HYBRID CONTROL OF A FOUR TANKS SYSTEM

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Abstract: Many processes, even being of a continuous nature, involve in its operation signals or rules different from the classical continuous variables represented by real variables and modelled by DAE. In practice they include on/off valves or other binary actuators, are subjected to logical operational rules, or are mixed with sequential operations. As a result, classical control does not fit very well with the overall operation of the plant. In this paper we consider the problem of hybrid control from a predictive control perspective, showing in a practical non trivial example with changing process structure, how the problem can be stated and solved.

Keywords: Hybrid control, predictive control, four tank system

1. INTRODUCTION

The topic of hybrid systems and hybrid control has received a lot of attention in the latest years, mainly in relation with complex distributed systems that combine continuous operating units of several nature with interconnections following logical rules. In the present paper this field is seen from a process control perspective where the core of the process is continuous, and the main variables can be represented by real numbers, but where there are also other elements that do not fit in this framework. These can be classified into four categories:

- Devices or elements that operates in an on/off way and that can be represented by binary variables instead of usual real ones. As typical examples we can mention on/off valves or motors.
- Process units that can operate or be switched off according to the production needs or constraints.
- Operational rules or constraints of logical nature that form part of the correct operation of a process. They are given usually in the form IF (situation) THEN (action).
- Process units that operate in batch mode according to a given sequence of stages. Here the timing and scheduling of the operation is a key factor.

In all these cases, the standard control approach, based on a continuous process model and continuous manipulated variables, fails due to the discrete (integer) or logical nature of the new elements. Nevertheless, in industrial practice, if we exclude the more simple cases of SISO control loops and we navigate towards plant wide control considering the problem of controlling a complex process unit or a section of a factory, it is very likely that the above mentioned elements are present in a certain degree. Then, it is important to reformulate the control problem finding adequate representations of these hybrid systems as well as practical paths to solve and analyse them.

There are several approaches to model hybrid systems. Some of them set hierarchical levels, leaving the continuous parts in the bottom and the discrete decision variables in the upper ones (Grossmann, *et al*., 1993). Other approaches take advantage of the fact that prepositional logic expressions can be formulated in a systematic way as linear inequalities of binary variables (Clocksin and Mellish, 1981). An important contribution in this line is (Bemporad and Morari, 1999), where MLD systems are defined and analysed. Other contributions can be seen in (Colmenares, *et al*., 2001; Zhu et al., 2000).

In this paper, within the framework of predictive control, a case study of a process with four interconnected tanks is presented. It is able to operate in different modes according to the value of a set of on/off valves. The paper is organised as follows: In section 2, a review of how to formulate hybrid models and the associated predictive control is presented. In section 3 the process is described, while in section 4 the specific formulation for the model and MPC controller is given. Results can be seen in section 5 and, finally, some brief conclusions are drawn.

2. MLD MODELS

A natural way to represent discrete elements with two or more states (on/off type) or process units that can be switched off, is by means of integer (0–1) variables. Logical operational rules can be translated into inequalities involving binary variables in a systematic way.

If P is a logical proposition that can have the values true or false, then associating an integer variable y (1-0) to it, conjunctions and disjunctions of propositions can be translated easily:

$$\begin{array}{ll} P_1 \wedge P_2 & y_1 + y_2 \geq 1 \\ P_1 \vee P_2 & y_1 \geq 1,\ y_2 \geq 1 \end{array} \quad (1)$$

More general expressions are first converted into the so called normal conjunctive pattern:

$$Q_1 \wedge Q_2 \wedge \ldots Q_n \quad (2)$$

where Q is a disjunctive proposition and then translated as before. The procedure for converting a proposition into this pattern follows three steps:

a) Replace the logical implications by its equivalent:

$$P_1 \Rightarrow P_2 \Leftrightarrow \overline{P_1} \vee P_2 \quad (3)$$

b) Apply Morgan's laws in order to move the negations inside

$$\overline{(P_1 \wedge P_2)} \Leftrightarrow \overline{P_1} \vee \overline{P_2} \qquad \overline{(P_1 \vee P_2)} \Leftrightarrow \overline{P_1} \wedge \overline{P_2} \quad (4)$$

c) Apply the distributed property in order to obtain the desired pattern

$$(\mathrm{P}_1 \wedge \mathrm{P}_2) \vee \mathrm{P}_3 \Leftrightarrow (\mathrm{P}_1 \vee \mathrm{P}_3) \wedge (\mathrm{P}_2 \vee \mathrm{P}_3) \quad (5)$$

Activation or de-activation of real variables x liked to the existence or operation of discrete elements can be formulated as products of the type xy, but this creates a non-linearity. An alternative is to formulate them in terms of linear inequalities of the type:

$$Ly \leq x \leq Uy \quad (6)$$

where L and U are lower and upper limits of x, while y is the associated integer variable. If y is 1, then the standard constraint on x remains active, but if y=0 the x variable is forced to 0. In (Floudas, 1995) a way of dealing with more complex situations can be seen.

A model integrating continuous dynamics, discontinuous variables and logical constrains results then in a set of equations such as:

$$\begin{array}{l} \dfrac{dx}{dt} = f(x,u,y) \\ h(x,u,y) = 0 \\ g(x,u,y) \leq 0 \\ x \in X \subset R^n \\ u \in U \subset R^m \\ y \in \{0,1\} \end{array} \quad (7)$$

being x continuous process variables, u real decision variables and y integer ones.

The predictive control problem is then to choose u and y over a given control horizon, so that a cost index is minimised along a given prediction horizon, repeating the problem every sampling period as part of a moving horizon strategy. Unfortunately, because of the presence of the integer variables, this is a mixed integer optimisation problem which implies a heavy computational burden.

3. PROCESS DESCRIPTION

The four tanks systems is part of a lab plant at UAB used as a test bed for this kind of problems. The system to be controlled is depicted in Fig. 1 and consists of two sections: the storage section, represented by the two upper tanks and the mixing one which includes the two bottom tanks.

Liquid flows from the storage tanks to the mixing ones through four pipes which have on/off valves (V) in order to activate or block the lines, and two speed pumps. Another flows q_{BM} are added into the mixing tanks in proportion to the main currents. Input flows q_{Ev} to the storage tanks, as well as the demands of the final products q_M , are subjected to strong and frequent changes, as coming from a batch section, and can be considered as the main disturbances to the plant.

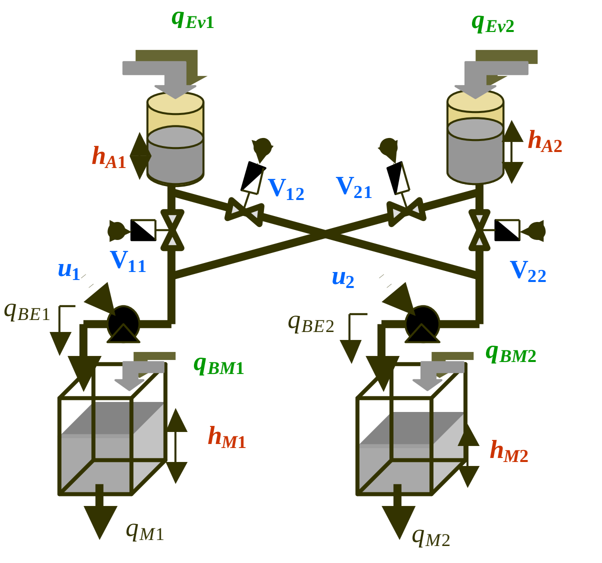

Fig. 1 Schematic Diagram

The purpose of this process is mixing the currents q_{BE} and q_{BM} in given proportions while maintaining the levels of the four tanks close to given setpoints and, on any case, within pre-specified ranges.

The storage phase can be described by the following equations:

$$S_{A1}\frac{dh_{A1}}{dt} = q_{Ev1} - q_{11} - q_{12} \qquad (8)$$

$$S_{A2}\frac{dh_{A2}}{dt} = q_{Ev2} - q_{21} - q_{22} \qquad (9)$$

$$q_{BE1} = ku_1 = q_{11} + q_{21} \qquad (10)$$

$$q_{BE2} = ku_2 = q_{12} + q_{22} \qquad (11)$$

where h represents the level in the tanks and the relation between the output flow of the pumps (q_{BE1}, q_{BE2}) and the input signal to them (u_1, u_2) is considered linear. The inflows q_{Ev1} and q_{Ev2} are measured disturbances and the four cross flows q_{ij} are depending on q_{BE1} and q_{BE2}.

$$q_{1i} = q_{BEi} \cdot f_1(h_{A1}, h_{A2}), \qquad i = 1{,}2 \qquad (12)$$

$$q_{2i} = q_{BEi} \cdot f_2(h_{A1}, h_{A2}), \qquad i = 1{,}2 \qquad (13)$$

where

$$f_1(h_{A1}, h_{A2}) = \frac{\sqrt{h_{A1}}}{\sqrt{h_{A1}} + \sqrt{h_{A2}}} \qquad (14)$$

and

$$f_1(h_{A1}, h_{A2}) = \frac{\sqrt{h_{A2}}}{\sqrt{h_{A1}} + \sqrt{h_{A2}}} \qquad (15)$$

A simplified linear expression of relations (12) and (13), can be obtained approximating (14) and (15) by its values at the nominal operating point $(\bar{h}_{A1}, \bar{h}_{A2})$:

$$q_{1i} = a_1 q_{BEi}, \qquad i = 1{,}2 \qquad (16)$$

$$q_{2i} = a_2 q_{BEi}, \qquad i = 1{,}2 \qquad (17)$$

being

$$\begin{aligned} a_1 &= f_1(\bar{h}_{A1}, \bar{h}_{A2}) \\ a_2 &= f_2(\bar{h}_{A1}, \bar{h}_{A2}) = 1 - a_1 \end{aligned} \qquad (18)$$

The other part of the process, the mixing tanks, is modelled by a similar set of equations:

$$S_{M1}\frac{dh_{M1}}{dt} = q_{BE1} + q_{BM1} - q_{M1} \qquad (19)$$

$$S_{M2}\frac{dh_{M2}}{dt} = q_{BE2} + q_{BM2} - q_{M2} \qquad (20)$$

$$q_{BM1} = Rq_{BE1} \qquad (21)$$

$$q_{BM2} = Rq_{BE2} \qquad (22)$$

The outflows q_{M1} and q_{M2} represent the demand of the final product which is also a known value. The equations (21) and (22) indicate that ratio control is apply in maintaining the value of q_{BM1} and q_{BM2}.

4. PREDICTIVE CONTROL

As mentioned above, the goal is to control the levels of the four tanks in spite of the disturbances, manipulating the signals (u_1, u_2) to the two pumps and the four on/off valves, V_{ij}, which interconnect the tanks. So, there are four controlled variables, two real manipulated variables and four integer ones, plus four disturbances.

Behind all approaches of predictive control there are a model of the plant used to predict the future evolution of the system. Based on this prediction, at each time step, the controller selects a sequence of future command inputs through an on line optimization procedure, which aims at maximizing the tracking performance subjected to given constraints. In our case, due to the on/off valves, the model is one of hybrid nature. So, in addition to continuous models used to describe the process ((8)-(22)), the behavior of the system must be completed with the operating modes imposed by the four on/off valves, which can be stated in terms of prepositional logic. In this way the process can be modeled through a MLD structure. There are several ways of doing it. Here, a particular one is presented.

4.1 Representation of Logic

Note that, when taking into account the on/off valves, equations (10) and (11) are not valid in all situations and the model must be modified. The outflows from the pumps depend also on the state of the on/off valves V_{ij}, according to the following rules:

$$q_{BE1} = \begin{cases} 0 & \text{if} \quad V_{11} = 0 \wedge V_{21} = 0 \\ ku_1 & \text{if} \quad V_{11} = 1 \vee V_{21} = 1 \end{cases} \qquad (23)$$

$$q_{BE2} = \begin{cases} 0 & \text{if} \quad V_{21} = 0 \wedge V_{22} = 0 \\ ku_2 & \text{if} \quad V_{21} = 1 \vee V_{22} = 1 \end{cases} \qquad (24)$$

So, considering the possible combinations of the valves states (0/1), for each left and right section of the process, four possibilities are generated. The first group of compound statements is:

$$V_{11} = 0 \wedge V_{21} = 0 \Rightarrow \begin{cases} q_{11} = 0 \\ q_{21} = 0 \end{cases} \rightarrow (P_{11}) \qquad (25)$$

$$V_{11} = 0 \wedge V_{21} = 1 \Rightarrow \begin{cases} q_{11} = 0 \\ q_{21} = ku_1 \end{cases} \rightarrow (P_{12}) \qquad (26)$$

$$V_{11} = 1 \wedge V_{21} = 0 \Rightarrow \begin{cases} q_{11} = ku_1 \\ q_{21} = 0 \end{cases} \rightarrow (P_{13}) \qquad (27)$$

$$V_{11}=1 \wedge V_{21}=1 \Rightarrow \begin{cases} q_{11}=ka_1u_1 \\ q_{21}=ka_2u_1 \end{cases} \rightarrow (P_{14}) \qquad (28)$$

The prepositional logic expressions P_{1j} (25)-(28) can be translated into a mathematical representation by associating a binary variable $y_{1i} \in \{0,1\}$ with each clause P_{1i}. The clause P being true or false corresponds to the values y=1 or y=0.

In this way the expression for the inlet flows to the left pump has a new mathematical form:

$$\begin{aligned} q_{11} &= ku_1y_{13} + a_1ku_1y_{14} \\ q_{21} &= ku_1y_{12} + a_1ku_1y_{14} \end{aligned} \qquad (29)$$

where the 0-1 y variables activate/de-activate continuous terms. As mentioned in section 2, a more efficient equivalent form of (29) is obtained by introducing inequality constraints instead:

$$\begin{aligned} 0y_{11} + 0y_{12} + q_{\min}y_{13} + a_1q_{\min}y_{14} &\le q_{11} \\ q_{11} &\le 0y_{11} + 0y_{12} + q_{\max}y_{13} + a_1q_{\max}y_{14} \end{aligned} \qquad (30)$$

$$\begin{aligned} 0y_{11} + q_{\min}y_{12} + 0y_{13} + a_2q_{\min}y_{14} &\le q_{21} \\ q_{21} &\le 0y_{11} + q_{\max}y_{12} + 0y_{13} + a_2q_{\max}y_{14} \end{aligned} \qquad (31)$$

where

$$\begin{aligned} q_{\max} &= kU_{\max} \\ q_{\min} &= kU_{\min} \end{aligned} \qquad (32)$$

with U_{max} and U_{min} the upper and lower bounds on the voltage of the pumps.

Notice that one and only one of the situations (25)-(28) can be active at a time, which implies the need of the prepositional logic expression (the exclusive-or condition):

$$P_{11} \oplus P_{12} \oplus P_{13} \oplus P_{14} \qquad (33)$$

which can be easily converted into a linear equality constraint in terms of the associated integer variables:

$$y_{11} + y_{12} + y_{13} + y_{14} = 1 \qquad (34)$$

In a similar way, other four 0/1 variables y_{2i} (i=1,4) are introduced in order to model the right section of the process. The corresponding constraints are:

$$\begin{aligned} 0y_{21} + 0y_{22} + q_{\min}y_{23} + a_1q_{\min}y_{24} &\le q_{12} \\ q_{12} &\le 0y_{21} + 0y_{22} + q_{\max}y_{23} + a_1q_{\max}y_{24} \end{aligned} \qquad (35)$$

$$\begin{aligned} 0y_{21} + q_{\min}y_{22} + 0y_{23} + a_2q_{\min}y_{24} &\le q_{22} \\ q_{22} &\le 0y_{21} + q_{\max}y_{22} + 0y_{23} + a_2q_{\max}y_{24} \end{aligned} \qquad (36)$$

$$\sum_{i=1}^{4} y_{2i} = 1 \qquad (37)$$

4.2 The optimization problem

The task of the predictive controller is minimizing at every sampling time the following finite horizon objective function:

$$\min_{\Delta u_i, y}\left(\sum_{i=1}^{4}\sum_{j=N1_i}^{N2_i} \left(g_i\left(\hat{h}_i(t+j) - r(t+j)\right)^2 \right) + \sum_{i=1}^{4}\sum_{j=0}^{Nu_i-1} b_i\left(\Delta u_i(t+j)\right)^2 \right) \qquad (38)$$

where $\hat{h}_i(t+j)$ are predicted values of the outputs (the levels of the four tanks) and $Du(t)=u(t)-u(t-1)$, subjected to the model previously developed and possible constraints on the process variables. Due to the presence of integer variables, the optimization procedure is a Mixed Integer Quadratic Programming (MIQP) problem. This is a hard task from a computational point of view, mainly in the non-linear case. So, in order to keep it as simpler as possible, that is, in linear form, the optimization problem was formulated in terms of the decision variable x (39), which includes current and future values of the inlet flows to the pumps as well as the eight integer variables y_{ij}, i=1÷2, j=1÷4, instead of the more natural V and u signals.

$$\mathbf{x} = \begin{pmatrix} \Delta q_{11}(t) \\ \vdots \\ \Delta q_{11}(t+Nu_1-1) \\ \Delta q_{21}(t) \\ \vdots \\ \Delta q_{21}(t+Nu_2-1) \\ \vdots \\ \vdots \\ \Delta q_{22}(t) \\ \vdots \\ \Delta q_{22}(t+Nu_2-1) \\ y_{11} \\ \vdots \\ y_{14} \\ y_{21} \\ \vdots \\ y_{24} \end{pmatrix} \qquad (39)$$

Then, in addition to (30), (31), (34)-(37), other constraints in the controlled and manipulated variables are also taken into account:

$$\begin{cases} \underline{U}_{ik} \le q_{ik}(t+j) \le \overline{U}_{ik} & 0 \le j \le Nu_i - 1;\ 1 \le i \le 4, 1 \le k \le 2 \\ \underline{D}_{ik} \le \Delta q_{ik}(t+j) \le \overline{D}_{ik} & \\ \underline{L}_i \le \hat{x}_i(t+j) \le \overline{L}_i & N3_i \le j \le N4_i;\ 1 \le i \le 4 \end{cases}$$

(40)

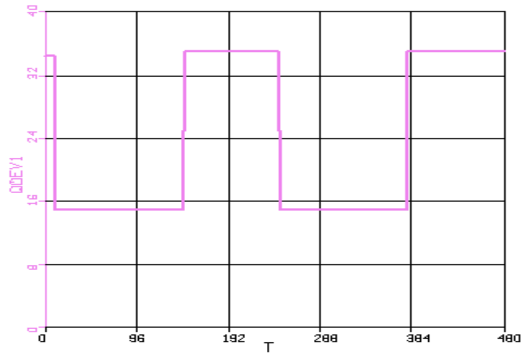

Fig.2 Evolution of inflow q_{Ev1}

Finally, a practical solution in terms of the two control signals u and the V positions can be obtained from the y_{ij} values and

$$u_1 = \frac{q_{11} + q_{21}}{k}; \quad u_2 = \frac{q_{12} + q_{22}}{k} \qquad (41)$$

At each time step, this problem involves $\sum_{i=1}^{4} Nu_i$ continuous variables, eight integer variables and $2 * \sum_{i=1}^{4} Nu_i + \sum_{i=1}^{4} (N_{4_i} - N_{3_i} + 1) + 6$ linear constraints.

The implementation of the Mixed-Integer Predictive Controller proposed in this paper has been obtained in C language by using the NAG package as a MIQP solver based on the branch and bound method.

5. RESULTS

Several tests have been carried out to investigate the performance of the controller. The nominal operating point is $(\bar{h}_{A1}, \bar{h}_{A2})$ = (8.37 cm, 11.16 cm) which in our plant leads to a_1=0.46 and a_2=0.54. The ratio factor from the mixing was chosen as R=3 and the coefficient k = 7.5. The sampling period was set to 5s, and the controller was tuned with the following design parameters:

- Prediction horizon: N_1={1,1,1,1}, N_2={10,10,10,10};
- Constraint horizon: N_3={1,1,1,1}, N_4={10,10,10,10};
- Weighting factor for the control term: b={0.01,0.01,0.01,0.01}

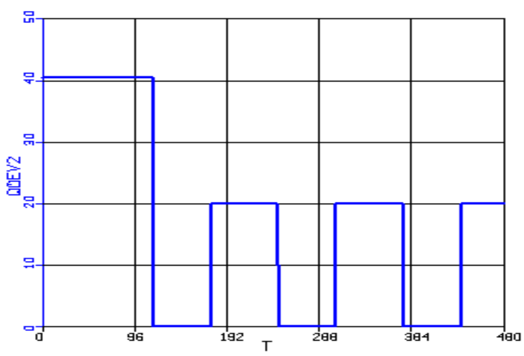

Fig. 3 Evolution of inflow q_{Ev2}

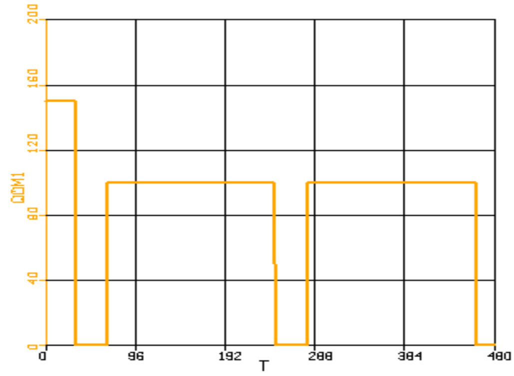

Fig. 4 Demand product outflow q_{M1}

- Weights of the controlled variables in the cost index g={1,1,15,15} which implies that preference was given to maintain the level of the mixing tanks.
- Set points for h_{A1}, h_{A2}, h_{M1} and h_{M2} were given the values: {8.37, 11.16, 20, 20 cm}, while the input and output constraints (40) were fix to:
- $\underline{U} = \{0,0,0,0\}$, $\overline{U} = \{75,75,75,75\}$ cm^3/s;
- $\underline{D} = \{-15,-15,-15,-15\}$; $\overline{D} = \{15,15,15,15\}$;
- $\underline{L} = \{0.14, 0.14, 0.038, 0.038\}$ cm
- $\overline{L} = \{26.5,\ 26.5,\ 30,\ 30\}$ cm.

The disturbances q_{Ev1} and q_{Ev2} representing the load to the storing tanks have the time evolution represented in Fig. 2 and 3, while the others two product outflows q_{M1} and q_{M2} have another periodic structure which is usual in cases where a batch section follows (Fig. 4, 5).

The first experiment considers the control horizon Nu={1,1,1,1} and the results are presented in Fig. 6. There we can see that the process operates according to the control objectives: keeping the levels of the mixing tanks on the set point (see the top half of the figure) and maintain the other two levels into the operating bounds (the bottom of the figure). Fig. 7 shows the manipulated variables, the two continuous signals to the pumps and the four on/off valves. A different response (Fig. 8, 9) of the process is obtained if the control horizon is increased to Nu={7,7,7,7}. The levels of the mixing tanks are closer to the set point and the control actions present a more active form.

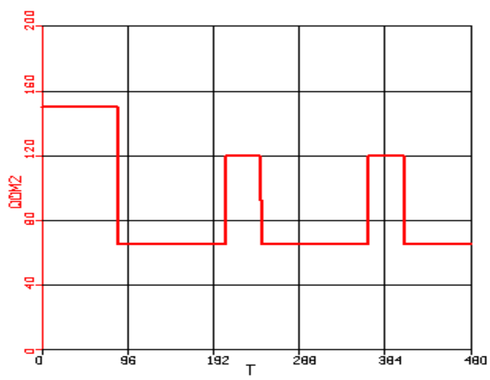

Fig. 5 Demand product outflow q_{M2}

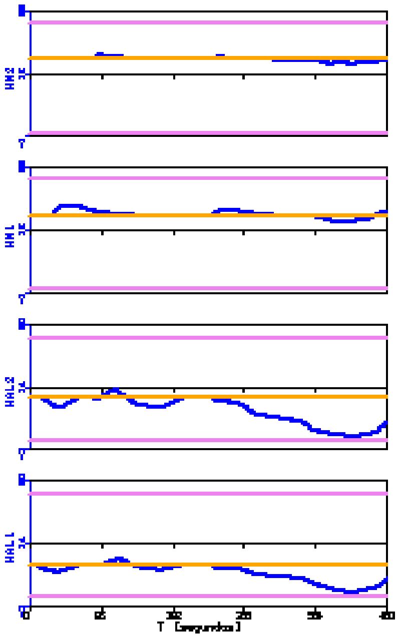

Fig. 6 Controlled variables. Short control horizon

The computation time corresponding to each sample time is approximately 0.01 seconds in a SUN workstation with 128 Mbytes of RAM.

6. CONCLUSIONS

In this paper an example of practical hybrid control have been presented. The results shows the feasibility of this approach but topics such as the best problem formulation, computational methods and closed loop properties are still open to further research.

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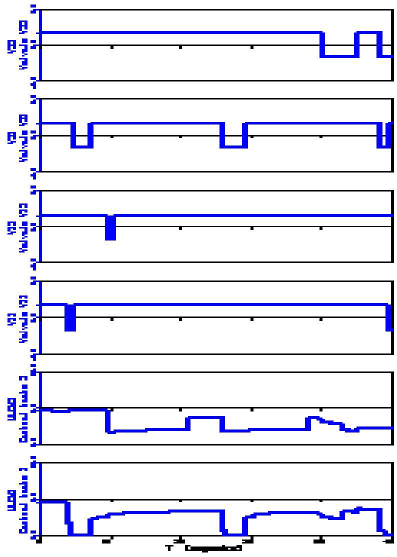

Fig. 7 Manipulated variables. *Nu*={1,1,1,1}

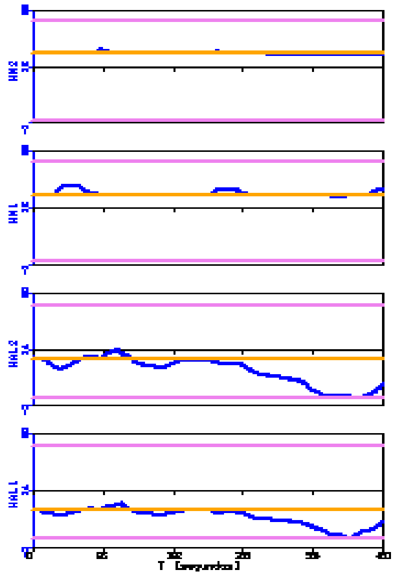

Fig. 8 Controlled variables. Longer control horizon

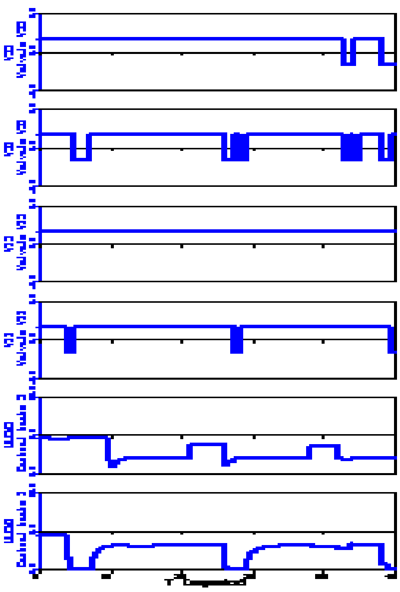

Fig. 9 Manipulated variables. *Nu*={7,7,7,7}