# Robust Stability Analysis for Descriptor Systems with State Delay and Parameter Uncertainty ‡

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**Abstract:** This paper considers the problem of robust stability analysis for continuous descriptor systems with state delay and structured uncertainties. A computationally simple approach to test stability of descriptor delay systems is proposed. Based on this, we developed a sufficient condition which guarantees that the perturbed descriptor delay system under consideration is regular, impulse-free and stable for all admissible uncertainties. An example is provided to demonstrate the application of the proposed approach.

**Keywords:** Continuous descriptor systems, robust stability, time-delay systems, uncertain systems.

## 1. Introduction

In the past years, much attention has been addressed to the study of stability analysis and controller design for time-delay systems since time delays are often the main causes for instability and poor performance of systems and encountered in various engineering systems such as chemical processes, long transmission lines in pneumatic systems, and so on [8] . When parameter uncertainty appears in a delay system, the problem of robust stability as well as robust stabilization has been dealt with and various approaches have been proposed [5, 16] .

On the other hand, it is known that descriptor systems provide a more natural description of dynamical systems than state-space systems and have attracted much interest in recent years. Descriptor systems are also referred to as singular systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems [4] . Applications of such systems can be found in dynamic models of chemical systems [2, 11] , mechanical engineering [9] , and other areas. There have been many research works on extending existing theories and results based on state-space systems to descriptor systems [4, 14] . Recently, there has been a growing interest in the study of robust stability analysis and robust control for descriptor systems [6, 7, 15, 21] . In [6] and [7] , upper bounds on structured perturbations ensuring robust stability for uncertain continuous and discrete descriptor systems were given, respectively. For descriptor systems with unstructured uncertainties, [20] and [22] studied the robust stability problem by extending the concept of "*quadratic stability*" for state-space systems, and some sufficient conditions for robust stability were obtained. Similar results for discrete-time descriptor systems were reported in [21] . Very recently, discrete descriptor systems with time delays as well as parametric uncertainties were studied in [18] , where both robust stability and robust D-stability results were presented. For continuous descriptor delay systems with unstructured uncertainties, sufficient conditions for both robust stability and robust stabilization were given in [19] . It is worth pointing out that when dealing with the robust stability problem for descriptor delay systems, similar to delay-free case [6, 7] , not only stability robustness, but also regularity and impulse immunity (for continuous descriptor systems) and causality (for discrete descriptor systems) should be considered simultaneously [18, 19] , while for state-space delay systems the latter two issues do not arise. For continuous descriptor delay systems, although robust stability results for unstructured uncertainties were obtained in [19] , when structured uncertainties appear, no results on robust stability are available in the literature, this issue is still open.

In this paper, we deal with the problem of robust stability for continuous descriptor systems with state delay and structured uncertainties. The purpose is to develope conditions such that the perturbed descriptor delay system under consideration is regular, impulse-free and stable for all admissible uncertainties. We first present a computationally simple stability condition for descriptor delay systems without parameter uncertainties. Then, by this and some properties of modulus matrix, a robust stability condition is proposed, which can be viewed as an extension of existing results on robust stability for descriptor systems without delay. Finally, an example is given to demonstrate the effectiveness of the proposed approach.

*Notation.* Throughout this paper, for matrices $X$, $Y \in \mathbb{R}^{n\times n}$, the notation $X \geq Y$ means that $X_{ij} \geq Y_{ij}$, $i$, $j = 1, 2, \ldots, n$, where $X_{ij}$, $Y_{ij}$ $(i, j = 1, 2, \ldots, n)$, are elements of $X$ and $Y$, respectively. $I$ is the identity matrix with appropriate dimension. The superscript "$T$" represents the transpose. $\mathbb{C}^+$ is the closed right-half plane. $\|x(t)\|$ denotes the Euclidean norm of vector $x$. $\rho(M)$ refers to spectral radius of matrix $M$ and $|M|_m$ is the modulus matrix of $M$. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

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## 2. Preliminaries and Problem Formulation

Consider the following linear continuous descriptor system with parameter uncertainties and state delay:

$$(\Sigma): \quad E\dot{x}(t) = (A+\Delta A)x(t) + (A_d + \Delta A_d)x(t-\tau) \quad (1)$$

$$x(t) = \phi(t), \; t \in (-\tau, 0] \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input. The matrix $E \in \mathbb{R}^{n\times n}$ may be descriptor, we shall assume that rank $E = r \leq n$. $A$ and $A_d$ are known real constant matrices with appropriate dimensions. $\tau > 0$ is a constant time delay of the system, $\phi(t)$ is the compatible continuous vector valued initial condition. $\Delta A$ and $\Delta A_d$ are time-invariant parameter uncertainties and are assumed to have the following properties [7, 13] :

$$|\Delta A|_m \leq M_A, \; |\Delta A_d|_m \leq M_d \quad (3)$$

where $M_A$ and $M_d$ are constant matrices whose elements are all nonnegative. The constant matrices $M_A$ and $M_d$ represent the highly structured information for the additive perturbation matrices $\Delta A$ and $\Delta A_d$. The parameter uncertainties $\Delta A$ and $\Delta A_d$ are said to be admissible if (3) holds.

The nominal descriptor delay system of (1) can be written as:

$$E\dot{x}(t) = Ax(t) + A_d x(t-\tau). \quad (4)$$

**Definition 1** [4, 14]

(I) The pair $(E, A)$ is said to be regular if $\det(sE - A)$ is not identically zero.

(II) The pair $(E, A)$ is said to be impulse-free if $\deg(\det(sE - A)) = \operatorname{rank} E$.

(III) The pair $(E, A)$ is said to be stable if all of its finite eigenvalues are in the open left-half plane.

The descriptor delay system (4) may have an impulsive solution, however, the regularity and the absence of impulses of the pair $(E, A)$ ensure the existence and uniqueness of an impulse-free solution to this system, which is shown in the following lemma.

**Proposition 1** [19] *Suppose the pair $(E, A)$ is regular and impulse free, then the solution to (4) exists and is impulse-free and unique on $(0, \infty)$.*

In view of this, we introduce the following definition for descriptor delay system (4).

**Definition 2** [19]

(I) The descriptor delay system (4) is said to be regular and impulse-free if the pair $(E, A)$ is regular and impulse free.

(II) The descriptor delay system (4) is said to be stable if for any $\varepsilon > 0$ there exists a scalar $\delta(\varepsilon) > 0$ such that, for any compatible initial conditions $\phi(t)$ satisfying $\sup_{-\tau\leq t\leq 0} \|\phi(t)\| \leq \delta(\varepsilon)$, the solution $x(t)$ of system (4) satisfies $\|x(t)\| \leq \varepsilon$. Furthermore,

$$x(t) \to 0, \; t \to \infty$$

The purpose of this paper is to develop robust $\alpha$-stability conditions for descriptor delay systems. To this end, it is worth pointing out that the regularity, impulse immunity as well as stability robustness should be considered simultaneously when dealing with the problem of robust stability analysis for uncertain descriptor delay systems [19] , which is similar to the robust stability analysis for uncertain descriptor systems without delay [6, 7] .

## 3. Main Results

In this section, a computationally simple robust stability condition for descriptor delay systems will be developed. We first present the following lemma which will play a key role in the derivation of our main results.

**Lemma 1** *Suppose the pair $(E, A)$ is regular, impulse-free and stable, then the descriptor delay system (4) is regular, impulse-free and stable if*

$$\rho\left[(sE - A)^{-1} A_d\right] < 1, \quad \forall\, s \in \mathbb{C}^+. \quad (5)$$

**Proof.** From the Definition 2, the regularity and impulse immunity of the pair $(E, A)$ implies that the descriptor delay system (4) is regular, impulse-free. To show the stability of system (4), we first note that from [4] the regularity and impulse immunity of the pair $(E, A)$ guarantees that there exist two invertible matrices $P$ and $Q$ such that

$$PEQ = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \; PAQ = \begin{bmatrix} A_1 & 0 \\ 0 & I \end{bmatrix} \quad (6)$$

where $A_1 \in \mathbb{R}^{r\times r}$. Since the pair $(E, A)$ is stable, we have that $sI - A_1$ is invertible for all $s \in \mathbb{C}^+$, which implies that $(sE - A)^{-1}$ is well defined for all $s \in \mathbb{C}^+$. Now, write

$$PA_dQ = \begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4} \end{bmatrix} \quad (7)$$

compatibly with (6). Noting

$$\lim_{s\to\infty} (sE - A)^{-1} A_d = Q \begin{bmatrix} 0 & 0 \\ -A_{d3} & -A_{d4} \end{bmatrix} Q^{-1}. \quad (8)$$

This together with (5) implies that

$$\rho(A_{d4}) < 1. \quad (9)$$

Now set $\xi(t) = Qx(t)$ and decompose

$$\xi(t) = \begin{bmatrix} \xi_1(t)^T & \xi_2(t)^T \end{bmatrix}^T$$

where $\xi_1(t) \in \mathbb{R}^r$ and $\xi_2(t) \in \mathbb{R}^{n-r}$. Then, noting (6) and (7), system (4) can be transformed to

$$\begin{aligned} \dot{\xi}_1(t) &= A_1\xi_1(t) + A_{d1}\xi_1(t-h) + A_{d2}\xi_2(t-h) \\ \xi_2(t) &= -A_{d3}\xi_1(t-h) - A_{d4}\xi_2(t-h). \end{aligned}$$

On the other hand, considering (5), it is easy to see

$$\det\left[I-(sE-A)^{-1}A_d e^{-s\tau}\right]\neq 0,\quad \forall\ s\in\mathbb{C}^+.$$

Using this and noting $\det(sE-A)\neq 0$ for all $s\in\mathbb{C}^+$, we have

$$\begin{aligned}&\det\left(sE-A-A_d e^{-s\tau}\right)\\ =&\det(sE-A)\det\left[I-(sE-A)^{-1}A_d e^{-s\tau}\right]\neq 0,\forall\ s\in\mathbb{C}^+.\end{aligned}\tag{10}$$

That is,

$$\det\begin{bmatrix} sI-A_1-A_{d1}e^{-s\tau} & -A_{d2}e^{-s\tau}\\ -A_{d3}e^{-s\tau} & -I-A_{d4}e^{-s\tau}\end{bmatrix}\neq 0,\quad \forall\ s\in\mathbb{C}^+.$$

From this and (9) and along the same lines as in the proof of Theorems A and B (page 384) in [10] we can show that

$$\xi_1(t)\to 0,\ \xi_2(t)\to 0, t\to\infty.$$

This implies

$$x(t)\to 0,\ t\to\infty.$$

Therefore, the descriptor delay system (4) is stable. □

The following lemmas will be used in the proof of our main results.

**Lemma 2** [7, 17] *For any $n\times n$ matrices $X$, $Y$ and $Z$ with $|X|_m\leq Z$, we have*

(a) $|XY|_m\leq |X|_m|Y|_m\leq Z|Y|_m$
(b) $|X+Y|_m\leq |X|_m+|Y|_m\leq Z+|Y|_m$
(c) $\rho(X)\leq\rho(|X|_m)\leq\rho(Z)$
(d) $\rho(XY)\leq\rho(|X|_m|Y|_m)\leq\rho(Z|Y|_m)$
(e) $\rho(X+Y)\leq\rho(|X+Y|_m)\leq\rho(|X|_m+|Y|_m)\leq\rho(Z+|Y|_m)$.

**Lemma 3** [12] *For any $n\times n$ matrices $X$, if $\rho(X)<1$, then $\det(I-X)\neq 0$.*

**Lemma 4** [1] *A regular pair $(E,A)$ is impulse-free if and only if $(sE-A)^{-1}$ is proper.*

**Lemma 5** [3] *Let $M(s)$ be a square rational matrix and be decomposed uniquely as $M(s)=M_p(s)+M_{sp}(s)$, where $M_p(s)$ is a polynomial matrix and $M_{sp}(s)$ is a strictly proper rational matrix. Then $M^{-1}(s)$ is proper if and only if $M_p^{-1}(s)$ exists and is proper.*

Suppose the pair $(E,A)$ is regular, impulse-free and stable, then we can write

$$(sE-A)^{-1}=G(s)+H\tag{11}$$

where $G(s)$ is a strictly proper rational matrix which is analytic in right-half $s$-plane and $H$ is a constant matrix.

**Lemma 6** [6] *If the pair $(E,A)$ is regular, impulse-free and stable, then*

$$\left|(sE-A)^{-1}\right|_m\leq L+|H|_m\tag{12}$$

*where*

$$L=\int_0^\infty |G(t)|_m\,dt.\tag{13}$$

*and $G(t)$ is the impulse response of $G(s)$ which is given in (11).*

Now we are in a position to present the robust stability result for uncertain discrete descriptor delay systems.

**Theorem 1** *Suppose the pair $(E,A)$ is regular, impulse-free and stable, then the uncertain descriptor delay system ($\Sigma$) is still regular, impulse-free and stable for all admissible uncertainties $\Delta A$ and $\Delta A_d$ if*

$$\rho\left[(L+|H|_m)M_A\right]+\rho\left[(L+|H|_m)(|A_d|_m+M_d)\right]<1\tag{14}$$

*where $H$ and $L$ are given in (11) and (13), respectively.*

**Proof.** From (14), it is easy to show that

$$\rho\left[(L+|H|_m)M_A\right]<1.\tag{15}$$

Then, by Lemma 2 and (11) we have

$$\begin{aligned}\rho\left[(sE-A)^{-1}\Delta A\right] &\leq \rho\left[\left|(sE-A)^{-1}\Delta A\right|_m\right]\\ &\leq \rho\left[\left|(sE-A)^{-1}\right|_m|\Delta A|_m\right]\\ &\leq \rho\left[(L+|H|_m)M_A\right]<1\end{aligned}\tag{16}$$

for all $s\in\mathbb{C}^+$. Therefore, it follows from Lemma 3 that

$$\det\left[I-(sE-A)^{-1}\Delta A\right]\neq 0,\quad \forall\ s\in\mathbb{C}^+.$$

Thus, $\forall\ s\in\mathbb{C}^+$,

$$\begin{aligned}&\det(sE-A-\Delta A)\\ =\ &\det(sE-A)\det\left[I-(sE-A)^{-1}\Delta A\right]\neq 0.\end{aligned}$$

This implies that the pair $(E,A+\Delta A)$ is regular for all admissible uncertainties. Next, we shall show that, for all admissible uncertainties, the pair $(E,A+\Delta A)$ is impulse-free. Applying Lemma 2 and noting (15), it can be seen that

$$\begin{aligned}\rho(H\Delta A) &\leq \rho(|H\Delta A|_m)\leq\rho(|H|_m M_A)\\ &\leq \rho\left[(L+|H|_m)M_A\right]<1.\end{aligned}$$

By Lemma 3, we have that $I-H\Delta A$ is invertible. Now, considering (11) we can write

$$\begin{aligned}&[sE-(A+\Delta A)]^{-1}\\ =\ &\left[I-(sE-A)^{-1}\Delta A\right]^{-1}(sE-A)^{-1}\\ =\ &\left[(I-H\Delta A)-G(s)\Delta A\right]^{-1}(sE-A)^{-1}.\end{aligned}\tag{17}$$

Taking into account $G(s)\Delta A$ is strictly proper and $I-H\Delta A$ is invertible, it then follows from Lemma 5 that $\left[(I-H\Delta A)-G(s)\Delta A\right]^{-1}$ is proper. Noting this and recalling that $(sE-A)^{-1}$ is proper, we have that

$$[sE-(A+\Delta A)]^{-1}$$

is proper too. Therefore, it follows from Lemma 4 that the pair $(E,A+\Delta A)$ is impulse-free. This together with the regularity of the pair $(E,A+\Delta A)$ implies that the uncertain descriptor delay system ($\Sigma$) is regular and impulse-free for all admissible uncertainties..

On the other hand, by Theorem 9.8.3 in [12] , it follows from (16) that for all $s \in \mathbb{C}^+$ we can write

$$\begin{aligned}\left[I-(sE-A)^{-1}\Delta A\right]^{-1} &= I+(sE-A)^{-1}\Delta A\\ &\quad +\left[(sE-A)^{-1}\Delta A\right]^2+\cdots.\end{aligned}$$

Using this and (15), we have

$$\begin{aligned}&\rho\left[\left|\left(I-(sE-A)^{-1}\Delta A\right)^{-1}\right|_m\right]\\ \leq\ &\rho\left[I+\left|(sE-A)^{-1}\Delta A\right|_m\right.\\ &\left.+\left|\left[(sE-A)^{-1}\Delta A\right]^2\right|_m+\cdots\right]\\ \leq\ &\rho\left[I+\left|(sE-A)^{-1}\right|_m|\Delta A|_m\right.\\ &\left.+\left[\left|(sE-A)^{-1}\right|_m|\Delta A|_m\right]^2+\cdots\right]\\ \leq\ &1+\rho\left[\left(L+|H|_m\right)M_A\right]\\ &+\rho\left(\left[\left(L+|H|_m\right)M_A\right]^2\right)+\cdots\\ =\ &1/\left(1-\rho\left[\left(L+|H|_m\right)M_A\right]\right).\end{aligned}$$

Hence,

$$\begin{aligned}&\rho\left[(sE-(A+\Delta A))^{-1}(A_d+\Delta A_d)\right]\\ =\ &\rho\left[\left(I-(sE-A)^{-1}\Delta A\right)^{-1}(sE-A)^{-1}(A_d+\Delta A_d)\right]\\ \leq\ &\rho\left[\left|\left(I-(sE-A)^{-1}\Delta A\right)^{-1}\right|_m\left|(sE-A)^{-1}\right|_m\right.\\ &\left.\times\left|(A_d+\Delta A_d)\right|_m\right]\\ \leq\ &\rho\left[\left|\left(I-(sE-A)^{-1}\Delta A\right)^{-1}\right|_m\right]\\ &\times\rho\left[\left|(sE-A)^{-1}\right|_m\left|(A_d+\Delta A_d)\right|_m\right]\\ \leq\ &\frac{\rho\left[\left(L+|H|_m\right)\left(|A_d|_m+M_d\right)\right]}{1-\rho\left[\left(L+|H|_m\right)M_A\right]}.\end{aligned}\qquad(18)$$

From (14), it can be easily shown that

$$\frac{\rho\left[\left(L+|H|_m\right)\left(|A_d|_m+M_d\right)\right]}{1-\rho\left[\left(L+|H|_m\right)M_A\right]}<1.$$

This together with (18) gives

$$\rho\left[(sE-(A+\Delta A))^{-1}(A_d+\Delta A_d)\right]<1.\qquad(19)$$

By recalling the pair $(E, A+\Delta A)$ is regular and impulse-free, noting (19) and using Lemma 1, we have the uncertain descriptor delay system $(\Sigma)$ regular, impulse-free and stable for all admissible uncertainties. □

**Remark 1** *Theorem 1 provides a simple method to test whether the uncertain descriptor delay system $(\Sigma)$ is regular, impulse-free and stable for all admissible uncertainties under the assumption that the pair $(E, A)$ is regular, impulse-free and stable. Note that in order to use Theorem 1, the computation of the matrices $L$ and $H$ is necessary. A simple method proposed in* [6] *can be resorted to and the matrices $L$ and $H$ can thus be easily computed.*

**Remark 2** *In the case when $A_d = 0$ and $M_d = 0$, that is, the time-delay system $(\Sigma)$ reduces to a descriptor system without delay, it is easy to verify that Theorem 1 coincides with Theorem 2.7 in* [6] *, therefore, Theorem 1 can be viewed as an extension of existing results on robust stability for descriptor systems with delay-free to descriptor delay systems.*

## 4. Example

Consider the uncertain continuous descriptor delay system $(\Sigma)$ with parameters as follows:

$$E=\begin{bmatrix}0&-1&0&0\\1&-1&0&1\\0&0&0&0\\0.5&-0.5&0&0.5\end{bmatrix},$$

$$A=\begin{bmatrix}0&6&0&0\\-5&5.5&0&-5\\0&1&0&-2\\-2.5&2.75&1&-2.5\end{bmatrix},$$

$$A_d=\begin{bmatrix}0&-0.6&0.1&0\\-0.4&-0.1&0.2&0\\0&0.1&-0.1&0.2\\0.1&-0.1&0&-0.5\end{bmatrix}$$

$$M_A=\begin{bmatrix}0.1&0.1&0.1&0\\0.1&0.1&0&0.2\\0&0.1&0.1&0.1\\0.1&0&0.2&0.1\end{bmatrix},$$

$$M_d=\begin{bmatrix}0.1&0.2&0.1&0.1\\0.1&0&0.1&0.1\\0.1&0.3&0&0.1\\0.2&0&0.1&0\end{bmatrix}.$$

The time delay is $\tau = 2$. It can be verified that there exist two invertible matrices

$$U=\left[\begin{array}{c}U_a\\ \hline U_b\end{array}\right]=\left[\begin{array}{cccc}-1&0&0&0\\0.5&-1&0&0\\ \hline 0&0.5&0&-1\\0&0&-1&0\end{array}\right]$$

$$V=\left[\begin{array}{c|c}V_a&V_b\end{array}\right]=\left[\begin{array}{cc|cc}0&-1&0&-0.5\\1&0&0&0\\0&0&-1&0\\0.5&0&0&0.5\end{array}\right]$$

such that

$$UEV=\left[\begin{array}{c|c}I&0\\ \hline 0&0\end{array}\right]=\left[\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\\ \hline 0&0&0&0\\0&0&0&0\end{array}\right],$$

$$UAV = \left[\begin{array}{c|c} A_1 & 0 \\ \hline 0 & I \end{array}\right] = \left[\begin{array}{cc|cc} -6 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right].$$

Therefore, the pair $(E, A)$ is regular, impulse-free. Now using the method in [7] , we obtain

$$\begin{aligned} L &= \int_0^\infty |G(t)|_m \, dt = \int_0^\infty \left|V_a e^{A_1 t} U_a\right|_m dt \\ &= \begin{bmatrix} 0.1 & 0.2 & 0 & 0 \\ 0.1667 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0833 & 0 & 0 & 0 \end{bmatrix} \\ |H|_m &= |V_b U_b|_m = \begin{bmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 1 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}. \end{aligned}$$

Then, we can calculate

$$\begin{aligned} \rho\left[(L + |H|_m) M_A\right] &= 0.3043 \\ \rho\left[(L + |H|_m)(|A_d|_m + M_d)\right] &= 0.6315 \end{aligned}$$

and

$$\begin{aligned} &\rho\left[(L + |H|_m) M_A\right] + \rho\left[(L + |H|_m)(|A_d|_m + M_d)\right] \\ = \;& 0.9358 < 1. \end{aligned}$$

Hence, from Theorem 1 it is seen that the uncertain descriptor delay system under consideration is regular, impulse-free and stable for all admissible uncertainties.

## 5. Conclusions

In this paper, the problem of robust stability analysis for continuous descriptor systems with state delay and structured uncertainties has been studied. A sufficient condition ensuring regularity, impulse immunity and stability for the perturbed descriptor delay system has been presented. The proposed approach is computationally simple to use. An example has been provided to demonstrate the effectiveness of the proposed approach.

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