CONSTRUCTING TAKAGI-SUGENO FUZZY MODEL BASED ON MODIFIED FUZZY CLUSTERING ALGORITHM

Xing Zongyi, Jia Limin, Shi Tianyun, Qin Yong, Jiang Qiuhua

China Academy of Railway Science 100081 Beijing China

Abstract: Gath-Geva fuzzy clustering algorithm is a nonparallel fuzzy clustering algorithm and is not easy to get a suitable and interpretable fuzzy set. The outputs of the Takagi-Sugeno fuzzy model can influence the input space partition. Neglecting this influence increases the identification error. In this paper, a modified Gath-Geva fuzzy clustering algorithm is introduced to solve these problems. Together with weighted least square method, we construct Takagi-Sugeno model to identify non-linear system. The identification of the glass oven demonstrated the effectiveness of the proposed method.

Keywords: fuzzy clustering, Takagi-Sugeno model, fuzzy modeling

1. Introduction

Fuzzy modelling and identification techniques have become an active research area due to its successful application to non-linear complex systems, where traditional methods are difficult to apply because of lack of sufficient knowledge. Among the different fuzzy modelling techniques, the Takagi-Sugeno (TS) fuzzy model has attracted most attention.

The TS fuzzy model consists a set of if-then rules that have a special format with a polynomial function type consequent. The TS fuzzy model approach tries to decompose the input space into fuzzy subspace and then approximate the system in each subspace by a simple linear regression. Without the time-consuming and mathematically intractable defuzzification operation, the TS fuzzy model is the most popular candidate for fuzzy modelling.

There are two main issues in the process of constructing a TS fuzzy model. The first is how to determine the premise structure, and the second is how to estimate the parameters of the TS fuzzy model. Fuzzy clustering and least square method have proved to be suitable techniques to create TS fuzzy model.

Fuzzy clustering algorithms like the algorithm by Gustafson and Kessel (GK), Gath and Geva (GG), or the fuzzy C-means algorithm partition the input space into adequate subspaces and detect linear local substructures. Therefore, these algorithms are very suitable to construct TS fuzzy model from data. A modified Gath-Geva algorithm is proposed in this paper.

This work is supported by NSFC (60174051/F03) and ICTF (DZYFYY0204)

The paper is organized as follows. In section 2, we formulate the TS fuzzy model. A modified Gath-Geva fuzzy clustering (MGG) algorithm is described in section 3. Identification of glass oven is provided in section 4 to illustrate the effectiveness of the modified Gath-Geva algorithm. Finally, Section 5 constrains some conclusions.

2. Takagi-Sugeno Fuzzy Model

The Takagi-Sugeno Fuzzy Model was proposed by Takagi, Sugeno in an effort to develop a systematic approach to generating fuzzy rules from a given input-output data set. A typical fuzzy rule has the form:

$\boldsymbol{R_i}$: if x_1 is $A_{i,1}$ and   and x_p is $A_{i,p}$ then y_i is $f_i(x)$

where

$$f_i(x) = a_{i,0} + a_{i,1}x_1 + \cdots + a_{i,p}x_p$$

in which i=1,   ,k, $x_i(1 \leqslant i \leqslant c)$, are the input variables, y_i is the output variables, $A_{i,j}$, $(1 \leqslant j \leqslant p)$, are fuzzy sets defined on the universe of discourse of the input. $f_i(x)$ is usually a linear polynomial function in the input variables.

In the TS fuzzy model, each fuzzy rule describes a local linear model. All these local models combine to describe a non-linear complex system, which is difficult to find a global model.

The outputs of the TS fuzzy model is computed using the normalized fuzzy mean formula:

$$y(k) = \frac{\sum_{i=1}^{c} A_i(x) * f_i(x)}{\sum_{i=1}^{c} A_i(x)}$$

where A_i is the level of fulfilment of the ith rule:

$$A_i(x) = A_{i,1}(x_1) \times A_{i,2}(x_2) \times \cdots \times A_{i,p}(x_p)$$

in this paper, Gaussian membership functions are used to represent the fuzzy sets $A_{i,j}$:

$$A_{i,j}(x_j) = \exp(-\frac{1}{2}\frac{(x_j - v_{i,j})^2}{\sigma_{i,j}^2})$$

where $v_{i,j}$ represents the centre and $\sigma_{i,j}^2$ the variance of the Gaussian function.

3. Modified Gath-Geva fuzzy clustering algorithm

The algorithm by Gath and Geva is an extension of the Gustasfon-Kessel algorithm that also takes the size and the density of the clusters into account. Contrary to the GK algorithm, the GG algorithm does not restrict the cluster s volumes and the clusters can be directly described by univariate parametric membership functions. So lower approximation error and more relevant consequent parameters can be obtained than GK algorithm can.

The Gath-Geva fuzzy clustering algorithm can briefly described as follows:

1) Choose c the number of the clusters and the weighting exponent $m > 1$;
2) Generate the matrix U with the membership degrees μ_{ik} randomly. U must satisfy condition $\sum_{k=1}^{N} \mu_{i,k} = 1$.
3) Compute the centre of the clusters:

$$v_i^{(l)} = \frac{\sum_{k=1}^{N} (\mu_{i,k}^{(l-1)})^m z_k}{\sum_{k=1}^{N} (\mu_{i,k}^{(l-1)})^m}$$

4) Compute the fuzzy covariance matrices:

$$F_i^{(l)} = \frac{\sum_{k=1}^{N} \mu_{i,k}^{(l-1)} (z_k - v_i^{(l)})(z_k - v_i^{(l)})^T}{\sum_{k=1}^{N} \mu_{i,k}^{(l-1)}}$$

5) Compute the distance between the data z_k and the centre of the clusters v_i:

$$D_{i,k}^2(z_k, v_i) = \frac{(2\pi)^{(\frac{n+1}{2})} \sqrt{\det(F_i)}}{\alpha_i} \exp(\frac{1}{2}(z_k - v_i^{(l)})^T F_i^{-1} (z_k - v_i^{(l)}))$$

with the a prior probability α_i

$$\alpha_i = \frac{1}{N} \sum_{k=1}^{N} \mu_{i,k}$$

6) Update the partition matrix U of the membership degrees:

$$\mu_{i,k}^{(l)} = \frac{1}{\sum (D_{i,k}(z_k, v_i) / D_{j,k}(z_k, v_i))^{2/(m-1)}}$$

7) Stop if $\| U^{(l)} - U^{(l-1)} \| < \varepsilon$ else go to the step 3

Univariate membership functions can often be assigned linguistic labels. This makes fuzzy systems transparent, i.e. easy to read and interpret by humans. But it is difficult to specify meaningful labels for membership functions with high dimensional domains. So it is necessary to decompose multi-dimension membership functions to univariate membership functions. Projection method was often utilized.

The projections of ellipses or ellipsoids, which are the clusters of the Gath-Geva fuzzy clustering algorithm, are rectangles that contain the ellipses (see Fig.1). In this transformation process, the information about the clusters rotation and the scaling of the axes is lost, and thus decomposition error is made. To circumvent this problem, we propose a new Gath-Geva fuzzy clustering method.

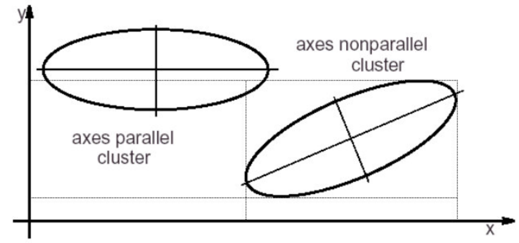

Fig.1 Axes parallel and nonparallel clusters

In Fig.1, the axes parallel cluster is illustrated. Obviously, axes parallel cluster has no rotation and thus reduce decomposition error. In Gath-Geva fuzzy clustering algorithm, if the covariance matrix is a diagonal matrix, only the axes are scaled and no rotation is performed.

For modification of the Gath-Geva algorithm, we assume the data are realization of p-dimensional normal distributions and each of these normal distributions is induced by p independent, one-dimension normal distribution.

The probability density function of the ith normal distribution is

$$g_i(x|\eta_i) = \frac{1}{(2\pi)^{p/2}} \cdot \frac{1}{\sqrt{\prod_{j=1}^{p} \sigma_j^{(i)}}} \cdot \exp(-\frac{1}{2}\sum_{j=1}^{p} \frac{(x_j - v_{i,j})^2}{\sigma_j^{(i)}})$$

where $\sigma_j^{(i)}$ is the jth element of the diagonal of the ith covariance A_i.

We introduce a fuzzification of the a posteriori probabilities in order to determine the parameter $\sigma_j^{(i)}$

$$p(\eta_i | x, y) = \prod_{j=1}^{p} (p_i g_i(x_j))^{(u_{i,j})^m}$$

We determine the maximum likelihood estimator for the formula to obtain the estimation for the parameter:

$$\sigma_j^{(i)} = \frac{\sum_{k=1}^{N} \mu_{i,k} (x_{j,k} - v_{j,k})^2}{\sum_{k=1}^{N} \mu_{i,k}}$$

the distance between the data z_k and the centre of the clusters v_i is modified as:

$$D_{i,k}^2 = \prod_{j=1}^{q} \frac{\sqrt{2\pi\sigma_{i,j}}}{\alpha_i} \exp\left(\frac{1}{2}\frac{(x_{j,k} - v_{i,j})^2}{\sigma_{i,j}}\right)$$

Thus, we gain the axes parallel version of Gath-Geva fuzzy clustering algorithm with diagonal covariance matrix.

The outputs of the TS model can influence the input space partition. Neglecting this influence, the Gath-Geva fuzzy clustering algorithm misses some data in the output space, and cannot get the most optimized clusters. Fig.2 illustrates such circumstance for a two-input single-output system. The input variables, x_1 and x_2, are divided into clusters, S_1 and S_2. After identification, S_1 projects to S^1, and S_2 projects to S^2. The dots denote the outputs of the system and the lines denote the linear consequents of TS model. Obviously, some data leak out of the ellipsoids, and identification error increased.

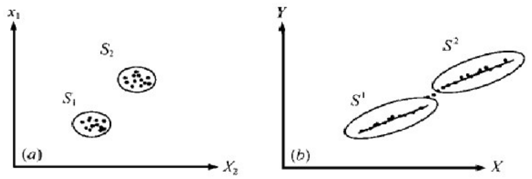

Fig.2 (a) input variables clustering (b) the actual outputs and the model outputs

To solve this problem, J.Abonyi introduce the error between actual output and model output, thus the formula of distance was modified as follow:

$$D_{i,k}^2 = \prod_{j=1}^{q} \frac{\sqrt{2\pi\sigma_{i,j}}}{\alpha_i} \exp\left(\frac{1}{2}\frac{(x_{j,k} - v_{i,j})^2}{\sigma_{i,j}}\right)^2 \cdot \sqrt{2\pi\sigma_i} \exp\left(\frac{(y_k - f_i(x_k,\theta_i))^T (y_k - f_i(x_k,\theta_i))}{\sigma_i}\right)$$

The second part of the $D_{i,k}^2$ considers the influence mentioned above, and in this way the performance of the TS model is proved.

In this paper, the weighted least-squares estimator is used to estimate the consequent parameters of TS model.

$$\theta_i = (X_e^T \Phi_i X_e)^{-1} X_e^T \Phi_i y$$

where $Xe = [X\ 1]$ and X is the input matrix. Φ_i is a matrix having the membership degrees on its main diagonal. y is the output of the system.

From above, we give the modified Gath-Geva algorithm used to construct TS model.

The modified Gath-Geva fuzzy clustering algorithm can briefly described as follows:

1) Choose c the number of the clusters and the weighting exponent $m > 1$, choose the termination tolerance $\varepsilon > 0$
2) Generate the matrix U with the membership degrees μ_{ik} randomly. U must satisfy condition $\sum_{k=1}^{N} \mu_{i,k} = 1$.
3) Compute the centre of the clusters and Compute the standard deviations of the membership functions:

$$v_i^{(l)} = \frac{\sum_{k=1}^{N} (\mu_{i,k}^{(l-1)})^m z_k}{\sum_{k=1}^{N} (\mu_{i,k}^{(l-1)})^m}$$

$$\sigma_j^{(i)} = \frac{\sum_{k=1}^{N} \mu_{i,k} (x_{j,k} - v_{j,k})^2}{\sum_{k=1}^{N} \mu_{i,k}}$$

4) Compute the consequent parameters of TS models:

$$\theta_i = (X_e^T \Phi_i X_e)^{-1} X_e^T \Phi_i y$$

5) Compute the distance between the data z_k and the centre of the clusters v_i:

$$D_{i,k}^2 = \prod_{j=1}^{q} \frac{\sqrt{2\pi\sigma_{i,j}}}{\alpha_i} \exp\left(\frac{1}{2}\frac{(x_{j,k} - v_{i,j})^2}{\sigma_{i,j}}\right) \cdot \sqrt{2\pi\sigma_i} \exp\left(\frac{(y_k - f_i(x_k,\theta_i))^T (y_k - f_i(x_k,\theta_i))}{\sigma_i}\right)$$

with the a prior probability α_i

$$\alpha_i = \frac{1}{N} \sum_{k=1}^{N} \mu_{i,k}$$

6) Update the partition matrix U of the membership degrees:

$$\mu_{i,k}^{(l)} = \frac{1}{\sum (D_{i,k}(z_k, v_i) / D_{j,k}(z_k, v_i))^{2/(m-1)}}$$

7) Stop if $\| U^{(l)} - U^{(l-1)} \| < \varepsilon$ else go to the step 3

4. Using modified Gath-Geva fuzzy clustering algorithm to construct TS model for glass oven

In this section, we will use the method mentioned above to construct Takagi-Sugeno model for glass oven. Other algorithms, including FMID and ANFIS, will also be used to identify the glass oven. The comparison of the result will prove the validity of the modified Gath-Geva fuzzy clustering algorithm.

The glass oven has 3 inputs (2 burners and 1 ventilator) and 6 outputs (temperature from sensors in a cross section of the furnace). The data have been pre-processed: detrending, peak shaving, delay estimates and normalization. The data set, including 1260 entries, is divided into a training subset and a test subset, each containing 600 samples. The number of clusters is 2.

The performance considered to evaluate the obtained model will be the root mean square error:

$$E = \frac{1}{P} \sum_{p=1}^{P} \sqrt{\frac{1}{N} \sum_{k=1}^{N} (y_k - \hat{y}_k)^2}$$

where P is the number of outputs and N is number of data, y_k is the actual output, and $\hat{y}_k$ if the model output.

Table 1 compares the performance of the model identified with these techniques, including FMID and ANFIS.

Table1. Comparison of the performance of the different algorithms

Method	Train data error	Test data error
MGG	0.8395	1.0732
FMID	0.8563	1.0489
ANFIS	0.7771	1.1403

The observation of this table indicates that modified Gath-Geva fuzzy clustering algorithm has slightly better performance than FMID in the training data set and slightly better performance than ANFIS in the test data set.

We explain the interpretability of the obtained TS fuzzy model of the 4th output as flows. For the number of clusters is two, there are also two rules.

R_1:

$$A_1 = \begin{bmatrix} 0.0309 & 0.0218 & 0.9892 \\ 1.0012 & 0.9924 & 0.0049 \end{bmatrix}$$

$$B_1 = [-0.2095 \quad 0.0620 \quad -1.9251 \quad 2.5638]$$

R_2:

$$A_2 = \begin{bmatrix} -0.0218 & -0.0099 & -1.0041 \\ 0.9960 & 1.0067 & 0.0071 \end{bmatrix}$$

$$B_2 = [-0.0413 \quad 0.0395 \quad -0.7477 \quad -1.4202]$$

The first row of A_i represents the centre v and the second row represents the variance σ^2 of the Gaussian function. B_i is the consequent parameters of the TS fuzzy model.

In Fig.3, we plot the curves of the actual outputs and the TS fuzzy model outputs of the 4th output of the glass oven. (a) is the result of training data and (b) is the result of test data.

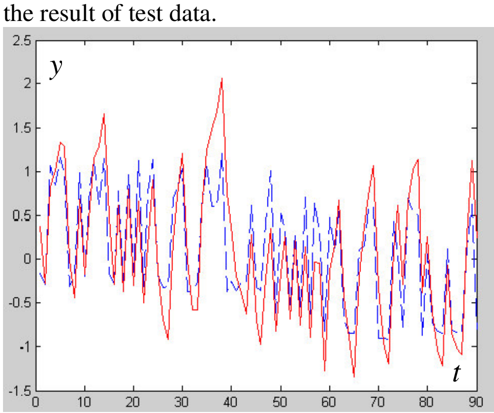

Fig.3 (a) the actual outputs (solid lines) and the TS model outputs (dotted lines) of the 4th outputs (training data, 90 samples).

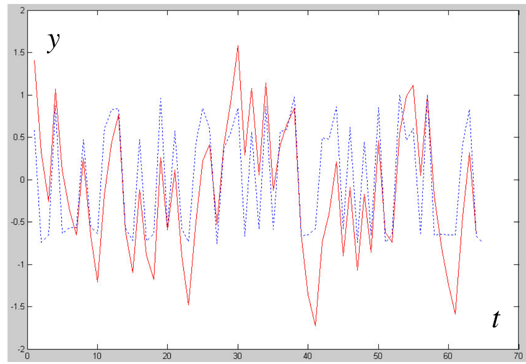

Fig.3 (b) the actual outputs (solid lines) and the TS model outputs (dotted lines) of the 4th output (test data, 65 samples).

This identification of the glass oven proves that the modified Gath-Geva fuzzy clustering algorithm can be used efficiently to construct Takagi-Sugeno fuzzy model.

5. Conclusions

We have proposed a modified Gath-Geva fuzzy clustering algorithm together with weighted least square method to create Takagi-Sugeno fuzzy model. Through minishing the projection error and considering the model outputs influence on input space partitions, we get interpretable and more accurate Takagi-Sugeno fuzzy model.

This method was used to identify the glass oven. The result proves that this method is sufficient to construct Takagi-Sugeno fuzzy model. We also compare other modelling methods, including FMID and ANFIS, with this method. The comparison shows its superiority.

REFERENCES

A.F. Gomez-Skarmeta (1999). About the use of fuzzy clustering techniques for fuzzy model identification *Fuzzy sets and systems* **106,**179-188.

Frank H?ppner and Frank Klawonn (1999). *Fuzzy Cluster Analysis*. John Wiley & Sons, LTD.

I.Gath and A.B. Geva (1989). Unsupervised optimal fuzzy clustering. *IEEE Transactions on Pattern Analysis and Machine Intelligence.* **11,** 773-781.

J.Abonyi and J.A Roubos. (2001). Compact TS-fuzzy models through clustering and OLS plus FIS model reduction. *FUZZ-IEEE. 2001*.

Jhy-shing Roger Jang (1997). *Neuro-Fuzzy and soft computing*. Prentice Hall

Tomohiro Takagi and Michio Sugeno (1985). Fuzzy identification of systems and it application to modeling and control. *IEEE Transactions on Systems, Man and Cybernetics.* **15,** 116-132.

Wang Hong-wei (2001). Improving fuzzy identifying method base on fuzzy clustering, *ACTA electronica sinica* . **29,** 436-438. (in Chinese).