THE EXPLICIT MODEL-BASED TRACKING CONTROL LAW VIA PARAMETRIC PROGRAMMING

Vassilis Sakizlis John D. Perkins Efstratios N. Pistikopoulos [1]

Centre for Process Systems Engineering, Department of Chemical Engineering, Imperial College London, South Kensington campus, London SW7 2AZ, U.K.

Abstract: In this paper two methods are presented for deriving the explicit model-based tracking optimal control law for constrained linear dynamic systems subject to persistent disturbances. The first scheme augments explicitly the model dynamics with a set of integral states that are then readily incorporated with a positive definite penalty in the system performance measure. The second scheme employs a state observer for estimating the value of the disturbance and then computes the new state target. Then it shifts accordingly the state and control values to ensure asymptotic tracking. The underlying controller structure in both approaches is derived off-line via parametric programming before any actual process implementation takes place. The proposed control schemes guarantee steady-state offset elimination and optimal performance in the presence of unknown constant uncertainties.

Keywords: Model predictive control, parametric programming, process control, integral action, observer.

1. INTRODUCTION

Contrary to conventional control design methods, model predictive control (MPC) (Lee and Cooley, 1997) is particularly effective for dealing with a broad class of complex multivariable constrained processes. MPC determines the optimal future control profile according to a prediction of the system behaviour over a receding time horizon. The control actions are computed by solving repetitively an on-line optimal control problem over a receding horizon every time a state measurement or estimate becomes available. The capabilities of MPC are limited mainly by the significant on-line calculations that make it applicable mostly to slowly varying processes. This shortcoming is surpassed by employing a different type of model-based controllers the so-called parametric controllers (see section 2)(Pistikopoulos *et al.*, 2002; Bemporad *et al.*, 2002*b*). These controllers are based on recently proposed novel parametric programming algorithms, developed in our research group at Imperial College, and succeed in obtaining the explicit mapping of the optimal control actions in the space of the current states. Thus, a state feedback control law for the system is derived off-line, hence avoiding the restrictive on-line computations.

However, the inevitable presence of persistent unmeasured disturbances, pertaining for instance to model inaccuracies, parameter drift, input variations, have largely been ignored while designing the parametric controllers. Consequently, the performance of this novel control technique may lead to infeasibilities and permanent offset from

[1] To whom correspondence should be addressed. Tel.: (44) (0) 20 7594 6620, Fax: (44) (0) 20 7594 6606, E-mail: e.pistikopoulos@ic.ac.uk

the target due to inaccurate forecasting of the process behaviour. The infeasibilities may result in situations such as off-spec production or hazardous plant operation. Hence, a modification of the explicit control law is necessary to ensure feasible and safe operation.

In this work novel methodologies are presented for designing tracking model based parametric controllers for general linear dynamic systems. The control policy is derived off-line as a function of the process states via our parametric programming based theory and techniques (Dua et al., 2002; Sakizlis et al., 2002a). The proposed control scheme achieves satisfactory disturbance attenuation, while eliminating any steady state offset from the target. This is achieved via two methods: (i) As described in section 3 the first method incorporates explicitly integral action in the control design formulation while (ii) the alternative approach presented in section 4 uses an observer for estimating the disturbance vector and a reference target computation for removing the offset. A comparison of the two schemes is performed in section 5 while a demonstrative example (section 6) is presented and some conclusions (section 7) are drawn in the next paragraphs.

2. PRELIMINARIES

For deriving the explicit model - based control law for a process system, the following receding horizon optimal control problem is formulated (Bemporad *et al.*, 2002):

$$\begin{aligned}
\hat{\phi}(x_{t|t}) = &\min_{v^N \in V^N} x_{t+N|t}^T P x_{t+N|t} \\
&+ \sum_{k=0}^{N-1} [y_{t+k|t}^T Q y_{t+k|t} + v_{t+k}^T R v_{t+k}] \\
\text{s.t.}& \\
x_{t+k+1|t} &= A_1 x_{t+k|t} + A_2 v_{t+k} \\
y_{t+k|t} &= B_1 x_{t+k|t} + B_2 v_{t+k} \\
0 \geq g(y_{t+k|t}, &x_{t+k|t}, v_{t+k}) = \\
&C_0 y_{t+k|t} + C_1 x_{t+k|t} + C_2 v_{t+k} + C_3 \\
&k = 0, 1, 2, ..N-1 \\
0 \geq \psi^e(x_{t+N|t}) &= D_1 x_{t+N|t} + D_2 \\
x_{t|t} &= x^*; \qquad (1)
\end{aligned}$$

where $x \in \Re^n$ are the states, $y \in \Re^m$, are the outputs and $v \in V \subseteq \Re^q$ are the controls; t is the time when a measurement is taken, k is the future time instants and N is the prediction horizon. The outputs are the variables that we aim to control, i.e. to drive to their set-point, (temperatures, concentrations) whereas the states are the variables that fully characterize the current process conditions (enthalpies, specific volume). $v^N = [v_{t+k}^T, v_{t+k+1}^T, ..., v_{t+k+N-1}^T]^T$ denotes the sequence of the control vector over the receding horizon. The constraints $g : \Re^m \times \Re^n \times V \mapsto \Re^r$, $\psi^e : \Re^n \times V \mapsto \Re^{Q_e}$, which may pertain to product specifications or actuator restrictions, and bounds on v define the feasible operating region. We assume that the pair (A_1, A_2) is stabilizable and the pair (A_1, B_1) detectable. By considering the current states x^* as parameters and eliminating the equalities in (1) by substituting $x_{t+k|t} = A_1^k x^* + \sum_{j=0}^{k-1} (A_1^j A_2 v_{t+k-1-j})$ for the states, problem (1) is recast as a multiparametric quadratic program (mp-QP). The solution of that problem (Dua *et al.*, 2002) consists of a set of affine control functions in terms of the states and a set of regions where these functions are valid. This mapping of the manipulating inputs in the state space constitutes a control law for the system. The mathematical form of the parametric controller is as follows:

$$v_t(x^*) = \mathcal{A}_c x^* + b_c; \text{ if } CR_c^1 x^* + cr_c^2 \leq 0 \quad \text{for } c = 1, ..., N_c; \qquad (2)$$

where N_c is the number of regions in the state space, $\mathcal{A}_c, CR_c^1$ and b_c, cr_c^2 are constant matrices and vectors respectively and the index c designates that each region admits a different control law. The vector $v_{t|0}|_c$ is the first element of the control sequence, whereas similar expressions are derived for the rest of the control elements.

The model-based parametric controller described here fails to address the impact of persistent, unmeasured disturbances on the process dynamic behaviour. These uncertainties (i) tend to cause a permanent deviation of the steady state output values from their target point and (ii) may also cause violation of constraints if the reference signal is close to the feasible region boundaries. For surpassing those shortcomings Bemporad *et al.* (2002*b*) treated the disturbance as an extra parameter, thus resulting in a control law that has an extra feedforward term in (2). This technique is valid provided the disturbance modelling is perfect and an accurate measurement of the uncertainty is available. Otherwise, it fails to address the issues stressed above. Other techniques are based on anti-windup (Bemporad *et al.*, 2002*a*) , avoiding the incorporation of reference tracking capabilities in the controller structure. In the next sections it is shown how to avert the impact of disturbances by modifying the nominal design of the model based parametric controller.

3. TRACKING PARAMETRIC CONTROLLER WITH INTEGRAL PENALTY

In conventional feedback control schemes, (Seborg *et al.,* 1989) integral action is incorporated for attenuating any permanent deviation of the output variables from their set-points (e.g. PI-controller). Here, the incorporation of the integral action for the same purpose is achieved by introducing an integral state in the plant dynamics that is equal to the accumulated deviations of the output from its reference point, usually the origin. This state is augmented as an additional penalty on the objective function. The open-loop control design optimization problem (1) over the nominal uncertainty scenario, after the incorporation of the integral state is modified as follows:

$$\phi(x^*, \mathbf{xq}^*) = \min_{v_{t+k}} x_{t+N|t}^T P x_{t+N|t} + \mathbf{xq_{t+N}^T P_1 xq_{t+N}}$$
$$+ \sum_{k=0}^{N-1} [y_{t+k|t}^T Q y_{t+k|t} + v_{t+k}^T R v_{t+k}$$
$$+ \mathbf{xq_{t+k}^T Q_1 xq_{t+k}}]$$
$$\text{s.t.}$$
$$x_{t+k+1|t} = A_1 x_{t+k|t} + A_2 v_{t+k} + W\theta_{t+k}^n$$
$$y_{t+k|t} = B_1 x_{t+k|t} + B_2 v_{t+k} + F\theta_{t+k}^n$$
$$\mathbf{xq_{t+k+1} = xq_{t+k} + y_{t+k|t}}$$
$$x_{t|t} = x^*, \quad \mathbf{xq_{t|t} = xq^*}$$
$$0 \geq g(y_{t+k|t}, x_{t+k|t}, v_{t+k}); \quad 0 \geq \psi^e(x_{t+N|t})$$
$$k = 0, 1, 2, ..N-1$$
$$\mathbf{xq_{t+k} \in Re^m} \tag{3}$$

where xq is the integral state; Q_1, P_1 are the quadratic costs corresponding to that state. By treating the *pure* and the *integral* states as parameters, problem (3) is recast as a multiparametric quadratic program. The solution of that problem derives a set of piecewise affine control functions in terms of the states and a set of critical regions where these expressions hold. These functions constitute a state feedback controller whose mathematical form is as follows:

$$v_t(x^*, xq^*) = \mathcal{A}_c \cdot x^* + b_c + \mathcal{D}_\mathbf{c} \cdot \mathbf{xq}^*;$$
$$\mathcal{CR}_c \equiv \{CR_c^1 \cdot x^* + cr_c^2 + \mathbf{CR_c^3} \cdot \mathbf{xq}^* \leq 0 \tag{4}$$
$$\text{for } \ c = 1, ...N_c\}$$

The piecewise affine multivariable controller represented by (4) contains a proportional part $\mathcal{A}_c \cdot x^*$ with respect to the states, an output integral part $\mathcal{D}_c xq^*$ and a bias b_c. The presence of the integral term guarantees off-set free tracking of the output set point giving rise to a tracking parametric controller. Note that the control law is partitioned to a set of regions that are completely defined by a set of constant matrices and vectors $[x^* \ \ xq^*]^T \in \mathcal{CR}_c, \mathcal{CR}_c \equiv \{CR_c^1, cr_c^2, CR_c^3\}$.

Theorem 3.1. The control law defined by (4) is asymptotically stable, thus it guarantees no steady state offset from the target point in the presence of constant disturbances $\theta_t \in \Theta \subset \Re^w$, where $\Theta \equiv \theta^L \leq \theta_t \leq \theta^U$ on condition that (i) the dimension of the controls is larger or equal to the output dimension $q \geq m$ (ii) the open-loop transfer matrix defined from the equation: $H(z) = B_1(zI - A_1)^{-1}A_2 + B_2$ poses no zeros at the origin, (iii) the quadratic cost matrix Q_1 that penalizes the integral error is positive-definite (iv) the terminal cost and the time horizon length are appropriately tuned according to the criteria in Rawlings and Muske (1993) and Chmielewski and Manousiouthakis (1996) respectively and (v) the reference point of attraction is an interior point of the feasible region space defined as:

$$\hat{y} \in Y, Y = \{y \in \Re^m \cup [y_{t+k|t} = B_1 x_{t+k|t} + B_2 v_{t+k} \tag{5}$$
$$x_{t+k+1|t} = A_1 x_{t+k|t} + A_2 v_{t+k} + W\theta_t, g(x_{t+k|t}, v_{t+k}) \leq 0,$$
$$\psi^e(x_{t+N|t}) \leq 0, v_{t+k} \in V, \quad \theta_t \in \Theta, \quad k = 1, ...N-1]\}$$

If the target point does not belong to the feasible region $y_{ref} = 0 \notin Y$ then the equilibrium point $\hat{y} \neq 0$ in terms of the control driven outputs will lie on the boundaries of the feasible region. Then Theorem 3.1 still holds provided that for the evaluation of the integral states, the error of the outputs is shifted according to the modified equilibrium point, i.e. $xq_{t+k+1|t} = xq_{t+k|t} + \underbrace{(y_{t+k|t} - \hat{y})}_{error}$.

4. TRACKING PARAMETRIC CONTROLLER WITH DISTURBANCE ESTIMATOR

The design of an offset free parametric controller integrated with a disturbance estimator is based on the work of Muske and Rawlings (1993) that has been extended recently by the work of Muske and Badgwell (2002) where different types of integrators were added into the plant representations. The steps of our method are:

1. Generate an input or output disturbance model. For that purpose a distinction is made between two systems (i) one being the real process plant and (ii) the other comprising a model that represents an estimate of the process behavior, as shown in Table 1: where vector w is the actual disturbance vector that enters the system, whereas θ is the disturbance estimate. Vector θ is modelled as a step disturbance, thus being constant over

Table 1. Real vs. Prediction Model

Real System
$\hat{x}_{t+1} = \hat{A}_1\hat{x}_t + \hat{A}_2 v_t + \hat{W}_1 w_t$
$\hat{y}_t = \hat{B}_1\hat{x}_t + \hat{B}_2 v_t + \hat{W}_2 w_t$
Prediction model
$x_{t+1} = A_1 x_t + A_2 v_t + W_1\theta_t$
$y_t = B_1 x_t + B_2 v_t + W_2\theta_t$

the receding horizon, but admitting a different realization every time a piece of information about the process plant is available.

2. Estimate the disturbance. Two different estimation schemes are used here:

- *Least squares recursive estimator.*

$$\zeta_{t+1} = \zeta_t + (\hat{y}_t - y_t)$$
$$\theta_t = (B_1 \cdot W_1 + W_2)^P \cdot \zeta_t \qquad (6)$$

  where $(B_1 \cdot W_1 + W_2)^P$ is the pseudo-inverse of matrix $B_1 \cdot W_1 + W_2$. This algebraic manipulation performs disturbance estimation via the least squares method (Stengel, 1994).
- *Augmented Kalman Filter Estimator* This scheme provides an estimation for the current states and the disturbances if they are not measured directly. Consider the augmented with the disturbance state space system.

$$\underbrace{\begin{bmatrix} x_{t+1} \\ \theta_{t+1} \end{bmatrix}}_{xe_{t+1}} = \underbrace{\begin{bmatrix} A_1 & W_1 \\ 0 & I \end{bmatrix}}_{\bar{A}_1} \cdot \begin{bmatrix} x_t \\ \theta_t \end{bmatrix} + \underbrace{\begin{bmatrix} A_2 \\ 0 \end{bmatrix}}_{\bar{A}_2} \cdot v_t$$
$$\bar{B}_1 = [B_1 \;\; W_2] \qquad (7)$$

The estimator equations are:

$$xe_{t+1} = \bar{A}_1(I - \bar{M}\bar{B}_1)xe_t + (\bar{A}_2 - \bar{A}_1\bar{M}B_2)v_t + \bar{A}_1\bar{M}\hat{y}_t$$
$$\bar{y}_t = \bar{B}_1(I - \bar{M}\bar{B}_1)xe_t + \bar{B}_1\bar{M}\hat{y}_t \qquad (8)$$

The input to that filter are the measurements from the plant, the inputs to the plant and the previous state/ disturbance estimate: $[\hat{y}_t, v_t, xe_t]$. The output from the filter are the output filtered estimate, the state estimate and the disturbance estimate: $\bar{y} = [xe, ye]^T$. $\bar{M}$ is the filter gain that is a function of (i) Q^e, R^e cost matrices of the estimator, (ii) the structure of the input noise to the system.

3. Based on the disturbance value a new steady state point $[x_s \;\; v_s]$ is computed. If the dimension of the output variables y is equal to the dimension of input controls v and no input and state constraints are violated in the new steady state, then this is done as follows:

$$\begin{bmatrix} I - A_1 & -A_2 \\ B_1 & B_2 \end{bmatrix} \cdot \begin{bmatrix} x_s \\ v_s \end{bmatrix} = \begin{bmatrix} W_1 \cdot \theta_t \\ -W_2 \cdot \theta_t \end{bmatrix} \qquad (9)$$

Otherwise if $q = \dim v \geq m = \dim y$ the evaluation of x_s, v_s is done via (Muske and Rawlings, 1993):

$$\min_{x_s, v_s} (v_s - v^o)^T R_s (v_s - v^o)$$
$$s.t. \begin{bmatrix} I - A_1 & -A_2 \\ B_1 & B_2 \end{bmatrix} \cdot \begin{bmatrix} x_s \\ v_s \end{bmatrix} = \begin{bmatrix} W_1 \cdot \theta_t \\ -W_2 \cdot \theta_t \end{bmatrix}$$
$$0 \geq g(y^o, x_s, v_s), \;\; 0 \geq \psi^e(x_s, v_s) \qquad (10)$$

In the case where there are more measurements than control inputs the evaluation of x_s, v_s is performed via solving:

$$\min_{x_s, v_s} (y^o - B_1 x_s - B_2 v_s - W_2\theta_t)^T Q_s (y^o - B_1 x_s - B_2 v_s - W_2\theta_t)$$
$$s.t. \;\; \begin{bmatrix} I - A_1 & -A_2 \end{bmatrix} \cdot \begin{bmatrix} x_s \\ v_s \end{bmatrix} = \begin{bmatrix} W \cdot \theta_t \end{bmatrix}$$
$$0 \geq g(y^o, x_s, v_s), \;\; 0 \geq \psi^e(x_s, v_s) \qquad (11)$$

where v^o are the prior-to-disturbance control nominal values, usually taken as $v^o = 0$; y^o are the output nominal set-points, usually $y^o = 0$. Note that problems (9) - (11) are constrained quadratic problems that can be recast as multiparametric quadratic programs (mp-QPs) by treating θ_t as parameters. The analytical solution (Dua *et al.*, 2002) of these problems:

$$v_s = \alpha_v^c \theta_t + \beta_v^c, \;\; x_s = \alpha_x^c \theta_t + \beta_x^c$$
$$\text{if} \;\; \tilde{CR}_c(\theta_t) \leq 0, \;\; c = 1, \cdots \tilde{N}_c \qquad (12)$$

4. The state and input constraints are shifted according to the new target point resulting in the following open-loop optimal control formulation (Rawlings, 2000):

$$\phi(\bar{x}_{t|t}) = \min_{\bar{v}^N} \bar{x}_{t|N}^T P \bar{x}_{t|N} + \sum_{k=0}^{N-1} [y_{t+k|t}^T Q y_{t+k|t} + \bar{v}_{t+k}^T R \bar{v}_{t+k}]$$
$$s.t.$$
$$\bar{x}_{t+k+1|t} = A_1\bar{x}_{t+k|t} + A_2\bar{v}_{t+k}$$
$$y_{t+k|t} = B_1\bar{x}_{t+k|t} + B_2\bar{v}_{t+k}$$
$$0 \geq g[y_{t+k|t}, (\bar{x}_{t+k|t} + x_s), (\bar{v}_{t+k} + v_s)] = C_0 y_{t+k|t} + C_1 \cdot (\bar{x}_{t+k|t} + x_s) + C_2 \cdot (\bar{v}_{t+k} + v_s) + C_3$$
$$0 \geq \psi^e(\bar{x}_{t+N|t} + x_s) = D_1 \cdot (\bar{x}_{t+N|t} + x_s) + D_2$$
$$v_s = \alpha_v^c \theta_t + \beta_v^c$$
$$x_s = \alpha_x^c \theta_t + \beta_x^c$$
$$\text{if} \;\; \tilde{CR}_c(\theta_t) \leq 0, \;\; c = 1, \cdots \tilde{N}_c$$
$$\bar{x}_{t|t} = \bar{x}^*; \;\; k = 0, 1, 2, ..N \qquad (13)$$

Note that the dynamic system is shifted ($\bar{x} = x - x_s$, $\bar{v} = v - v_s$) to bring the system to the output target point, however, the constraints remain unaltered. As such, (13) can be viewed as a set of $\tilde{N}_c$ quadratic programs each one pertaining to every individual region of (12). Hence, once θ_t

and $\bar{x}^*$ are treated as a parameters, each one of the N_c programs (13) is recast as an mp-QP. The solution of these problems can be unified resulting the following control law:

$$\bar{v}_t(\bar{x}^*, \theta_t) = \{\{\mathcal{A}_{c,j}\bar{x}^* + \mathcal{B}_{c,j}\theta_t + b_{c,j} \\ \text{if } CR^1_{c,j}\bar{x}^* + CR^2\theta_t + cr^3_{c,j} \leq 0 \\ \text{for } c = 1, ..., N_c\} \text{ if; } \tilde{CR}(\theta_t) \leq 0, \;\; j = 1, \cdots \tilde{N}_j\} \tag{14}$$

5. REMARKS

- Note in controller (4) that the input and state constraints do not include any integral states. Hence, if the current states lie in a critical region where at least one of those constraints is active the corresponding control functions in (4) do not include any integral term. Thus, when constraint saturation occurs the integral action is switched off automatically by our controller. Hence, our compensator features explicit *anti-reset windup* properties as they are defined in the scheme of Kothare *et al.* (1994).
- In controller (14) note that both estimators incorporate an integrator: ζ in the least squares, θ in the Kalman filter estimator. The compensator can be viewed as the proportional part of the controller and the estimator as the integral part (Vogel and Downs, 2002).
- The main advantage of the tracking parametric controller (4) vs. (14) is that it does not necessitate the existence of a disturbance or uncertainty estimator.
- The tracking parametric controller (4) is easier to tune since its tunings can readily be obtained via a modified Ziegler Nichols or IMC approach.
- The tracking parametric controller with estimator (14) can provide improved performance and feasibility provided the estimator represents exactly the plant and the disturbance profile. In fact in the unlikely case where the model is perfect and the disturbance is estimated exactly, controller (14) ensures constraint satisfaction over the transient as well as asymptotic system behaviour.
- Controller (14) does not wind-up and exhibits less overshoots and aggressiveness than controller (4).

6. ILLUSTRATIVE EXAMPLE

A 2-state MIMO example is presented here. The problem is concerned with deriving the explicit tracking control law for an evaporator process studied in a sequence of works starting from (Newell and Lee, 1989). The constraints and the nominal values of the system outputs are: $C_2^n = 25\%$, $25\% \leq C_2 \leq 30\%$, $P_2^n = 50.57KPa$, $40KPa \leq P_2 \leq 80KPa$. Similarly for the control inputs: $P_{100}^n = 193.37KPa$, $0KPa \leq P_{100} \leq 400KPa$, $F_{200}^n = 207.52kg/min$, $0 \leq F_{200} \leq 400kg/min$. Two parametric tracking controllers are designed (Δt =1 min, N = 3) following the procedure described in sections 3 and 4. The controller with integral penalty is partitioned to 97 critical regions for a wide range of variations of the pure and integral states. For example in the region defined by the following set of inequalities:

$$4.5 \leq 0.2C_2 \leq 6$$

$$\vdots$$

$$1.210 \cdot 10^5 \leq 4.887 \cdot 10^3 + 0.3296P_2 + 4.8868$$

The control functions are the following:

$$P_{100} = -16.575C_2 + 3.8956P_2 + 414.18;$$

$$F_{200} = -97.842C_2 + 675.86P_2 - 0.63C_{2q} + 45.63P_{2q} \\ -3.149 \cdot 10^4$$

The control law consists of the assembly of these control functions. Note that the prescribed function of the first control variable P_{100} is not affected at all by the integral states. The reason is that when the states lie on that region the system operates in the neighborhood or on the boundary of the constraint $C_2 \geq 25\%$. Thus, the control variable P_{100} that largely affects C_2 is readjusted to ensure constraint satisfaction and does not feature target tracking capabilities. Whereas, when the states enter the region where none of the constraints is active the control activity features integral action and the corresponding expressions are:

$$P_{100} = -3.11 \cdot 10^1 C_2 + 3.89P_2 - 1.45 \cdot 10^1 C_{2q} + 7.75 \cdot 10^2$$

$$F_{200} = -1.72 \cdot 10^2 C_2 + 6.75 \cdot 10^2 P_2 - 7.56 \cdot 10^1 C_{2q} \\ +1.45 \cdot 10^2 P_{2q} - 2.96 \cdot 10^4$$

Thus, the values of the integral coefficients in the control functions are alternating according to how close the constraints are to saturation. This characteristic clearly manifests, therein, the *anti-windup* properties of our proposed tracking controller. The tracking parametric controller with the disturbance estimator is derived in a similar fashion.

The execution of the tracking controllers is compared with the nominal controller. The system is initially perturbed to $C_{2,t=0} = 26\%$ and $P_{2,t=0} = 51.57KPa$ and as it is driven back to the origin a sequence of non-vanishing persistent step disturbances in C_1, F_1 occur. The disturbances have overall a magnitude of $\Delta F_1 \simeq \pm 0.16kg/min$ and

$\Delta C_1 \simeq \pm 0.4\%$. The profiles of disturbance F_1 and of output C_2 corresponding to the action of the nominal parametric controller (nominal parco), the tracking parametric controller with integral penalty and with estimator (tracking parco / integral, tracking parco / estimator) are displayed in Figure 1. A Kalman Filter estimator is used for the simulation of the corresponding tracking controller and its estimation of disturbance F_1 is also shown in Figure 1. The nominal controller exhibits severe constraint violations since $C_2(t) < 25\%$. The tracking controllers however, respect the constraints over the complete envelope of operation because they bring the system into the interior of the feasible region after every disturbance step.

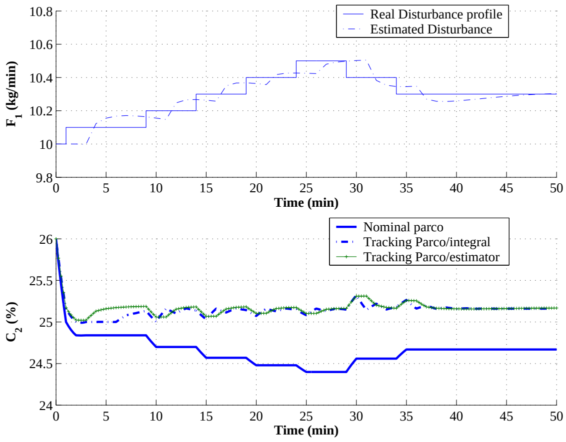

Fig. 1. Output and control profiles for nominal and tracking parametric controller

7. CONCLUSIONS

In this paper a novel framework is presented for designing model-based tracking parametric controllers for linear dynamic systems that are subject to input disturbances and uncertainties. Two control schemes are developed that consist of piecewise affine expressions for the control variables in terms of the states. The implementation of the control action is achieved by simple linear function evaluations, thus avoiding any expensive on-line computations. The controller guarantees effectively disturbance attenuation and offset elimination.

8. REFERENCES

Bemporad, A., A.R. Teel and L. Zaccarian (2002*a*). l_2 anti-windup via receding horizon optimal control. In: *American Control Conference*. Anchorage, Alaska.

Bemporad, A., M. Morari, V. Dua and E.N. Pistikopoulos (2002*b*). The explicit linear quadratic regulator for constrained systems. *Automatica* **38**(1), 3–20.

Chmielewski, D. and V. Manousiouthakis (1996). On constrained infinite-time linear quadratic optimal control. *Systems & Control Letters* **29**, 121–129.

Dua, V., N.A. Bozinis and E.N. Pistikopoulos (2002). A multiparametric programming approach for mixed integer and quadratic engineering problems. *Comp. Chem. Eng.* **26**(4-5), 715–733.

Kothare, M.V., P.J. Campo, M. Morari and C.N. Nett (1994). A unified framework for the study of anti-windup designs. *Automatica* **30**(12), 1869–1883.

Lee, J.H. and B. Cooley (1997). Recent advances in model predictive control and other related areas. In: *Proc. of Chemical Process Control - V: assessment and new directions for research* (J.C. Kantor, C.E. Garcia and B. Carnahan, Eds.). AIChE. pp. 201–216.

Muske, K.R. and J.B. Rawlings (1993). Model predictive control with linear models. *AIChE J.* **39**(2), 262–287.

Muske, K.R. and T.A. Badgwell (2002). Disturbance modeling for offset-free linear model predictive control. *J. of Proc. Control* **12**, 617–632.

Newell, R.B. and P.L. Lee (1989). *Applied Process Control - A Case Study*. Prentice-Hall. Sydney.

Pistikopoulos, E.N., V. Dua, N.A. Bozinis, A. Bemporad and M. Morari (2002). On-line optimization via off-line parametric optimization tools. *Comput. Chem. Eng.* **26**(2), 175–185.

Rawlings, J.B. (2000). Tutorial overview of model predictive control. *IEEE Control SYstems Magazine* **20**(3), 38–52.

Rawlings, J.B. and K.R. Muske (1993). The stability of constrained receding horizon control. *IEEE Trans. Automatic Contr.* **38**(10), 1512–1516.

Sakizlis, V., J.D. Perkins and E.N. Pistikopoulos (2002). *Recent Developments in Optimization and Control in Chem. Eng.*. Chap. Multiparametric Dynamic Optimization of Linear Quadratic Optimal Control Problems: Theory and Applications, pp. 195–216. Editor: R. Luus, Research Signpost. Kerala, India.

Seborg, D.E., T. F. Edgar and D.A. Mellichamp (1989). *Process Dynamics and Control*. Wiley and Sons.

Stengel, R.F. (1994). *Optimal Control and Estimation*. Dover Publications. Mineola, New York.

Vogel, E.F. and J.J. Downs (2002). Industrial experience with state-space model predictive control. In: *CPC-VI Proceedings* (J. Rawlings, T. Ogunnaike and J. Eaton, Eds.). Vol. 98 of *AIChE Symposium Series*. AIChE. pp. 438–442.