A STATE-SHARED MODELING APPROACH TO TRANSITION CONTROL

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Abstract: A rigorous theoretical derivation of a state-shared model structure for multiple-input multiple-output (MIMO) systems is proposed. When a nonlinear system transitions in a large operating space, this state-shared modeling approach can be used to approximate the nonlinear system, such that effective model-based controllers can be applied. A MIMO nonlinear reactor system illustrates the proposed approach.

Keywords: Adaptive identifier, reduced-order model, multiple models, parameter adaptation, nonlinear reactor

1. INTRODUCTION

In the chemical industry, it has become quite common for plants to produce more than one grade of product, the choice being dictated by market forces. This is particularly true for polymer industries, where product and grade transitions occur frequently. This necessitates the use of a controller that can successfully regulate the plant not only at the operating points, but also during the transition.

Useful measures to evaluate a transition control strategy are no violation of constraints, speed of response, satisfactory performance, and closed-loop stability (Narendra *et al.*, 1995).

Nonlinear behavior is not an uncommon characteristic during the transition. This feature serves to not only heighten the control problem but also to require a nonlinear dynamic model of the process for control studies. However, due to a lack of available high fidelity nonlinear models, because nonlinear control methods are not well-understood, and closed-loop stability arguments are difficult to prove, nonlinear strategies are very difficult to implement (Tian and Hoo, 2002*b*). Additionally, the maintenance costs of nonlinear control algorithms are usually substantially higher than those of linear control algorithms for the same process (Eker and Nikolaou, 2002).

A popular option is to use multiple linear models that together represent the nonlinear system. Thus, many transition control strategies are based on linear models with fixed parameters so that linear control theory can be applied. For large operating spaces, the issue of how many fixed model/controller pairs are needed remains unanswered. A variety of the multiple model strategies employ a conditional probability to determine how to combine a set of fixed models to generate a new model that better represents the plant outputs (Aufderheide and Bequette, 2001). Others adapt the existing models and controller parameters using a model reference adaptive control (MRAC) framework (Narendra *et al.*, 1995; Gundala *et al.*, 2000).

It is well known that different control strategies can be attempted once the estimation model is chosen. Whether a more robust controller can be used to assure stability and improved perfor-

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mance, is an intriguing question. Sun and Hoo (1999a, 1999b) presented a robust dynamic transition control structure for time-delayed systems using a set of fixed models and controllers. Tian and Hoo (2000b) used H_∞ controllers based on fixed and adaptive models.

In this work, a state-shared model framework represents multiple fixed and adaptive models. The state-shared model consists of a non-minimal realization and a non-minimal identifier. The former is developed at the known operating points, while the latter is used at any other point. The parameters in the measurement equation are adapted. Thus, all models, adaptive and fixed, can be cast into such a structure, in which all the models share the same states but the parameters in the measurement equation represent different operating points.

The rationale for using adaptive and fixed models is to ensure that there is at least one model with parameters sufficiently close to those of the unknown plant to provide accurate controller response. The fixed models together with stable switching provide speed, while the intelligent adaptive models and tuning provide accuracy.

From a system identification point of view, the state-shared model has attractive properties such as the uniqueness of the identified parameterization and convergence of the adaptable parameters.

The coefficient matrices in the state-shared model are selected to be controllable by the designer. The existence and uniqueness criteria for this type of parameterized model are based on a uniquely identifiable parameterization known as a matrix fraction description (MFD) (Kailath, 1980). The convergence of the parameter adaptation is also proven. For an m-input m-output linear MIMO system, the total number of identified parameters is derived to be $N_\theta = n(m^2+1)$, where n is the upper bound of the McMillan degree of all nominal linear models.

The organization of the paper is as follows. Section two begins with a brief review of some relevant results and properties of linear MIMO systems. Section three develops the state-shared model and presents the existence and uniqueness criteria. The convergence of the identifier is also analyzed. Section four demonstrates the construction of the state-shared model on a MIMO nonlinear system studied by (Gundala *et al.*, 2000) that undergoes a production rate transition. Lastly, section five summarizes the findings.

2. PRELIMINARIES

An m-input p-output linear time-invariant (LTI) system has a transfer function $H(s) \in R^{p\times m}(s)$, the set of $(p \times m)$ matrices with polynomial elements (Antsaklis and Michel, 1997; Kailath, 1980).

Definition 1. A rational transfer function matrix $H(s)$ is said to be *proper* if

$$\lim_{s\to\infty} H(s) < \infty$$

and *strictly proper* if

$$\lim_{s\to\infty} H(s) = 0$$

Theorem 1. (Antsaklis and Michel, 1997) H(s) is realizable as the transfer function matrix of a linear time-invariant system given by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Vu \end{aligned}$$

if and only if $H(s)$ is a proper rational matrix. If V is the null matrix, then $H(s)$ is a strictly proper rational matrix.

Rewrite $H(s)$ as

$$\begin{aligned} H(s) &= \frac{N(s)}{d(s)} \\ d(s) &= s^n + d_1 s^{n-1} + \cdots + d_n \end{aligned}$$

where $d(s)$ is the least common multiple of the denominators of $H(s)$ and deg $d(s) = n$. Thus,

$$H(s) = D^{-1}(s)N(s), \quad D(s) = d(s)I_p \quad (1)$$

and define the *degree* of the denominator matrix as

$$\deg D(s) \equiv \deg\det(D(s)) = np$$

The pair $\{N(s),\ D(s)\}$ is called a *left* matrix fraction description (*left* MFD) (Kailath, 1980).

Proposition 1. Given any left MFD of $H(s) = D^{-1}(s)N(s)$, a state-space realization of order

$$\deg\det(D(s)) \equiv \deg \text{ left MFD}$$

can always be found.

Kailath (1980) provides a procedure to obtain the realization from the left MFD.

Lemma 1. If H(s) is a strictly proper (proper) transfer function and the left MFD is given by Eq (1), then every row of N(s) has degree strictly $<$ ($\leq$) that of the corresponding row of D(s).

The proof is similar to that of Kailath (1980).

3. MIMO STATE-SHARED MODEL

The aim is to achieve a non-minimal realization/non-minimal identifier, such that while all the models share the same state space representation, each model has unique input/output parameter set.

3.1 A non-minimal realization

Theorem 2. Any controllable and observable m-input m-output LTI system given by

$$Y_p(s) = H(s)U(s) = D^{-1}(s)N(s)U(s) \quad (2)$$

with $D(s) = d(s)I_m$ and $d(s)$, a polynomial of degree n, is input-output equivalent to the LTI system described by the differential equations

$$\begin{aligned} \dot{\omega}_1 &= F\omega_1 + GU & \omega_1 \in R^{nm\times 1} \\ \dot{\omega}_2 &= F\omega_2 + GY_p & \omega_2 \in R^{nm\times 1} \\ Y_p &= \Theta^T\omega & \end{aligned} \quad (3)$$

by suitable choice of the parameter vector $\Theta \in R^{2nm\times m}$, and Θ is uniquely determined by the given $H(s)$. The pair (F, G) should be controllable with $F \in R^{nm\times nm}$, an asymptotically stable matrix, and $G \in R^{nm\times m}$. The (F, G) can be arbitrarily chosen under the given restrictions.

Proof

Consider Figure 1. The transfer functions from $U(s)$ to v_1 and Y_p to v_2 are given by

$$\begin{aligned} &\Theta_1^T(sI-F)^{-1}G & \Theta_1 \in R^{nm\times m} \\ &\Theta_2^T(sI-F)^{-1}G & \Theta_2 \in R^{nm\times m} \end{aligned}$$

respectively. Define, $\Phi_I \equiv (sI-F)^{-1}G$. Then, the transfer function from the input U to the output Y_p can be expressed as

$$\begin{aligned} H(s) &= \left(I - \Theta_2^T\Phi_I\right)^{-1}\left(\Theta_1^T\Phi_I\right) \equiv D^{-1}(s)N(s) \\ &= (D(s)/\Omega_I(s))^{-1}(N(s)/\Omega_I(s)) \end{aligned} \quad (4)$$

where, $\Omega_I(s) \equiv \det(sI-F) = s^n + a_n s^{n-1} + \cdots + a_2 s + a_1$. From Eq (4), the following equivalent relations can be obtained

$$I - \Theta_2^T\Phi_I = \frac{D(s)}{\Omega_I(s)} \qquad \Theta_1^T\Phi_I = \frac{N(s)}{\Omega_I(s)} \quad (5)$$

Eq (5) includes $n(m^2+1)$ linear equations with the same number of unknown variables. It is not difficult to show that the coefficient matrix of this system of linear equations is nonsingular, which means that Θ_1 and Θ_2 can be determined uniquely. It then follows that any linear time-invariant plant can be parameterized as shown in Eq (3). *QED*

Remark 1: Theorem 2 implies the existence of parameter vector Θ such that the transfer function of the state-space model, given in Eq (3), is equivalent to $H(s)$, given in Eq (2).

Remark 2: The value of Θ depends on the pair (F, G) and the coefficients of $H(s)$. Since (F, G) are under the influence of the designer, Θ is uniquely determined.

Remark 3: $D(s)$ is block diagonal with the same elements in each block. As a result, there are only n elements to be identified to determine Θ_2; the others are zeros. There are nm^2 elements in Θ_1, the total number of parameters to be identified is $n(m^2+1)$.

It is assumed that the space of operating conditions is large such that there are many possible operating states. At known operating states, a suitable linear model can be developed or identified and such a model will have fixed parameters in its input/output form.

3.2 A non-minimal identifier

At any unknown operating point, it is not difficult to construct a non-minimal identifier (not interpolation). Assume the non-minimal identifier has the same form as the non-minimal realization. Because the operating point is not known *a priori*, let the parameters in the measurement equation be adapted to obtain an accurate representation. The state-space equation of the non-minimal identifier will be driven by the measured signals, U and Y_p.

The non-minimal identifier is given by (3)

$$\begin{aligned} \dot{\omega}_1 &= F\omega_1 + GU & \omega_1 \in R^{nm\times 1} \\ \dot{\omega}_2 &= F\omega_2 + GY_p & \omega_2 \in R^{nm\times 1} \\ \hat{Y} &= \hat{\Theta}^T\omega & \end{aligned} \quad (6)$$

3.2.1. Parameter adaptation The adaptation of the parameters must be done in a stable fashion. In this work, a normalized least-squares for an m-input m-output system is proposed. For details see (Tian and Hoo, 2002*a*).

Let the model-plant mismatch be given by $\tilde{Y} \equiv \hat{Y} - Y_p$. It can be shown that

$$\lim_{t\to\infty} \tilde{Y}(t) = 0, \qquad \lim_{t\to\infty} \hat{\Theta}(t) = \Theta$$

It is understood that the state-shared model is both the non-minimal realization of the plant and the adaptive non-minimal identifier. From the procedure described in the previous section, it can be concluded that the parameters of the state-shared model are uniquely determined by the transfer functions of the input/output models. Note that any such realization necessarily fulfills the requirement that the output of model j, Y_j be an asymptotically correct estimate of output of the plant Y_p if the process model transfer function were $H(s)$, i.e. $Y_j \to Y_p$.

3.3 Model reduction

For control implementable solutions, it is desirable to use low-order controllers whenever possible. One means of reducing the order of the controller is to generate a reduced-order approximation of the plant before designing the controller (Mahadevan and Hoo, 2000; Zheng and Hoo, 2002).

Reduced-order approximation of plant dynamics is not an uncommon engineering practice. In general, mathematical models are reduced-order approximations of the true system generated by ignoring *minor* effects during modeling. Clearly, the number of parameters to be adapted affects the rate of convergence and the computational burden.

From the previous section, it is known that the order of state-shared model for a MIMO system is $2nm$. The input/output number, m, cannot be changed. To reduce the order of the state-shared model, the order of $d(s)$ (n) must be reduced. In this work, a balanced truncation approach is used (Burl, 1999).

4. EXAMPLE: NONLINEAR REACTOR

The chemical reactor consists of a continuous-stirred tank reactor in which a single, isothermal, irreversible reaction given as $A(g) + C(g) \rightarrow D(l)$, occurs in the vapor phase (Ricker, 1993). Components A and C are non-condensible gases and component D, the product, is a non-volatile liquid.

The molar balance of each component in the system is given by,

$$\begin{aligned}
\frac{dN_A}{dt} &= y_{A1}F_1 + F_2 - \frac{N_A}{N_3}F_3 - r_D \\
\frac{dN_B}{dt} &= y_{B1}F_1 - \frac{N_B}{N_3}F_3 \\
\frac{dN_C}{dt} &= y_{C1}F_1 - \frac{N_C}{N_3}F_3 - r_D \\
\frac{dV_L}{dt} &= (r_D - F_4)/\rho_L
\end{aligned} \quad (7)$$

$$\begin{aligned}
N_k &= \sum_{i=A}^{C} N_{ik} \qquad k = 1, \ldots, 4 \\
p_i &= \frac{N_i}{\sum_{j=A}^{C} N_{jk}} P \qquad i = A, B, C \\
r_D &= k_0 p_A^{v_1} p_C^{v_2} \\
F_4 &= (\chi_4 + u_4) c_{v4} \sqrt{P - P_r} \\
u_4 &= K_c (V_L^* - \frac{V_L}{V_{L,max}} 100\%)
\end{aligned}$$

where r_D is the reaction rate (kmol/h) that depends on the partial pressures of components A and C, N_i is the number of moles of component i, and y_{ij} is the mole fraction of component i in stream j. There are two feed streams (F_1, F_2) a purge (F_3), and a product stream (F_4) with units of kmol/h. The ideal gas law is assumed to be valid and the liquid density (ρ_L) is constant. Measured outputs include the reactor pressure (P), the liquid volume (V_L), and the mol% of unreacted A in F_3 (y_{A3}).

For economic, safety, and operational considerations, F_4, P, and y_{A3} should be controlled. The three manipulated variables are F_1, F_2 and F_3. The product rate is adjusted by a proportional feedback controller in response to variations in the liquid inventory. The control signal from a liquid inventory controller is u_4. There are safety and production constraints. The reactor pressure must be maintained below the shutdown limit of 3000 kPa, and F_1 and F_2 cannot be larger than their maximum values of 330.46 kmol/h and 22.46 kmol/h, respectively. More details can be found in (Tian and Hoo, 2002*a*).

It is desired to transition the reactor between two production rates, OPI: 100 and OPII: 130 kmol h^{-1}. An analysis of the model shows that both states are feasible, stable operating states.

4.1 Modeling

Linearization at the known operating states yields a linear model in the form,

$$\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx \\
x &= x_p - \bar{x} \qquad y = y_p - \bar{y}_p
\end{aligned} \quad (8)$$

where $\bar{x}$ and $\bar{y}_p$ are the steady state values of x_p and y_p, respectively; $A = \nabla_{x_p} f$, $B = \nabla_{u_p} f$, and C is found from the measurement equation. An eigenvalue analysis shows that the matrix A, at both OPI and OPII, is asymptotically stable. Additionally, the linear systems at both operating states are stable and output controllable implying that the nonlinear system at these operating states are at least locally stable. The system is also observable.

At each operating point, a 2-state, 3-input 3-output reduced-order model corresponding to the 4-state, 3-input 3-output full-order linear model is obtained by the model reduction method of balance truncation.

Assume that at any point in the operating space, the nonlinear system can be approximated by a reduced-order linear model of order 2 (the upper bound of the McMillan degree), but each model may be different at each operating state. The aim of the state-shared model structure is to represent all the linear models by one state-shared framework. Their measurement equations represent their differences. From the theory presented in §3, the state-shared model will have order $nm = 6$, with $n(m^2 + 1) = 20$ adaptable parameters. Let the pair (F, G) be given by,

$$F_j = \begin{bmatrix} 0 & 1 \\ & \\ -a_1 & -a_2 \end{bmatrix} \quad G_j = \begin{bmatrix} 0 \\ \\ 1 \end{bmatrix} \quad j = 1, 2, 3$$

with $a_1 = a_2 = 1$ such that the pair (F, G) are controllable. It then follows that,

$$F = \begin{bmatrix} F_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & F_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_3 \end{bmatrix}_{6\times 6} \quad G = \begin{bmatrix} G_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & G_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & G_3 \end{bmatrix}_{6\times 3}$$

At any operating point, given $N(s)$ and $D(s)$, the measurement equation parameters, (Θ_1, Θ_2), can be calculated.

4.2 Model validation

In a neighborhood of OPI, random input disturbance signals (zero mean and standard deviations of 15% of their nominal values at OPI) is introduced to both the nonlinear system and linear models. To quantify the differences between the model responses, define the Average Relative Error (ARE) of the j^{th} measurement after k sample points by,

$$\mathrm{ARE}(j) = k^{-1} \sum_{i=1}^{k} \left| \frac{y_p(i,j) - \bar{y}_p(j) - y(i,j)}{y_p(i,j)} \right| \quad (9)$$

Here, $y_p(i,j)$ is the j^{th} output of the nonlinear system, $\bar{y}_p(j)$ is the j^{th} nominal value of the nonlinear model, and $y(i,j)$ is the j^{th} model response.

Table 1 lists the AREs among the different models. The largest errors are associated with F_4 but they are $\leq 1\%$. Similar results were obtained at OPII.

4.3 Transition control

The system is forced to transition from 100 kmol/h to 130 kmol/h, while satisfying all other constraints. First order reference trajectories (dotted lines in the figures) are selected for the three outputs. A model predictive controller is used with the state-shared model and measurement equations to achieve the transition. A controller horizon of 4 and a prediction horizon of 10 are selected. To represent a preference among the controlled variables, output weights of 3, 1, and 10 for P, y_{A3}, and F_4, respectively are selected. Equal weighting of the rate of change in the inputs is used. The system has constraints on the outputs and inputs. No special attempts were used to determine optimal values for these parameters.

Figure 3 shows the closed-loop responses of F_4 and P. The production rate achieves its set point in about 10 hours. There is no violation of the pressure constraint. The production rate can be made to reach its the set point within 5 hours without violation of any constraint, but the controller action is more aggressive.

Figure 4 shows the the closed-loop responses of F_4 and P when the system has unmeasured disturbances as it transitions. Here, unmeasured output disturbances, with a signal noise ratio of 10:1, are introduced into the system. A first order filter is used to filter out any noise. There are no constraint violations, and although F_4 does not track closely the reference trajectory, the set point change is achieved within 8 hours.

In practice, the composition measurement can not always be obtained in a timely fashion. Assume y_{A3} must be inferred from the other measures signals. Closed-loop performance based on estimates of y_{A3} are shown in Figure 5. The transition is achieved within 8 hours. There is no violation of the pressure constraint (not shown).

5. SUMMARY

In this work, a method to construct a state-shared model for MIMO systems is developed and its properties analyzed. This approach can represent the plant in such a fashion that all the unknown parameters of the plant appear as the elements of a single matrix in the measurement equation of the state-shared model. A solution of the parameter vector is obtained when the transfer function is known. Existence and uniqueness for the parameterization are proven. The parameter vector can be adapted by least-squares, such that the adaptive model can be obtained with the same state-space representation.

A nonlinear chemical reactor system that transitions from one production rate to another was used to demonstrate the concept of the state-shared model in a model predictive control framework. Satisfactory closed-loop performance was achieved even in the face of unmeasured output disturbances and when composition was estimated. Future work will be to apply the state-shared model framework to a plant wide problem such as the Tennessee Eastman challenge problem.

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Table 1. AREs (%).

	P	y_{A3}	F_4
Nonlinear-Full linear	0.02	0.01	0.07
Nonlinear-Reduced linear	0.02	0.29	0.93
Nonlinear-State Shared	0.25	0.32	0.93
Full linear-Reduced linear	0.01	0.30	0.83
State Shared-Reduced linear	0.01	0.04	0.18

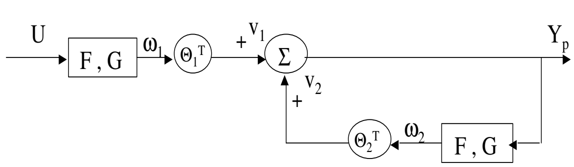

Fig. 1. Non-minimal realization.

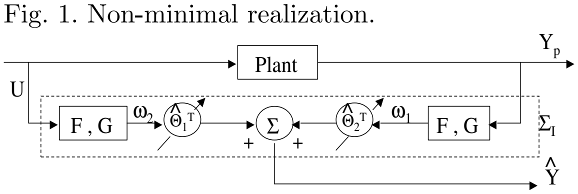

Fig. 2. Non-minimal identifier.

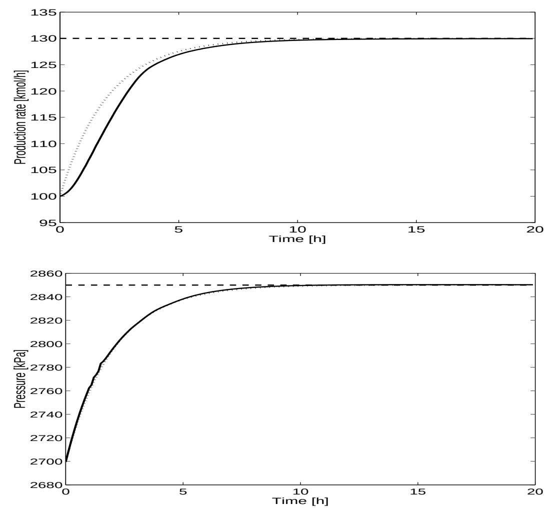

Fig. 3. Transition: ideal conditions.

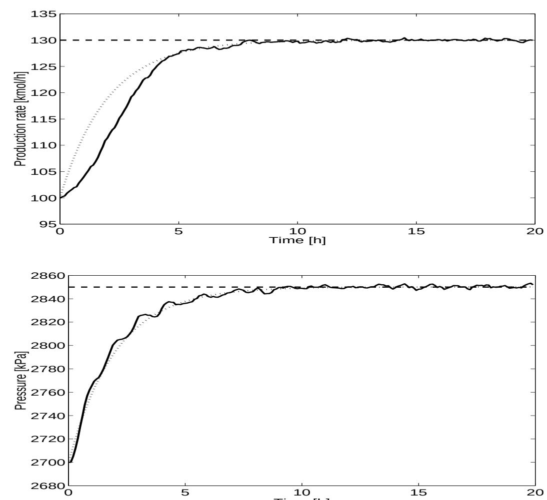

Fig. 4. Transition: unmeasured output disturbances.

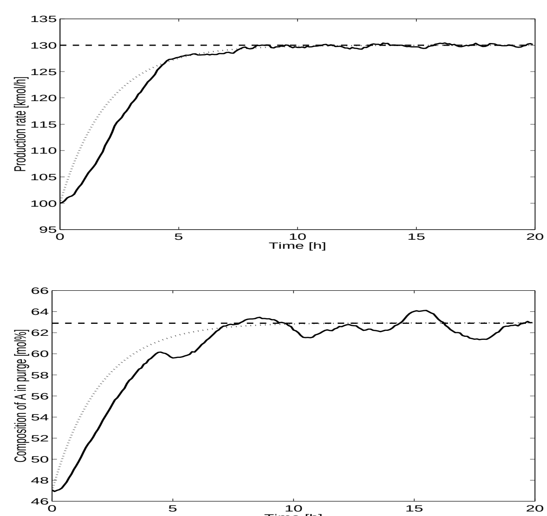

Fig. 5. Transition: estimate composition in the purge.