VARIANCE ESTIMATION IN MULTISENSOR FUSION ALGORITHM

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Abstract: In weighted fusion algorithm for multisensor, the weights are only determined by noise variance and the precision of the variance estimation will affect the performance of the fusion algorithm. An approach of variance estimation for multisensor is presented and proves unbiased in this paper. The recurrence formula for the algorithm is also proposed, and moreover, there is no need for initial values, for which the approach is adaptive and can be used in real-time estimation. A numerical example is given to show the usefulness of the approach.

Keywords: multisensor integration, data fusion, variance, estimation theory, Gaussian noise.

1. INTRODUCTION

Modern industry adopts a great variety of sensors to monitor and control production in order to obtain a satisfactory control performance of the industrial process (Yang and Yuzo, 2000), and thus some appropriate methods are required. Multisensor data fusion is defined as the process of integrating information from multiple sources to produce the most specific and comprehensive unified data about an entity, activity or event (Raol and Girija, 2002). The process is supposed to achieve improved accuracy and more specific inferences than could be achieved by the use of a single sensor alone. In the field of measurement, weighted fusion algorithm is widely used for multisensor fusion process. The weight of each sensor is determined only by its own variance (Ling, et al., 2000; Yifeng and Leung, 1997). The precision of the variance estimation will affect the performance of the fusion algorithm seriously and the accuracy of the fusing results as well.

The variance of sensor is determined by both internal noise and environmental interference. Most of the variance estimation methods used in weighted fusion algorithm are based on experience or the sensor's variance parameter and the environmental noise is not included in consideration, which results in the distortion of the variance and the imprecision of the fusing results (Zhong, et al., 2002).

An algorithm of variance estimation for multisensor is presented and proves unbiased in this paper. No *a priori* information about each sensor and environment noise is needed in this algorithm and the real-time variance estimation can be achieved only by the observations of sensors. Simulated data given in this paper indicate the usefulness of the algorithm.

2. WEIGHTED FUSION ALGORITHM FOR MULTISENSOR

The observation of state can be modeled by using the linear system (Zhong, et al., 2002):

$$Y = Hx + e \tag{1}$$

where Y is the (n, 1) observation vector with $Y = [y_1 \quad y_2 \quad \cdots \quad y_n]^T$, x is the (1, 1) state, H is a known (n, 1) vector with $H = [1 \quad 1 \quad \cdots \quad 1]^T$, and e is the (n, 1) vector of measurement error (noise, including internal and environmental) with $e = [e_1 \quad e_2 \quad \cdots \quad e_n]^T$, a zero-mean Gaussian white noise sequence and independent of each other. It is also assumed that the noise sequence is a stationary process with ergodic property. Therefore,

$$E(e_i) = E(y_i - x) = 0$$

$$i = 1,2,\cdots,n \tag{2}$$

$$E(e_i^2) = E[(y_i - x)^2] = R_i$$

$$i = 1,2,\cdots,n \tag{3}$$

where $E(\cdot)$ is the expected value operator, and R_i denotes the noise variance of sensor i. According to the result (Gao, et al., 1999; Yifeng and Leung, 1997), The estimate of the state is：

$$\hat{x} = (H^T W^{-1} H)^{-1} H^T W^{-1} Y = \frac{\sum_{i=1}^{n} \frac{y_i}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}} \tag{4}$$

The state estimation variance (Ling, et al., 2000) is

$$E[(x - \hat{x})^2] = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}} \tag{5}$$

From the foregoing, it can be seen that the weight of each sensor is determined only by its own variance. The accuracy of the results obtained from data fusion process will be determined by the precision of the variance estimation directly.

3. VARIANCE ESTIMATION FOR MULTISENSOR

The mean of measurements from n sensors is:

$$\bar{y} = \frac{1}{n} \sum_{k=1}^{n} y_k \tag{6}$$

where $\bar{y}$ is the unbiased estimation of x obviously. From (3), the variance of sensor j is:

$$R_j = D(y_j - x) = E[(y_j - x)^2]$$

$$j = 1,2,\cdots,n \tag{7}$$

where $D(\cdot)$ is the variance operator. In fact, it is impossible to obtain the actual state x. Here let $\bar{y}$, the unbiased estimation of x, replace x in form, then the following form can be obtained:

$$E(y_j - \bar{y}) = 0 \tag{8}$$

$$R'_j = D(y_j - \bar{y}) = D\left(y_j - \frac{1}{n} \sum_{k=1}^{n} y_k \right)$$

$$= D\left[\frac{(n-1) y_j - \sum_{\substack{k=1 \\ k \neq j}}^{n} y_k}{n} \right]$$

$$= D\left[\frac{(n-1)e_j - \sum_{\substack{k=1\\k\neq j}}^{n} e_k}{n}\right]$$

$$= \frac{(n-1)^2}{n^2} R_j + \frac{1}{n^2}\sum_{\substack{k=1\\k\neq j}}^{n} R_k$$

$$j = 1,2,\cdots,n \tag{9}$$

R'_j denotes the variance of the difference between the measurement from sensor j and the mean of the measurements from n sensors. The relation between R'_j and R_k （$k = 1,2,\cdots,n$）is given in （9）. Sum up R'_j （$j = 1,2,\cdots,n$）:

$$\sum_{j=1}^{n} R'_j = \sum_{j=1}^{n}\left[\frac{(n-1)^2}{n^2} R_j\right] + \sum_{j=1}^{n}\left(\frac{1}{n^2}\sum_{\substack{k=1\\k\neq j}}^{n} R_k\right)$$

$$= \frac{(n-1)^2}{n^2}\sum_{j=1}^{n} R_j + \frac{n-1}{n^2}\sum_{j=1}^{n} R_j$$

$$= \frac{n-1}{n}\sum_{j=1}^{n} R_j \tag{10}$$

According to (9) and (10), the following form of the variance of sensor j is obtained:

$$R_j = \frac{n}{n-2}\left[R'_j - \frac{1}{n(n-1)}\sum_{k=1}^{n} R'_k\right] \tag{11}$$

Considering the condition of measuring N times by using n sensors, y_{ij} is the i-th measurement from sensor j and e_{ij} is the error. Based on the ergodic property of stochastic process，incorporation of (8) and (9) gives the estimation of R'_j:

$$\hat{R}'_j = \frac{1}{N}\sum_{i=1}^{N}(y_{ij} - \bar{y}_i)^2$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(y_{ij} - \frac{1}{n}\sum_{k=1}^{n} y_{ik}\right)^2 \tag{12}$$

By using (11), the variance estimation of sensor j is:

$$\hat{R}_j = \frac{n}{n-2}\left[\hat{R}'_j - \frac{1}{n(n-1)}\sum_{k=1}^{n}\hat{R}'_k\right] \tag{13}$$

What should be given attention to is that the foregoing algorithm is invalid when measuring with only two sensors because of the lack of the redundant information. The method of variance estimation is applicable when the number of sensors is larger than 2.

4. THE UNBIAS OF VARIANCE ESTIMATION

Based on the assumption of noise and (12), the following equation is obtained:

$$E(\hat{R}'_j) = E\left[\frac{1}{N}\sum_{i=1}^{N}\left(y_{ij} - \frac{1}{n}\sum_{k=1}^{n} y_{ik}\right)^2\right]$$

$$= \frac{1}{N}\sum_{i=1}^{N} E\left(y_{ij} - \frac{1}{n}\sum_{k=1}^{n} y_{ik}\right)^2$$

$$= \frac{1}{Nn^2}\sum_{i=1}^{N} E\left(\sum_{k=1}^{n}(y_{ij} - y_{ik})\right)^2$$

$$= \frac{1}{Nn^2}\sum_{i=1}^{N}\left\{\sum_{k=1}^{n}\sum_{m=1}^{n} E[(y_{ij} - y_{ik})(y_{ij} - y_{im})]\right\}$$

$$= \frac{1}{Nn^2}\sum_{i=1}^{N}\left\{\sum_{k=1}^{n}\sum_{m=1}^{n} E[(e_{ij} - e_{ik})(e_{ij} - e_{im})]\right\}$$

$$= \frac{1}{Nn^2}\sum_{i=1}^{N}[(n-1)^2 \cdot R_j + \sum_{\substack{k=1\\k\neq j}}^{n} R_k]$$

$$= \frac{(n-1)^2}{n^2} R_j + \frac{1}{n^2}\sum_{\substack{k=1\\k\neq j}}^{n} R_k \tag{14}$$

Considering (9) and (14), the conclusion that $\hat{R}'_j$ is the unbiased estimation of R'_j can be reached. By using (13), R_j also proves to be unbiased.

5. IMPLEMENTATION OF ALGORITHM AND THE SIMULATED INSTANCE

5.1 Implementation of algorithm

Assume that $\hat{R}'_{ij}$ denotes the variance estimation of the i-th measurement from sensor j, then (12) can be described by the following recursion:

$$\hat{R}'_{ij} = \begin{cases} 0 & i=0 \\ \dfrac{1}{i}\left[(i-1)\cdot \hat{R}'_{(i-1)j} + (y_{ij} - \dfrac{1}{n}\sum_{k=1}^{n} y_{ik})^2\right] & i=1,2,\cdots \end{cases} \tag{15}$$

Using (15) in (13), the variance estimation of each sensor based on i times sampling is obtained. However, a smaller number of sampling times than needed will lead to an inaccurate estimation and even results in the negative variance estimation. In order to ensure that the variance estimation is strictly positive, (13) is reformed as follows:

$$\hat{R}_j = \frac{n}{n-2}\left|\hat{R}'_j - \frac{1}{n(n-1)}\sum_{k=1}^{n}\hat{R}'_k\right| \tag{16}$$

In practice, (15) and (16) are used to estimate the variance of each sensor.

The algorithm of variance estimation presented in this paper can be used in real-time estimation because of its small amount of calculations by using recursive algorithm. In the mean time, it is also an adaptive algorithm and there is no need to set its initial value. With sample size increasing, the variance estimation of each sensor tends to be stable and approaches the true variance gradually.

5.2 Simulated Instance

Consider a system with 8 sensors. It is assumed the noise of each sensor is composed of internal noise and environmental noise and the noise is independent of each other. Suppose that the internal noise of each sensor is zero-mean white Gaussian noise with standard deviation 0.10, 0.20, 0.05, 0.40, 0.50, 0.30, 0.24 and 0.10 respectively and environmental interference zero-mean white Gaussian noise with standard deviation 1.0, 0.8, 1.5, 2.0, 0.8, 2.5, 3.0 and 1.3 respectively. In Table 1, algorithm 1 refers to the algorithm presented in this paper. The algorithm in which the weight is determined only by the sensor's own variance is algorithm 2, and the algorithm of averaging measurement is algorithm 3. In algorithm 4, each sensor's weight is determined by its true variance. Based on the foregoing assumption, the state modeled by $y(t)=t$ is sampled 2000 times, with the sampling interval T=1. The simulation results are shown in Figure 1, Table 1, and Table 2.

As can be seen from Figure 1, the estimated variance of each sensor gradually approaches its true variance as the sampling times increasing in algorithm 1. Data given in Table 1 and Table 2 indicate that algorithm 1 is better than algorithms 2 and algorithm 3. Algorithm 1 is a little inferior to algorithm 4 which is the optimal algorithm theoretically (see Table 1). Mean of estimation errors in each sampling range given in Table 2 can be regarded as the state estimation variance of each algorithm. According to (5), the optimal estimated variance is 0.1951. Data given in Table 2 show that mean of the square of state estimation errors approaches the true value as the sampling times increase.

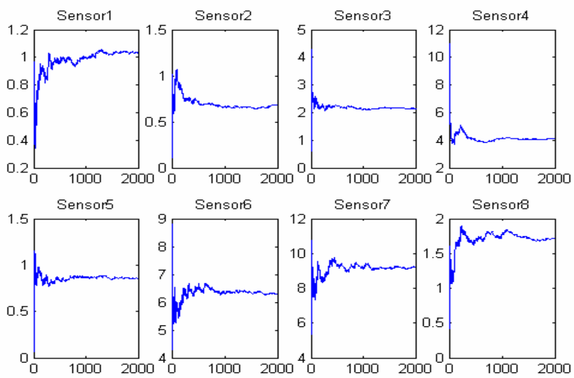

Fig. 1 The curve of sensors' variance estimation. As can be seen from the figure, the estimates of the variance converge to the actual values gradually.

Table 1 The absolute value distribution of estimation error

Error Rang	Algorithm 1	Algorithm 2	Algorithm 3	Algorithm 4
0-0.1	360	171	229	364
0.1-0.3	653	325	485	645
0.3-0.7	747	597	725	749
0.7-1.5	236	681	519	238
1.5-3	4	221	42	4
>3	0	5	0	0

Table 2 The mean square error of state estimation

Sampling Rang	Algorithm 1	Algorithm 2	Algorithm 3	Algorithm 4
1-50	0.3059	1.0778	0.3947	0.2190
51-500	0.2106	0.9094	0.3926	0.2090
501-2000	0.1954	0.9052	0.4266	0.1952

6. CONCLUSION

The algorithm of variance estimation for multisensor is discussed in this paper and proves unbiased. The approach is adaptive and can be used in real-time estimation due to the presentation of the recurrence formula. A numerical example is given to illustrate the results and to show the usefulness of the approach.

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