CLOSED LOOP CONTINUOUS-TIME FOPTD IDENTIFICATION USING TIME-FREQUENCY DATA FROM RELAY EXPERIMENTS

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Abstract: In this work the identification of first-order plus dead-time models from a relay experiment is considered. The relay excitation is applied to the closed-loop. Alternative techniques for identification are examined and simple algorithms are proposed for dealing with the dead-time. Simulation examples are used to illustrate the techniques.

Keywords: Continuous-time identification; Process identification; Closed-loop identification; Time delay process; Relay excitation.

1. INTRODUCTION

The estimation of continuous-time models from sampled data has received some attention in last years, motivated by the need of such models to recover physical parameters or to allow the use of design techniques developed for continuous-time controllers. An extensive list of references on the subject can be found in (Mensler, 1999), in which a detailed survey discusses the advantages of a direct approach in relation to the indirect estimation of a discrete-time model plus a later transformation into a continuous-time model. Several papers have been presented in recent conferences (for instance, 13th IFAC Symposium on System Identification (SYSID 2003) and 16th IFAC World Congress 2005) to report new developments and applications.

The continuous-time results reported in the literature mainly address finite-dimensional systems. But dead-time is present in several industrial processes so that simple models such as first and second order dead-time continuous time one are widely used to tune industrial controllers. In the design of PID controllers the process model that receives most attention is first-order plus dead-time model (FOPDT) (Sudaresan and Krishnaswamy, 1977). There are a few methods to estimate parameters for this model. Among them one can mention the graphics and the area methods (Åström and Hägglund, 1995). A method less sensitive to noise is proposed in (Wang *et al.*, 1999) which uses least-squares method to estimate the parameters of FOPDT model. Variants of this methods are used in (Wang and Zhang, 2001) and (Wang *et al.*, 2000). Other method for open loop unstable processes is presented in (Marchetti and Lewin, 2001). For such simple models the results are remarkably good and motivated the present work. Methods of closed-loop identification have been used in industrial applications (Forssell and Ljung, 1999). The closed-loop identification doesn't cause stops in system operation comparing with the open-loop identification. Besides this reason, there are others to do experiments of closed-loop identification which are:

demands of safety in the operation of the process or because the process has unstable behavior in open-loop, which are found in many industrial processes (Ljung, 1999). There are situations where the plants are stable but restrictions in production are strong reasons not to allow experiments in open-loop. An additional consideration to accomplish experiments in closed-loop is that the dynamic exhibited by the plant with the old controller must be more important to design a high performance controller than the dynamic of the plant in open-loop.

In this paper three techniques for the estimation of continuous-time systems from discrete-time measurements with data obtained from relay based closed-loop experiments are compared.

The first technique is the one presented in ((Coelho and Barros, 2003)) where least-squares minimization is used with a search for a initial dead-time estimate. The second technique uses an approximated model. Both techniques use only time-domain data.The third one is a constrained least-squares minimization which uses frequency data obtained from a relay experiment.

This paper is organized as follows. In Section 2, the problem statement is presented. The relay closed loop experiment used to obtain time and frequency information is presented in Section 3. The continuous-time identification of FOPTD techniques are presented in Section 4. In Section 5 the techniques are compared using simulations of examples and, finally, conclusions are presented in Section 6.

2. THE PROBLEM STATEMENT

In this paper it is considered the identification of first-order plus dead-time (FOPDT) continuous-time models represented by

$$G(s) = \frac{b}{s+a} e^{-Ls}. \tag{1}$$

It is assumed closed-loop operation and that the excitation is generated from a relay-based experiment. In this paper is considered a closed-loop with transfer function $T(s)$, process transfer function $G(s)$, controller $C(s)$, and loop gain $L(s)$. Although it is desired to estimate a continuous-time model, the available data to the estimation is discrete-time. The aim of the paper is to evaluate the improvements obtained, for such simple models, with the introduction of frequency domain information as constraints in the minimization problem.

The frequency domain information is obtained by using a relay based test as described in the sequel.

3. THE LOOP GAIN RELAY EXPERIMENT

A basic procedure for the estimation of a general frequency point of the loop gain transfer function using a relay feedback is presented in (de Arruda and Barros, 2003). The feedback structure applied for loop transfer function estimation is presented in Fig. 1. The conditions of the limit cycle operation are defined by the following proposition.

Consider the closed loop relay system shown in Fig (1). Assume that for a stable closed loop $T(s)$ and a real positive number r, the transfer function

$$F(s) = \frac{2}{r}\frac{T(s)}{T(s)\left(\frac{1-r}{r}\right)+1} - 1 \tag{2}$$

is also stable. Then if a limit cycle is present it oscillates at a frequency ω_o such that

$$|L(j\omega_o)| \approx r\ .$$

See (de Arruda and Barros, 2003).

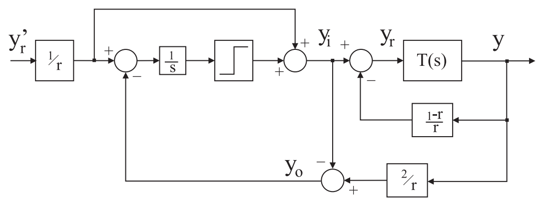

Fig. 1. Relay Closed Loop Experiment for Loop Transfer Function Estimation.

Selecting $r = 1$, the current loop gain crossover frequency ω_g can be estimated. This estimate is denoted $\hat{\omega}_g$. In this case the scheme reduces to the one presented in (Schei, 1992).

The setpoint $y_r(t)$ is the loop gain experiment excitation applied to the closed loop $T(s)$ formed by the process $G(s)$ with controller $C(s)$. The process transfer function at the crossover frequency is estimated computing the DFT of one period of the process input u and output y when the relay oscillation is present and steady. This loop gain relay excitation is used at all examples.

4. TECHNIQUES USED FOR THE IDENTIFICATION OF FOPDT MODELS

In this Section the identification techniques are described.

4.1 Technique 1: Identification of FOPDT Model with Dead-Time Search

This technique is the one presented in ((Coelho and Barros, 2003)) for a closed-loop step response

In this paper the excitation used is the one obtained from the loop gain relay experiment with $r = 1$.

Under mild conditions the process model (1)can be written as

$$y(t) = -a\int_0^t y(\tau)\,d\tau + b\int_0^{t-L} u(\tau)\,d\tau. \quad (3)$$

It can also be rewritten as

$$y(t) = -a\int_0^t y(\tau)\,d\tau + b\int_0^t u(\tau)\,d\tau - b\int_{t-L}^t u(\tau)\,d\tau. \quad (4)$$

Then, define

$$\phi(t) = \left[-\int_0^t y(\tau)\,d\tau \;\; \int_0^t u(\tau)\,d\tau \;\; -\int_{t-L}^t u(\tau)\,d\tau \right]^T,$$
$$\theta = \left[\, a \;\; b_1 \;\; b_2 \,\right]^T. \quad (5)$$

and Eq.(4) can be written in regression form

$$y(t) = \phi(t)\,\theta. \quad (6)$$

Unfortunately, the value of L is not known. In this case, a straightforward procedure is to search for the best fit among several values of L. An algorithm presented in ((Coelho and Barros, 2003)) is used to avoid estimate a L which is a multiple of the sampling period. Its motivation comes from the fact that $b_1 = b_2$ for the true value of L. Is choose a range for the dead-time, $[L_{\min}, L_{\max}]$, with $L_{\min} = k_{\min}T_s$ and $L_{\max} = k_{\max}T_s$. Using a regression model the parameters for each value of k in $[k_{\min}, k_{\max}]$ are estimated. For each value $i = k - k_{\min} + 1$ estimate $\hat{\theta}^i$ is computed

$$\begin{bmatrix} a^i \\ b_1^i \\ b_2^i \end{bmatrix} = \begin{bmatrix} \hat{\theta}^i(1) \\ \hat{\theta}^i(2) \\ \hat{\theta}^i(3) \end{bmatrix}. \quad (7)$$

an estimate of L, say L_1, is recovered

$$L_1 = \hat{k}T_s \quad \text{with} \quad \hat{k} = \min_i \left| b_1^i - b_2^i \right|.$$

So, applying the estimator to the regression vector

$$\phi(t) = \begin{bmatrix} -\int_0^t y(\tau)\,d\tau \\ \int_0^t u(\tau)\,d\tau \\ -\frac{1}{L_1}\int_{t-L_1}^t u(\tau)\,d\tau \end{bmatrix}, \quad (8)$$
$$\theta = \left[\, a \;\; b \;\; \beta \,\right]^T, \quad (9)$$

the final estimate $\left\{\hat{a}, \;\; \hat{b}, \;\; \hat{L} = \hat{\beta}/\hat{b}\right\}$ are obtained and the corresponding model $G_{LS1}(s)$.

4.2 Technique 2: Identification of FOPTD Model using Approximation

In this second technique, the following approximation for model 1 is used:

$$G(s) = \frac{b(1 - sL)}{s + a} \quad (10)$$

the process model (10) can be written as

$$y(t) = -a\int_0^t y(\tau)\,d\tau + b\int_0^t u(\tau)\,d\tau - bLu(t). \quad (11)$$

Define

$$\phi(t) = \left[-\int_0^t y(\tau)\,d\tau \;\; \int_0^t u(\tau)\,d\tau \;\; u(t) \right]^T,$$
$$\theta = \left[\, a \;\; b \;\; \beta \,\right]^T. \quad (12)$$

This case is equivalent to choose $L_1 = T_s$ in the first technique. The final estimate $\left\{\hat{a}, \;\; \hat{b}, \;\; \hat{L} = \hat{\beta}/\hat{b}\right\}$ are obtained and the corresponding model $G_{LS2}(s)$.

4.3 Technique 3: Identification of FOPTD Model with Frequency Domain Constraints

In the third technique, equality constraints are used with the least-squares minimization ((Nelles, 2001)). The procedure solves a time least-squares problem submitted to a constraint on frequency. The constraint is obtained through the process frequency response on the first harmonic of the relay experiment signal. The frequency response is obtained computing the DFT of process input and output. In this frequency, the loop gain has approximately magnitude one. Assuming the data is grouped in a vector from yielding matrices Y and $\hat{\theta}$. The least-squares optimization problem is given by

$$J = \left(Y - \Phi\hat{\theta}\right)^T \left(Y - \Phi\hat{\theta}\right) \quad (13)$$

submitted to the

$$M\theta = \gamma. \quad (14)$$

which express the equality constraints in the time and frequency domains in a linear form.

The equality constraint is defined through the following regression vector which is obtained using the linear form 14 given by:

$$\hat{z} = x^T(\omega)\,\hat{\theta}$$

with

$$\hat{z} = j\omega\hat{G}(j\omega)\,; \quad x^T(j\omega) = \left[\, -\hat{G}(j\omega) \;\; 1 \;\; -j\omega \,\right]$$
$$\theta = \left[\, a \;\; b \;\; \beta \,\right]^T$$

where ω is the crossover frequency estimated using the relay experiment. More frequencies points may have been used.

In this case, the least-squares optimization problem with constraint is equivalent to minimize in relation to $\hat{\theta}$ and λ the cost function given by

$$J = \left(Y - \Phi\hat{\theta}\right)^T \left(Y - \Phi\hat{\theta}\right) + \lambda(\gamma - M\theta) \quad (15)$$

By defining

$$E = 2\Phi^T \Phi$$
$$F = 2\Phi^T Y$$

The optimal solution has a closed-form

$$\begin{bmatrix} E & -M^T \\ M & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \lambda^T \end{bmatrix} = \begin{bmatrix} F \\ \gamma \end{bmatrix}$$

which can be solved explicitly as

$$\lambda^T = \left[ME^{-1}M^T\right]^{-1}\left[\gamma - ME^{-1}F\right]$$
$$\hat{\theta} = [E]^{-1}\left(F + M^T\lambda^T\right)$$

The final estimate $\left\{\hat{a}, \quad \hat{b}, \quad \hat{L} = \hat{\beta}/\hat{b}\right\}$ are obtained and the corresponding model $G_{LS3}(s)$.

5. SIMULATION EXAMPLES

In this section the closed loop identification algorithms are applied to three processes. The cost function used to compare the estimates is

$$\varepsilon = \frac{1}{N}\sum_{k=0}^{N-1}\left[y\left(kT_s\right) - \widehat{y}\left(kT_s\right)\right]^2$$

where $y\left(kT_s\right)$ is the actual process output (with noise), while $\widehat{y}\left(kT_s\right)$ is the estimated process output from a closed loop simulation with the same controller and under the same step setpoint. In all experiments $T_s = 0.1s$, $k_{\max} = 50$ samples ($= 5s$), $k_{\min} = 5$ samples ($= 0.5s$), and the controller used is $C_1 = 1 + \frac{0.1}{s}$. White noise is added only to the output of the process. The processes and the results are shown below.

5.1 Example 1

In the first example it is used a FOPDT model

$$G_1(s) = \frac{0.14}{s + 0.12}e^{-0.95s}.$$

The noise variance is 0.02. The estimates are

$$G_{LS1}(s) = \frac{0.1329}{s + 0.1099}e^{-0.6397s}$$
$$G_{LS2}(s) = \frac{0.1354}{s + 0.1144}e^{-0.8381s}$$
$$G_{LS3}(s) = \frac{0.1368}{s + 0.1164}e^{-0.8326s}.$$

The mean squared errors are

$\varepsilon_1 = 7.549e{-}05,\ \varepsilon_2 = 1.562e{-}05,\ \varepsilon_3 = 1.009e{-}05.$

The estimated crossover frequency is $\widehat{w}_g = 0.1017$ and the processes have the following magnitudes in this frequency

$$|G_1(j\widehat{w}_g)| = 0.8901$$
$$|G_{LS1}(j\widehat{w}_g)| = 0.8878$$
$$|G_{LS2}(j\widehat{w}_g)| = 0.8848$$
$$|G_{LS3}(j\widehat{w}_g)| = 0.8851.$$

All techniques yields good results. The nyquist plot is shown in Fig. (2).

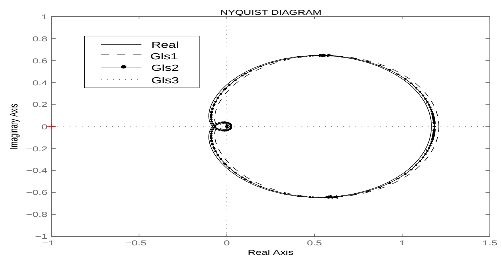

Fig. 2. Nyquist plot for process 1

In other simulation for the same $G_1(s)$, the noise variance is increased to 0.05. The estimates are

$$G_{LS1}(s) = \frac{0.1255}{s + 0.1001}e^{-0.3302s}$$
$$G_{LS2}(s) = \frac{0.1335}{s + 0.1133}e^{-0.9368s}$$
$$G_{LS3}(s) = \frac{0.1379}{s + 0.1179}e^{-0.9043s}.$$

The mean squared errors are

$\varepsilon_1 = 2.83e{-}04,\ \varepsilon_2 = 2.93e{-}05,\ \varepsilon_3 = 2.532e{-}06.$

The estimated crossover frequency is $\widehat{w}_g = 0.1018$. The process magnitude in this frequency are

$$|G_1(j\widehat{w}_g)| = 0.8901$$
$$|G_{LS1}(j\widehat{w}_g)| = 0.8786$$
$$|G_{LS2}(j\widehat{w}_g)| = 0.8762$$
$$|G_{LS3}(j\widehat{w}_g)| = 0.8849.$$

Increasing the noise variance the third technique provides a better fitting in the crossover frequency and the decreasing of the quadratic error. The loop gain experiment and nyquist plot are shown in Fig. (3) and (4).

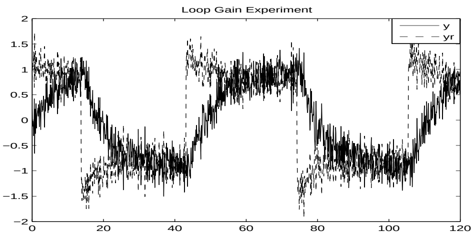

Fig. 3. Loop Gain Experiment for process 1.

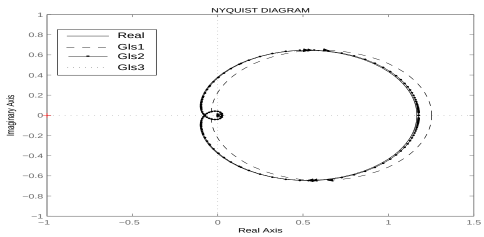

Fig. 4. Nyquist plot for process 1

5.2 Example 2

The process is now given by

$$G_2(s) = \frac{0.14}{(s+0.12)(s+1)} e^{-0.95s}.$$

The noise variance is 0.02. The estimates are

$$G_{LS1}(s) = \frac{0.1314}{s+0.1093} e^{-1.5325s}$$
$$G_{LS2}(s) = \frac{0.1276}{s+0.1057} e^{-1.4075s}$$
$$G_{LS3}(s) = \frac{0.1299}{s+0.1074} e^{-1.3854s}.$$

The mean squared errors are

$$\varepsilon_1 = 7.693e{-05},\ \varepsilon_2 = 1.255e{-04},\ \varepsilon_3 = 1.365e{-04}.$$

The estimated crossover frequency is $\widehat{w}_g = 0.0983$ and the process magnitudes

$$|G_2(j\widehat{w}_g)| = 0.8981$$
$$|G_{LS1}(j\widehat{w}_g)| = 0.8939$$
$$|G_{LS2}(j\widehat{w}_g)| = 0.8836$$
$$|G_{LS3}(j\widehat{w}_g)| = 0.8922.$$

Although there was a better fitting in the crossover frequency if compared with the second technique, the error have increased because the use of the constraint. The nyquist plot is shown in Fig. (5).

In other simulation for the same $G_2(s)$, the noise variance is increased to 0.05. The estimates are

$$G_{LS1}(s) = \frac{0.1254}{s+0.1017} e^{-1.2603s}$$
$$G_{LS2}(s) = \frac{0.1275}{s+0.1076} e^{-1.5736s}$$
$$G_{LS3}(s) = \frac{0.1319}{s+0.1104} e^{-1.5193s}.$$

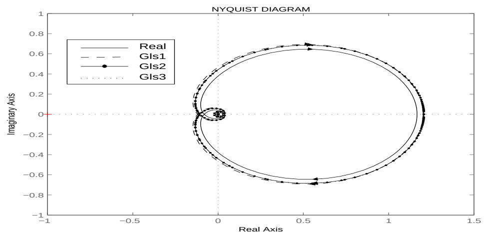

Fig. 5. Nyquist plot for process 2.

The mean squared errors are

$$\varepsilon_1 = 2.143e{-04},\ \varepsilon_2 = 7.22e{-05},\ \varepsilon_3 = 7.975e{-05}.$$

The estimated crossover frequency is $\widehat{w}_g = 0.0985$. The process magnitude in this frequency are

$$|G_2(j\widehat{w}_g)| = 0.8975$$
$$|G_{LS1}(j\widehat{w}_g)| = 0.8858$$
$$|G_{LS2}(j\widehat{w}_g)| = 0.8741$$
$$|G_{LS3}(j\widehat{w}_g)| = 0.8914.$$

Increasing the noise variance the third technique provides a better fitting in the crossover frequency and the decreasing of the quadratic error if compared with the first technique. The loop gain experiment and nyquist plot are shown in Fig. (6) and (7).

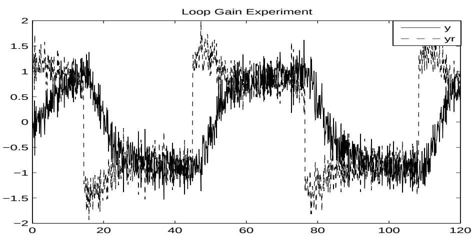

Fig. 6. Loop Gain Experiment for process 2.

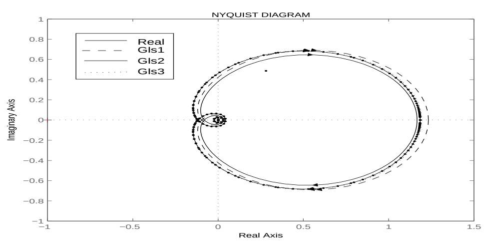

Fig. 7. Nyquist plot for process 2

5.3 Example 3

The process is now given by

$$G_3(s) = \frac{1}{(s+1)^8}.$$

The noise variance is 0.001.The estimates are

$$G_{LS1}(s) = \frac{0.2978}{s + 0.2929} e^{-4.8819s}$$
$$G_{LS2}(s) = \frac{0.1529}{s + 0.1357} e^{-2.8747s}$$
$$G_{LS3}(s) = \frac{0.1737}{s + 0.1569} e^{-2.3721s}.$$

The mean squared errors are

$$\varepsilon_1 = 0.0021, \; \varepsilon_2 = 0.0124, \; \varepsilon_3 = 0.0154.$$

In this case, the estimated crossover frequency is $\widehat{w}_g = 0.0985$ and the process magnitudes are

$$|G_3(j\widehat{w}_g)| = 0.9621$$
$$|G_{LS1}(j\widehat{w}_g)| = 0.9639$$
$$|G_{LS2}(j\widehat{w}_g)| = 0.9120$$
$$|G_{LS3}(j\widehat{w}_g)| = 0.9377.$$

The constrained least-square minimization if compared with the second technique produces a data fitting with a larger quadratic error despite a closer model in the crossover frequency to the real process. The loop gain experiment and nyquist plot are shown in Fig. (8) and (9).

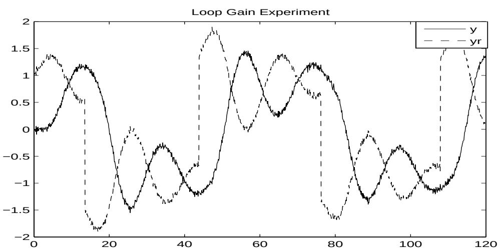

Fig. 8. Loop Gain Experiment for process 3.

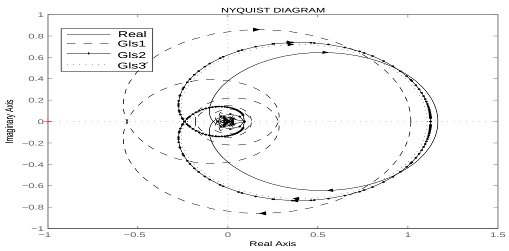

Fig. 9. Nyquist plot for process 3.

6. CONCLUSIONS

In this paper three techniques for the identification of continuous-time FOPTD models from closed loop step response was presented. One of the techniques use frequency domain information as equality constraints. Structures for identification in closed loop were also discussed. The use of constrains provided a better fitting in the crossover frequency, a good issue in closed loop identification and controller design.

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