PREDICTIVE CONTROL OF ASYMMETRICAL PROCESSES

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Abstract: This paper deals with the control of processes that present different dynamic responses for equal increments and decrements of its manipulate variable, showing a non symmetric response. Being non-linear systems, instead of using non linear general methods directly, the paper explores two alternative formulations based on an MPC approach that take advantage of its structure. An application example is provided showing the behaviour of the proposed methods.

Keywords: asymmetrical processes, predictive control, non linear programming, MIQP problem

1. INTRODUCTION

There are many processes in which, when a positive change in the manipulated variable is performed, one obtains a dynamic response that is quite different, either in shape or magnitude, to the one that can be obtained with a similar, but opposite change in the same manipulated variable (Fig.1).

We will denote these kind of processes as asymmetric. Examples of them can be found in several processes due, for instance, to different heating/cooling systems in chemical reactors. Other examples include the relation between the gas phase concentration and the pressure in a flash tank, or the one between head temperature and reflux flow in a distillation column. Clearly, they are non-linear and, as such, they pose a challenging problem to the control engineer. If tuned according to the "slow" response, they will present oscillations and overshoots in some operating conditions and if tuned according to the "fast" response, the closed loop will be also very slow in other cases.

The answer is, of course, to take into account the non-linear characteristic of the process and to design a controller applying general methods for this type of systems. Nevertheless, one can ask himself if it is possible to gain advantage of the special structure of the system in order to obtain easier solutions, either in terms of modeling or in the controller.

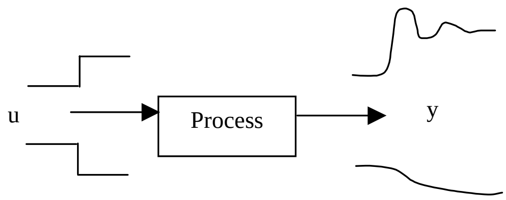

Fig. 1 Typical response of an asymmetric process to positive and negative steps inputs

A control technique that has gained wide acceptance in industry is model predictive control (MPC). As it is well known, it is based in the use of an internal model for computing predictions of the model responses, over a given time horizon, as functions of the present and future values of the manipulated variables (Fig. 2).

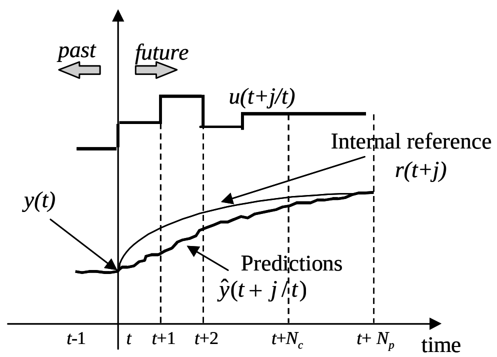

Fig. 2 MBPC strategy

The manipulated variables are chosen by minimization of a cost function of the quadratic errors between the output predictions and a desired internal reference, under the constraint imposed by the model prediction equations. Usually, other additional constraints are included on the range and speed of the manipulated inputs, as well as on the admissible range of the outputs, and, perhaps, some others in order to guarantee some stability properties.

If the internal model is linear, than the solution is obtained solving a QP optimization problem every sampling time. If not, the solution of a NLP problem is usually required, which increased considerably the complexity and computation time of the controller.
In the literature, very few contributions can be found devoted specifically to this topic. Previous contributions to the mentioned problem appear for instance in (Doyle *et al*., 1995) and (Camacho and Bordons, 2000) but only from the point of view of general non-linear systems. In (Tan *et al*., 1998) the use of PID controller has focused on the control of processes with severe asymmetry where an automatic tuning procedure for gain-scheduled is described.

This paper follows the non-linear MPC approach, but, at the same time, tries to re-formulate the model and the solution, in such way that, and this is the main contribution of the paper, for this particular kind of problems, an alternative and efficient formulation is obtained. The main point of the proposed method is the direct use of the linear models that could represent each of the two asymmetric dynamics of the process, instead of a more complex, perhaps first principles, non-linear model that could explain the whole system behavior, or the use of multiple linear models characteristic of other approaches. Using this internal model, two optimization algorithms are proposed that exploit its structure.

The paper is organized as follows: after the introduction, section II describes the internal model of the controller, then section III gives the first algorithm that includes a non-linear constraint. Section IV describes an alternative that leads to a mix integer algorithm. Finally, section V provides an application example showing the advantages of the proposed approach.

2. INTERNAL PROCESS MODEL

The use of linear models or a combination of them for representing a non-linear process is always an attractive approach because it leads very often to less complex controllers. In the so-called multi-model representation, a collection of linear models, each of which describing an operating point, are combined according to some fuzzy rules. The control action is computed for each of the models and the actual control applied to the process is a combination of the control actions computed from the different models.

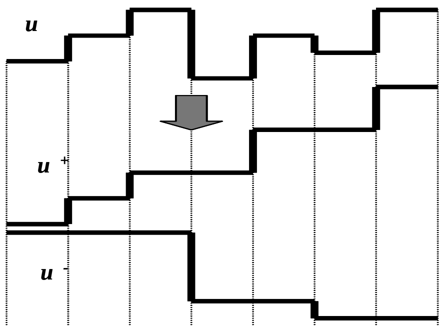

Fig. 3 Signal u(t) decomposed as u^+(t) and u^-(t)

Existing methods can differ in implementation details. One of the main problems of this approach is how to establish the partitions between the different operating regimes.
In our case, the nature of the problem is a different one, where the source of non-linearity is not the operating point but the direction of the changes in the manipulated input. It is assumed that the system responses to both, positive and negative inputs, can be characterized by discrete transfer functions that will be denoted as $G^+(q^{-1})$ and $G^-(q^{-1})$ respectively, as in Fig.1.
The key modeling idea for combining both models is depicted in Fig. 3, where the discrete control signal $u(t)$ being applied to a process can always be considered as the sum of two sequences $u^+(t)$ and $u^-(t)$, that have only increasing or decreasing changes correspondingly.

Every positive or negative change in $u(t)$ at a sampling time, is translated into the same change in $u^+(t)$ or $u^-(t)$ with the condition that they can not take place simultaneously. In this way, the moves in the control signal can be written as:

$$\begin{aligned} &\Delta u(t+j) = \Delta u^+(t+j) + \Delta u^-(t+j) \\ &\begin{cases} \Delta u^+(t+j) \ge 0 \\ \Delta u^-(t+j) \le 0 \\ \Delta u^+(t+j)\cdot \Delta u^-(t+j) = 0 \end{cases} \end{aligned} \quad (1)$$

Now the model output can be formulated as:

$$y(t) = G^+(q^{-1})u^+(t)(q^{-1}) + G^-(q^{-1})u^-(t) + v(t) \quad (2)$$

where $v(t)$ represents a non-stationary stochastic disturbance, or alternatively as:

$$\begin{aligned} A(q^{-1})y(t) = {} & B^+(q^{-1})u^+(t) + B^-(q^{-1})u^-(t) \\ & + \frac{T(q^{-1})}{\Delta}\xi(t) \end{aligned} \quad (3)$$

where $\Delta = 1 - q^{-1}$, A, B^+, B^-, T are polynomials in the backward operator q^{-1}, $\xi(t)$ is a zero-mean white noise signal and u^+, u^- are constrained by (1).

3. MPC CONTROLLER

Model (3), even if constrained by (1), has a nice structure that allows us to formulate closed expressions for the predictions of the process outputs. Taking into account that (1) refers only to the inputs, we can apply the superposition principle with our model (3) and develop prediction equations in the usual way in linear MPC.

Future values of the output at times *t+j, (j=1,…,N_2* sampling periods) can be considered as the sum of two terms: the so called free and forced responses:

$$y(t+j) = y_f(t+j) + y_c(t+j) \qquad (4)$$

where the free response corresponds to:

$$y_f(t+j) = \frac{B^+}{A} u_f^+(t+j) + \frac{B^-}{A} u_f^-(t+j) + \frac{T}{A\Delta}\xi(t+j) \qquad (5)$$

and u^+_f and u^-_f can be computed according to:

$$\begin{aligned} &\text{if } u(t-1)-u(t-2)>0 \quad \begin{cases} u_f^+(t+j) = u(t-1) \\ u_f^-(t+j) = 0 \end{cases} \\ &\text{if } u(t-1)-u(t-2)<0 \quad \begin{cases} u_f^+(t+j) = 0 \\ u_f^-(t+j) = u(t-1) \end{cases} \end{aligned} \qquad (6)$$

so they are known variables at time *t*. Then, the predictions are obtained using the usual procedures, either based on filters or in Diophantic equations. As they are well known, they will not be repeated here. See for instance (Clarke, 1987).

On the other hand, the forced response can be computed using:

$$\begin{aligned} &y_c(t+j) = G_j^+(q^{-1})\Delta u_c^+(t+j) + G_j^-(q^{-1})\Delta u_c^-(t+j) \\ &G_j^+(q^{-1}) = g_1^+ q^{-1} + \ldots + g_j^+ q^{-j} \\ &G_j^-(q^{-1}) = g_1^- q^{-1} + \ldots + g_j^- q^{-j} \end{aligned} \qquad (7)$$

with g_j being the step response coefficients of each transfer functions and

$$\begin{aligned} &\Delta u_c^+(t+j) = \Delta u^+(t+j) \geq 0 \\ &\Delta u_c^-(t+j) = \Delta u^-(t+j) \leq 0 \end{aligned} \qquad (8)$$

Once output predictions are available, the optimal control actions of our MPC controller can be obtained as the ones that minimize a cost function (9) of the squared errors between these predictions and a desired set point, including a penalty on the control moves, along a given time horizon:

$$I(t) = \sum_{j=N1}^{N2} \left(r(t+j) - \hat{y}(t+j)\right)^2 + \beta \sum_{j=1}^{Nu} \left(\Delta u(t+j-1)\right)^2 \qquad (9)$$

Notice that (1) allow us to write the equivalent problem:

$$\begin{aligned} \min_{\substack{\Delta u^+(t),\ldots,\Delta u^+(t+Nu-1), \\ \Delta u^-(t),\ldots,\Delta u^-(t+Nu-1)}} I(t) = &\sum_{j=N_1}^{N_2} \left(r(t+j) - \hat{y}(t+j)\right)^2 + \\ &+ \beta_1 \sum_{j=1}^{Nu} \left(\Delta u^+(t+j-1)\right)^2 + \beta_2 \sum_{j=1}^{Nu} \left(\Delta u^-(t+j-1)\right)^2 \end{aligned} \qquad (10)$$

$$\begin{aligned} &\hat{y}(t+j) = \hat{y}_f(t+j) + \\ &+ G_j^+(q^{-1})\Delta u^+(t+j) + G_j^-(q^{-1})\Delta u^-(t+j) \end{aligned} \qquad (11)$$

where we take advantage of the cost function structure that allows different weighting of the positive and negative moves.

Defining the matrices:

$$\mathbf{G}^+ = \begin{bmatrix} g_{N_1}^+ & g_{N_1-1}^+ & \cdots & g_1^+ & 0 & \cdots & 0 \\ g_{N_1+1}^+ & g_{N_1}^+ & \cdots & \cdots & g_1^+ & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots & \vdots & \cdots & \vdots \\ g_{N_2}^+ & g_{N_2-1}^+ & \cdots & \cdots & \cdots & \cdots & g_{N_2-Nu+1}^+ \end{bmatrix} \qquad (12)$$

$$\mathbf{G}^- = \begin{bmatrix} g_{N_1}^- & g_{N_1-1}^- & \cdots & g_1^- & 0 & \cdots & 0 \\ g_{N_1+1}^- & g_{N_1}^- & \cdots & \cdots & g_1^- & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots & \vdots & \cdots & \vdots \\ g_{N_2}^- & g_{N_2-1}^- & \cdots & \cdots & \cdots & \cdots & g_{N_2-Nu+1}^- \end{bmatrix} \qquad (13)$$

and the vectors:

$$\mathbf{u}^+ = \left[\Delta u^+(t) \quad \Delta u^+(t+1) \quad \ldots \quad \Delta u^+(t+Nu-1)\right]^T \qquad (14)$$

$$\mathbf{u}^- = \left[\Delta u^-(t) \quad \Delta u^-(t+1) \quad \ldots \quad \Delta u^-(t+Nu-1)\right]^T \qquad (15)$$

$$\mathbf{E_0} = \left[e_0(t+N_1) \quad e_0(t+N_1+1) \quad \ldots \quad e_0(t+N_2)\right]^T \qquad (16)$$

where

$$e_0(t+j) = r(t+j) - \hat{y}(t+j) \qquad (17)$$

it is possible to reformulate *I(t)* as a quadratic function that the controller solves:

$$\min_{\mathbf{x}} I(t) = \min_{\mathbf{x}} \left(\frac{1}{2}\mathbf{x}^T \mathbf{H} \mathbf{x} + \mathbf{c}^T \mathbf{x}\right) \qquad (18)$$

with

$$\mathbf{H} = \left[\begin{array}{c:c} \mathbf{G^+}^T\mathbf{G}^+ + \beta_1 \mathbf{I} & \mathbf{G^+}^T\mathbf{G}^- \\ \hdashline \mathbf{G^-}^T\mathbf{G}^+ & \mathbf{G^-}^T\mathbf{G}^- + \beta_2 \mathbf{I} \end{array}\right] \qquad (19)$$

$$\mathbf{c}^T = \left[-\mathbf{E_0}^T\mathbf{G}^+ \;\vdots\; -\mathbf{E_0}^T\mathbf{G}^-\right] \qquad (20)$$

$$\mathbf{x}^T = \left[\mathbf{u}^+ \quad \mathbf{u}^-\right] \qquad (21)$$

and the constraints (1) every sampling time. Due to constraint

$$\Delta u^+(t+j)\cdot\Delta u^-(t+j)=0\,, \qquad (22)$$

a non-linear programming problem with $2N_u$ decision variables, a quadratic cost function and the linear constraints (8) has to be solved. Notice that adding additional constraints on the manipulated and controlled variables, or on their speed of change, this assertion does not modify.

$$U_{\min} \le \sum_{i=0}^{j}\Delta u^+(t+j)+\sum_{i=0}^{j}\Delta u^-(t+j)\le U_{\max}$$
$$U_{\min}=U_m-u(t-1) \text{ and } U_{\max}=U_M-u(t-1) \qquad (23)$$
$$0\le j\le Nu-1$$

$$D_m \le \Delta u(t+j)=\Delta u^+(t+j)+\Delta u^-(t+j)\le D_M \qquad (24)$$

$$L_m-\hat{y}_f(t+j)\le \sum_{i=0}^{\min(j,Nu-1)} g^+_{j-i}\Delta u^+(t+i)+$$
$$+\sum_{i=0}^{\min(j,Nu-1)} g^-_{j-i}\Delta u^-(t+i)\le L_M-\hat{y}_f(t+j) \qquad (25)$$
$$N_3\le j\le N_4$$

where U_x, D_x, L_x, refers to the low and upper limits of these variables.

As such, it must be solved with an appropriate NLP solver. The fact that the cost function, with adequate values of β, is convex and the most of the equations are linear, can help in finding the optimal solution, but there is no way to avoid the non-convex constraint (22).

4. A HYBRID PREDICTIVE CONTROL ALTERNATIVE

Because of this difficulty, an alternative solution to the problem described by (1), (18), (23)-(25) is proposed, in the form of a hybrid MPC problem.

With this purpose, we have introduced N_u new binary variables z_j, $j = 1,\ldots, N_u-1$. It is not difficult to realize that the set of constraints (1) is equivalent to

$$0\le \Delta u^+(t+j)\le D\,z_j$$
$$(z_j-1)D\le \Delta u^-(t+j)\le 0 \qquad (26)$$

where D is a large positive number and the variable z_j has 0/1 value. Notice that $z_j = 1$ means that a positive change will be implemented at time instant $t+j$ and $z_j = 0$ means that a negative change will be the one implemented at that time instant.

The interest of substituting the set of constraints (1) by (26) is that, even if new binary variables z_j are introduced, the set (26) is a linear one, eliminating the non-convex equations (22), which facilitates the finding of the optimal control moves.

Now, the associated controller optimization problem can be formulated as an MIQP problem considering the cost index (18), the definitions (19)-(21) and the constraints (26). The optimization problem is solved every sampling period, for which efficient algorithms as Branch & Bound can be found.

5. APPLICATION EXAMPLE

In order to test the proposed method, an asymmetric process like the one depicted in Fig. 4 was considered. The two dynamics have been identified and the corresponding transfer functions are given by:

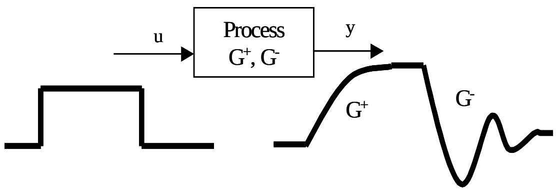

Fig. 4 Asymmetric process of our example

$$G^+(s)=\frac{2}{5s+1} \quad \text{and} \quad G^-(s)=\frac{0.5}{s^2+0.6s+1} \qquad (27)$$

with the step responses like the ones shown in Fig. 5 and 6.

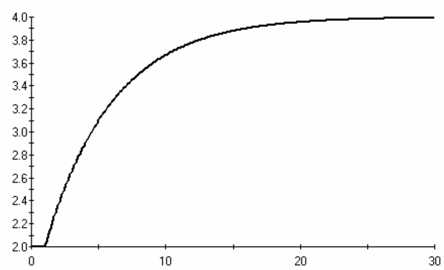

Fig. 5. Step response of $G^+(s)$

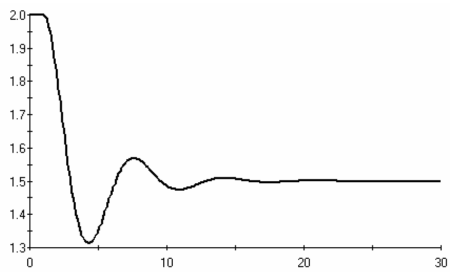

Fig. 6. Step response of $G^-(s)$

The proposed hybrid controller was applied to this asymmetric process and a series of experiments were performed.

The controller had the following parameters: $N_1 = 1$, $N_2 = 15$, $N_u = 2$, and, initially, the same control weights $\beta_1 = 0.1$, $\beta_2 = 0.1$, were applied to each positive or negative movements of the control variable u. The experiments consist of several step changes in the set point of the controller. Fig. 7 shows the process response and the set point in the lower graph as well as the control signal in the upper one. As we can see it behaves very well following the reference and with sensible control efforts.

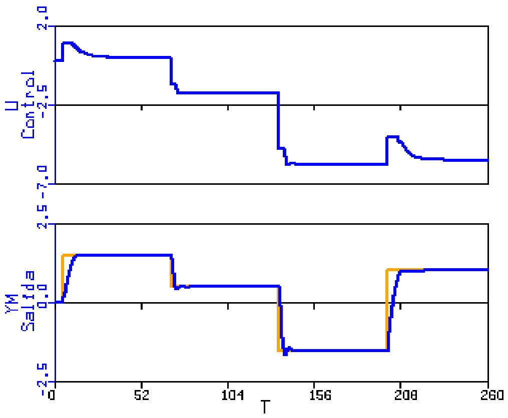

Fig. 7. Closed loop response of the asymmetric process to several step changes in the set point. Upper graph: control signal. Lower graph: set point and process output.

A more symmetric process response can be obtained weighting the control moves differently. For instance, with β_1 = 0.1, β_2 = 10, it is possible to obtain the response of Fig. 8.

Additional constraints in both inputs and outputs can be added easily. Fig. 9 and 10 show responses of the process when the control signal was constraint to be in the range [-1, 0.7] (Fig. 9.) and the process output was constraint also to be in the range [-1, 1] (Fig. 10). Notice that the set point is outside this range.

In order to test the advantages of the proposed controller, it was compared with a linear MPC using a fix internal model. Fig. 11 shows the case where the internal model is given by $G^{+}(s)$ with the same tuning parameters as in the hybrid controller. As we can see, the response is very poor, with high control moves and oscillatory process output. This response can be improved by tuning of the control weights, compromising between the positive and negative responses. For instance, with β = 5, it is possible to obtain the responses of Fig. 12, which is still not very good.

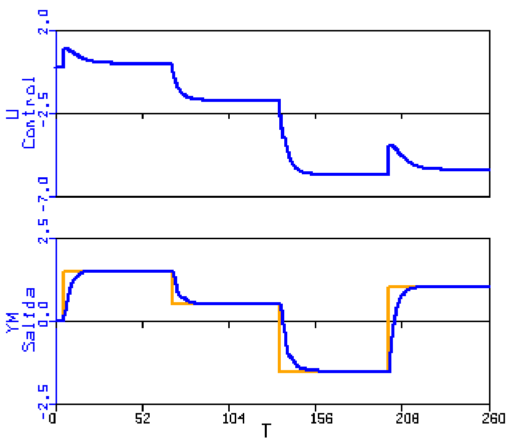

Fig. 8. Closed loop response of the asymmetric process to several step changes in the set point with different weights. Upper graph: control signal. Lower graph: set point and process output.

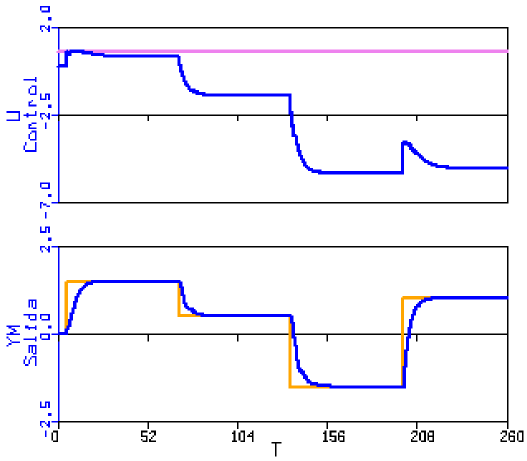

Fig. 9. Closed loop response of the asymmetric process to several step changes in the set point. Limits on *u*. Upper graph: control signal. Lower graph: set point and process output.

In the same way, Fig. 13 shows the case where the internal model is given by $G^{-}(s)$ with the same tuning parameters as in the hybrid controller. Similar conclusions can be obtained. Moreover the computation time for the whole experiment is only a bit smaller (0.313 seconds) than the case of the hybrid control (0.344 seconds). The both experiments were computed in a Pentium 2.53 GHz and 524 kB RAM.

As a final test, we compared the hybrid and the non-linear MPC model. The responses were very similar to the ones of Fig. 7, but the computation time was almost five times faster in the case of the hybrid controller and moreover the guarantee of optimality is given by the fact that the hybrid problem is convex.

For save of simplicity, no terminal penalty term has been added to the cost functions but it could be easily included in order to stabilize the closed system.

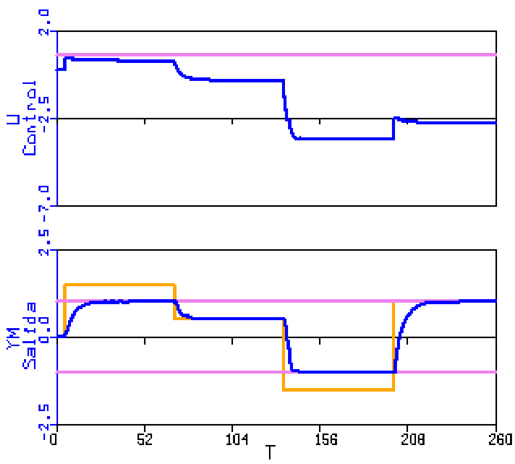

Fig. 10. Closed loop response of the asymmetric process to several step changes in the set point. Limits on *u* and *y*. Upper graph: control signal. Lower graph: set point and process output.

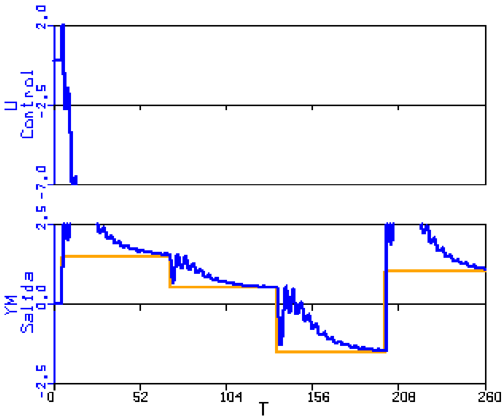

Fig. 11. Closed loop response of the asymmetric process to several step changes in the set point. Internal model G^+. Upper graph: control signal. Lower graph: set point and process output.

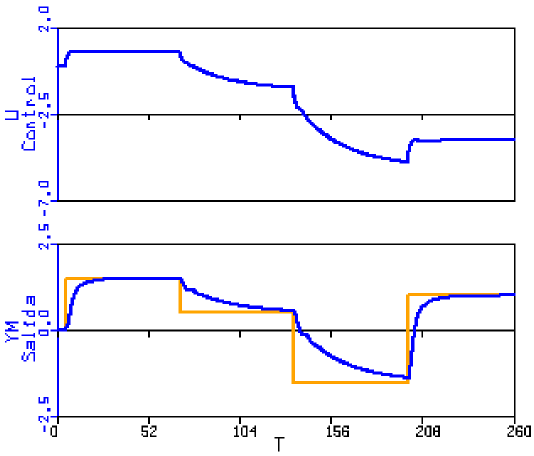

Fig. 12. Closed loop response of the asymmetric process to several step changes in the set point. Internal model G^+ and $\beta = 5$. Upper graph: control signal. Lower graph: set point and process output.

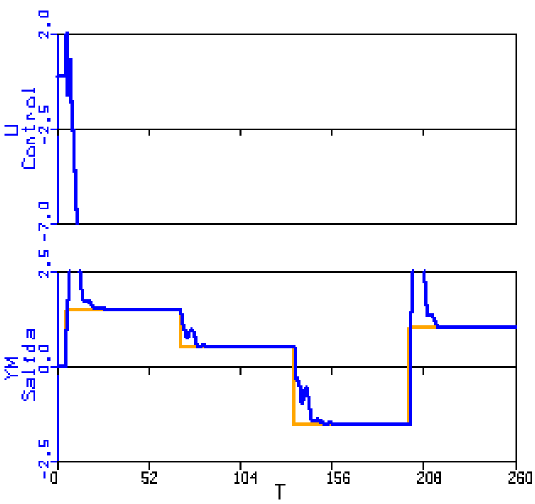

Fig. 13. Closed loop response of the asymmetric process to several step changes in the set point. Internal model G^- , $\beta = 0.1$. Upper graph: control signal. Lower graph: set point and process output.

6. CONCLUSIONS

In this paper two alternative formulations for a MPC algorithm to deal with asymmetrical processes are presented instead of using a full non linear model strategy. Both algorithms take advantage of the use of two linear internal models and a quadratic cost function in spite of the non linearity of the process. The first proposed method involves NLP because of the constraints and the second one has the form of a hybrid MPC problem with only linear constraints, some of them including binary variables, leading to a MIQP problem.

Simulations using an example of a process with severe asymmetry were performed. Comparing with a linear MPC using a fix internal model, the new controllers show a better performance and an important advantage raises with the hybrid one where the computation time is similar to a classic linear MPC.

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