; it relies on the knowledge of the steady-state gain matrix, very simple to measure.
Therefore in literature many researchers tried to extend the basic RGA definition, with several modifications.

A frequency-dependent RGA matrix can be introduced as (Grosdidier, 1986, Witcher, 1977):

$$\Lambda(s)=P(s)\otimes(P(s)^T)^{-1}=P\otimes P^{-T} \qquad (2)$$

Unfortunately, there are some arguments against the use of this frequency-dependent RGA: an ideal mathematical model of the process is usually unknown and it is very sensitive to modelling errors. Furthermore a classical frequency-based analysis will require to consider and analyse n(n+1) Bode plots, which is very time-consuming.

A different approach (Karlsmose, 1994) combines the frequency-dependent RGA method with the singular value decomposition of the transfer matrix representing the process, for a quantitative analysis of the results in the frequency domain.

A different method for measuring interaction is based on the Niederlinski index (Bristol, 1966, Niederlinski, 1971, Chiu, 1991), defined as:

$$N(s)=\frac{\prod_{i=1}^{n} P_{ii}(s)}{\det P(s)} \qquad (3)$$

This index is unitary, if P(s) is diagonal or triangular; an application of (3) to example 3 is able to select the correct pairing. Niederlinski index cannot discriminate the correct pairing in the case of example 2.

In practice it is possible to use more criteria sequentially, for selecting correct pairings; for example a first analysis can be performed using the classic RGA method, then the Niederlinski index can be applied, after discarding the negative pairings. If necessary it can be used a singular value decomposition.

In industrial applications seldom process engineers cope with so complex and unusual problems, with a high number of input/output variables. Furthermore the knowledge of the process at a physical-chemical level suggests often in a natural way the selection of the pairing between controlled and manipulated variables. Nevertheless the problem of the pairing maintains a remarkable interest, both from a theoretical point of view and in practice.

In the following section a different solution of this problem is addressed, suggesting a new index of dominance and the method for its practical application.

3. ROmA INDEX

In the design of a decentralized control system, standard proportional-integral-derivative (PID) controllers have remained the most popular ones in the industry since the 1950s, due to their simplicity and immediate way of operation. Relay feedbacks and auto-tuning techniques (Semino, 1998) are simple, powerful, and commonly used methods of finding system parameters useful for designing and tuning PID controllers: their parameters can be easily set from the knowledge of the ultimate gains (k_{ij}'s) and frequencies (ω_{ij}'s) (Loh, 1993, Dhen, 1994). The auto-tuning method ATLS of Loh and Shen ranks the loops according to their speed and the fastest loop is to be tuned first (Toh, 2002).

This procedure is in a good agreement (Leonhard, 2001) with the practice of the experts of cascade-control, especially in the field of high-performance electrical drives, where the synthesis proceeds closing first the inner faster loop and proceeding toward the slowest outer loops.

If we consider the generic loop between the i-th output variable and the j-th input variable, it will be described by the $P_{ij}(s)$ transfer function; if we consider the insertion of a standard $C_{ij}(s)$ controller, the open loop transfer function is $G_{ij}(s) = P_{ij}(s)\, C_{ij}(s)$.

The ideal closed loop transfer function, keeping all other loops open is:

$$W_{ij} = G_{ij}(s)/(1+ G_{ij}(s)) \qquad (4)$$

At the critical frequency ω_{ij}, we obtain:

$G_{ij}(\omega_{ij}) = -m_{ij}$,

and

$W_{ij}(\omega_{ij}) = -m_{ij}/(1-m_{ij})$.

In single-input single-output systems the critical frequency ω_{ij} remains unchanged in the passage from open loop to closed loop; m_{ij} represents the relative gain margin, strictly less than one for guaranteeing the loop stability. Of course this property holds also for multi-input multi-output systems if the systems are decoupled.

Consider now the introduction of a new interaction measure, taking into account dynamics. It relies on the classic definition RGA-like, where the measure is expressed as the ratio of a variable correlated to a single loop of the process under test with all other outputs uncontrolled, and the same variable when all other outputs are perfectly controlled. In this last situation the transfer function between the i-th output variable and the j-th input variable is modified, due to the interaction of all other control loops. For RGA case the variable under test is the steady-state gain, but in our proposal we consider the critical frequencies ω_{ij}.

Define $\hat{W}_{ij}(s)$ as the new transfer function in case of perfect control and $\hat{\omega}_{ij}$ the corresponding new critical frequency. For non interacting systems, $\hat{\omega}_{ij} = \omega_{ij}$ so that, by mimicking the RGA procedure, the ratios $\omega_{ij}/\hat{\omega}_{ij}$ are considered for creating a new matrix $F=\{\omega_{ij}/\hat{\omega}_{ij}\}$, as a different dominance index.

Dominance is guaranteed if a ratio $\omega_{ij}/\hat{\omega}_{ij}$ tends to one. The pairings can be easily verified introducing the matrix ROmA (Relative Omega Array), in a way analogous to the RGA definition:

$$\Psi = F \otimes F^{-T} \qquad (5)$$

Note that the ROmA matrix retains all the properties of the RGA matrix.

<u>Example 1</u>: Wood and Berry process (Ogunnaike, 1994):

$$P_1(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}$$

Operating conditions: a relay and a fixed delay of 4 s are inserted in the control loop. The delay is inserted for maintaining physical causality in all following tests. In plain words all other loops are open, only the loop under test is closed for detecting its critical frequency.

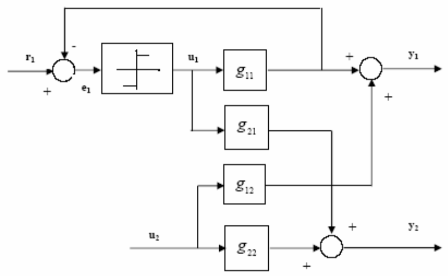

Fig. 1. A 2x2 system with a relay test in the first loop.

Results of the tests are:
A) closing the first loop y_1-u_1 the critical frequency is ω_{11}=0.3553 rad/s.
B) closing the second loop y_2-u_2 the critical frequency is ω_{22}=0.2688 rad/s.
Note that to induce a stable oscillation in the second loop the sign of the input u_2 has to be inverted.
This means that pairing is possible only by changing sign to u_2 ; therefore the test procedure continues inverting the sign of this input.
The system under test considering the added delay and the correct signs becomes:

$$P_1^{'}(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{19.4e^{-s}}{14.4s+1} \end{bmatrix} e^{-4s}$$

C) closing the loop y_1-u_2 the critical frequency is ω_{12}=0.192 rad/s
D) closing the loop y_2-u_1 the critical frequency is ω_{21}=0.2567 rad/s

Consider now the oscillating conditions in the hypothesis of a perfect control.
A1) the loop y_1-u_1 is closed in the hypothesis the an ideal controller in the other loop may guarantee y_2 = 0. Then:

$$\begin{cases} y_{1CL} = \left(g_{11} - \dfrac{g_{12}g_{21}}{g_{22}} \right) u_1 \\ y_{1OL} = g_{11}u_1 \end{cases}$$

and, after simple computations:

$$y_{11CL} = \left(12.8\frac{e^{-s}}{(16.7s+1)} - 6.43\frac{(1+14.4s)e^{-7s}}{(1+10.9s)(1+21s)} \right) u_1$$

Pre-multiplying by e^{-4s}:

$$y_{11CL} = \left(12.8\frac{e^{-5s}}{(16.7s+1)} - 6.43\frac{(1+14.4s)e^{-11s}}{(1+10.9s)(1+21s)} \right) u_1$$

The critical frequency $\hat{\omega}_{11}=0.4012$ rad/s is detected.

B1) In a similar way the loop 2-2 is closed:

$$\begin{cases} y_{2CL} = \left(g_{22} - \dfrac{g_{12}g_{21}}{g_{11}} \right) u_2 \\ y_{2OL} = g_{22}u_2 \end{cases}$$

$$y_{22CL} = \left(-19.4\frac{e^{-3s}}{(14.4s+1)} + 9.7453\frac{(1+16.7s)e^{-9s}}{(1+10.9s)(1+21s)} \right) u_2$$

.

Pre-multiplying by e^{-4s}:

$$y_{22CL} = \left(-19.4\frac{e^{-7s}}{(14.4s+1)} + 9.7453\frac{(1+16.7s)e^{-13s}}{(1+10.9s)(1+21s)} \right) u_2$$

The critical frequency $\hat{\omega}_{22}=0.3185$ rad/s is measured.

C1) For the loop 1-2:

$$y_{12CL} = \left(-18.9\frac{e^{-3s}}{(21s+1)} + 37.6242\frac{(1+10.9s)e^{3s}}{(1+16.7s)(1+14.4s)} \right) u_1$$

Pre-multiplying by e^{-4s}:

$$y_{12CL} = \left(-18.9\frac{e^{-7s}}{(21s+1)} + 37.6242\frac{(1+10.9s)e^{-s}}{(1+16.7s)(1+14.4s)} \right) u_1$$

The critical frequency $\hat{\omega}_{12}=1.582$ rad/s is measured.

D1) For the loop 2-1:

$$y_{21CL} = \left(6.6\frac{e^{-7s}}{(10.9s+1)} - 13.1386\frac{(1+21s)e^{-s}}{(1+16.7s)(1+14.4s)} \right) u_2$$

Pre-multiplying by e^{-4s}:

$$y_{21CL} = \left(6.6\frac{e^{-11s}}{(10.9s+1)} - 13.1386\frac{(1+21s)e^{-5s}}{(1+16.7s)(1+14.4s)} \right) u_2$$

The critical frequency $\hat{\omega}_{21}$=0.4095 rad/s is measured.

Therefore the matrix of the relative frequencies is:

$$F = \frac{\omega_{OL}}{\omega_{CL}} = \begin{pmatrix} \frac{0.3553}{0.4012} & \frac{0.192}{1.582} \\ \frac{0.2567}{0.4095} & \frac{0.2688}{0.3185} \end{pmatrix}$$

The relative omega array (ROmA) index is:

$$\Psi = F \otimes F^{-T} = \begin{bmatrix} 1.1133 & -0.1133 \\ -0.1133 & 1.1133 \end{bmatrix} = ROmA$$

Pairing suggested by the ROmA matrix is in a perfect agreement with the RGA rule:

$$\Lambda = A \otimes A^{-T} = \begin{bmatrix} 2.0094 & -1.0094 \\ -1.0094 & 2.0094 \end{bmatrix} = RGA \ .$$

<u>Example 2.</u> Consider now the process (Meeuse, 2002):

$$P_2(s) = \begin{pmatrix} \frac{e^{-s}}{1+s} & \frac{1}{1+s} \\ \frac{-1}{1+s} & \frac{e^{-2s}}{1+s} \end{pmatrix}$$

Introducing the new scaled input variables $v_1 = u_2$, $v_2 = - u_1$ the new matrix under test is:

$$P_2' = \begin{bmatrix} \frac{1}{1+s} & -\frac{e^{-s}}{1+s} \\ \frac{e^{-2s}}{1+s} & \frac{1}{1+s} \end{bmatrix}$$

Adding a common delay in all loops, e.g., e^{-3s} for guaranteeing the oscillations, the new matrix under test is:

$$P_2'' = \begin{bmatrix} \frac{1}{1+s} & -\frac{e^{-s}}{1+s} \\ \frac{e^{-2s}}{1+s} & \frac{1}{1+s} \end{bmatrix} e^{-3s}$$

Note that all operations introduced do not vary the results of the RGA or ROmA analysis, because of its invariance property under scaling.
The matrix of relative critical frequencies is now:

$$F = \frac{\omega_{OL}}{\omega_{CL}} = \begin{pmatrix} \frac{0.67}{2.101} & \frac{0.8566}{0.5196} \\ \frac{0.8566}{0.5196} & \frac{0.5522}{1.471} \end{pmatrix}$$

The ROmA index is:

$$\Psi = F \otimes F^{-T} = \begin{pmatrix} -0.0461 & 1.0461 \\ 1.0461 & -0.0461 \end{pmatrix} = \text{ROmA}$$

By comparing this matrix with the traditional RGA matrix:

$$RGA = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

It may be observed that the suggested pairings is the off-diagonal one: y_1-u_2, y_2-u_1. This result is in a good agreement with the results of Meeuse [8], in a critical case where steady-state RGA does not prefer any pairing.

<u>Example 3.</u> Consider the process studied in (Seider, 1999):

$$P_3(s) = \begin{pmatrix} \frac{2.5e^{-5s}}{(1+15s)(1+2s)} & \frac{1}{1+4s} \\ \frac{1}{1+3s} & \frac{-4e^{-5s}}{1+20s} \end{pmatrix}$$

In this case the common delay chosen is 6 s.
The matrix of relative critical frequencies is now:

$$F = \frac{\omega_{OL}}{\omega_{CL}} = \begin{pmatrix} \frac{0.1573}{1.823} & \frac{0.3786}{0.09454} \\ \frac{0.3992}{0.4133} & \frac{0.174}{3.143} \end{pmatrix}$$

Matrix ROmA is:

$$\Psi = F \otimes F^{-T} = \begin{pmatrix} -0.0012 & 1.0012 \\ 1.0012 & -0.0012 \end{pmatrix}$$

An off-diagonal pairing is suggested, opposite to the wrong pairing given by the steady-state RGA:

$$RGA = \begin{pmatrix} 0.9091 & 0.0909 \\ 0.0909 & 0.9091 \end{pmatrix}.$$

4. SENSITIVITY

In the previous examples an additional delay has been introduced in order to assure the oscillating conditions in presence of a relay. Indeed for a given matrix of transfer function it is possible that no oscillation occurs. In practice if the control is networked or is remote we must always take into account some delay. In the previous examples the amount of delay introduced is somewhat arbitrary. Of course if the oscillations are obtained on the field by suitable relay testing, we must consider that greater is the delay more expensive and time consuming are the tests; by reducing the delay there may occur a limiting value leading to a non-oscillating condition.

It is interesting to evaluate the sensitivity of the proposed procedure to the amount of delay θ introduced.

By computing for different delays the ROmA array for Wood and Berry process we get the following results, where λ is the element 1-1 of the ROmA matrix:

Table 1 . Element 1-1 of the ROmA matrix varying the introduced delay θ.

Θ (s)	4	5	6	7	8	50	100
λ	1.113	1.31	1.398	1.496	1.580	8.656	15.545

The results displayed in the Table show that the information given by ROmA are quite insensitive to the delay; of course for very large delay the pairing suggested is still correct but the larger value of λ denote a possible greater difficulty in control.

4. CONCLUSIONS

In this work a new approach for the selection of the pairings between input and output variables in decentralized MIMO control schemes. The ROmA index is based on the ratio between the critical frequencies of the loops closed with a relay, keeping all other loops open and the critical frequencies of the loops closed with a relay, in case of a perfect control on the other loops. It maintains all properties of static RGA index, but it considers dynamics of the process under test with simple autotuning tests. All critical frequencies can be measured from autotuning tests, they may be easily performed on line and the knowledge of the model of the process in not necessary. Examples of applications were selected to illustrate the effectiveness of the new indicator in different critical cases, also in presence of different delays; other examples of applications to MIMO processes have always shown the capability of the new approach in choosing the correct pairing.

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