# ADAPTIVE EXTREMUM SEEKING CONTROL OF CONTINUOUS STIRRED TANK BIOREACTORS [1]

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Abstract: In this paper, we present an adaptive extremum seeking control scheme for continuous stirred tank bioreactors. We assume limited knowledge of the growth kinetics. An adaptive learning technique is introduced to construct a seeking algorithm that drives the system states to the desired set-points that maximizes the value of an objective function. Lyapunov's stability theorem is used in the design of the extremum seeking controller structure and the development of the parameter learning laws. A simulation experiment is given to show the effectiveness of the proposed approach.

Keywords: Extremum seeking, Lyapunov function, adaptive learning, persistence of excitation

## 1. INTRODUCTION

The goal of extremum seeking is to find the operating set-points that maximize or minimize an objective function. Since the early research work on extremum control in the 1920's (Leblanc 1922), many successful applications of extremum control approaches have been reported (e.g., (Vasu 1957), (Astrom and Wittenmark 1995), (Sternby 1980) and (Drkunov *et al.* 1995)). Recently, Krstic et. al ((Krstic 2000), (Krstic and Deng 1998)) presented several extremum control schemes and stability analysis for extremum-seeking of linear unknown systems and a class of general nonlinear systems ((Krstic 2000) and (Krstic and Deng 1998)).

In this study, we investigate an alternative extremum seeking scheme for continuous stirred tank bioreactors. The proposed scheme utilizes an explicit structure information of the objective function that depends on system states and unknown plant parameters. However, it is assumed that the objective function is not available for measurement. Furthermore, no explicit knowledge of the microbial growth kinetics are assumed. A Lyapunov-based adaptive learning control technique is used to approximate the unknown kinetics and to steer the system to its unknown extremum. The technique ensures convergence of the system to an adjustable neighbourhood of its unknown optimum that depends on the approximation error. We also show that a certain level of persistence of excitation (PE) condition is necessary to guarantee the convergence of the extremum-seeking mechanism. The paper is organized as follows. Section 2 presents some notations and the problem formulation. In Section 3, an parameter estimation algorithm is developed. Sec-

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tion 4 presents the adaptive extremum seeking controller and the stability and convergence of the closed-loop extremum seeking system. Numerical simulation is shown in Section 5 followed by brief conclusions in Section 6.

## 2. PROBLEM

Consider the following microbial growth models

$$\dot{x} = \mu(s)x - ux \tag{1}$$

$$\dot{s} = -k_1\mu(s)x + u(s_0 - s) \tag{2}$$

$$y = k_2\mu(x, s)x \tag{3}$$

where states $x \in [0, +\infty)$ and $s \in [0, +\infty)$ denote biomass and substrate concentrations, respectively, $u \geq 0$ is the dilution rate, $y$ is the production rate of the reaction product, $s_0$ denotes the concentration of the substrate in the feed, and $k_1, k_2 > 0$ are yield coefficients. We consider the case where only $s$ and $y$ are measurable, the biomass concentration $x$ is not available for feedback control.

In this work, we consider the extremum seeking problem for plant (1)-(2) with an unknown substrate-dependent growth rate expression $\mu(s)$. One of the most common growth rate model is Monod's model given by

$$\mu(x, s) = \mu(s) = \frac{\mu_m s}{K_s + s} \quad \text{(Monod)} \tag{4}$$

where $\mu_m > 0$ is the maximum value of the specific growth rate, and positive constant $K_s$, $K_c$ and $K_0$ to $K_2$ denote the coefficients for different growth rate models. While this simple model form is very useful in practice, a wide variety of growth patters and characteristics exist where the Monod expression is not applicable.

The extremum-seeking control of plants described by the Monod model have been investigated in (Zhang *et al.* 2002). In this study, we extend the result to a broad class of uncertain plants with unknown growth rate representations. As in (Zhang *et al.* 2002), the control objective is to design a controller, $u$, such that the production rate $y$ achieves its maximum.

The strategy developed in this paper consists in approximating the growth rate expression using a neural network approximation technique. In this paper, radial basis function (RBF) neural networks presented in (Sanner and Slotine 1992) shall be used to approximate a continuous function $\phi(z) : R^p \to R$

$$\phi(z) = W^{*T}S(z) + \mu_l(t) \tag{5}$$

with NN approximation error $\mu_l(t)$, and basis function vector

$$S(z) = [s_1(z), s_2(z), \cdots, s_l(z)]^T$$
$$s_i(z) = \exp\left[\frac{-(z-\varphi_i)^T(z-\varphi_i)}{\sigma_i^2}\right], \qquad i = 1, 2, .. l \tag{6}$$

where $\varphi_i$ is the center of the receptive field, and $\sigma_i$ is the width of the Gaussian function. The ideal weight $W^*$ in (5) is defined as

$$W^* := arg \min_{W \in \Omega_w} \left\{ \sup \left| W^T S(z) - \phi(z) \right| \right\} \tag{7}$$

where $\Omega_w = \left\{ W \;\middle|\; \|W\| \leq w_m \right\}$ with positive constant $w_m$ to be chosen at the design stage. Universal approximation results stated in (Funahashi 1989) (Sanner and Slotine 1992) indicate that, if $l$ is chosen sufficiently large, then $W^T S(z)$ can approximate any continuous function to any desired accuracy on a compact set.

We apply eq.(5) to develop an approximation of the growth rate expression given by

$$\mu(s(t)) = W^{*T}S(s(t)) + \mu_l(t) \tag{8}$$

where $W^*$ and $S$ are as defined in eqs.(6)-(7). Additionally, we make the following assumption about the approximation error $\mu_l(t)$.

**Assumption 1**: the NN approximation error satisfies $|\mu_l(t)| \leq \bar{\mu}_l$ with constant $\bar{\mu}_l > 0$ over a compact set in the state space.

We first calculate the system's equilibria corresponding to a constant dilution rate $u_e$. By setting the right-hand side of (1)-(2) to zero, we obtain two equilibria. The first is $x_e = 0$ and $s_e = s_0$ which is called the wash-out equilibrium. The second is

$$x_e = \frac{s_0 - s_e}{k_1}$$

where $s_e$ is a positive solution of the equation

$$u_e = \mu(s_e).$$

At the steady-state, the production rate can be expressed by

$$y_e = \frac{k_2}{k_1}\mu(s_e)(s_0 - s_e) \tag{9}$$

Following eq.(8), the steady-state production rate is approximated by

$$y_e = \frac{k_2}{k_1}W^{*T}S(s_e)(s_0 - s_e) \tag{10}$$

From (2) and (4), we have

$$\frac{\partial y_e}{\partial s_e} = \frac{k_2}{k_1} W^{*T} \Big( dS(s_e)(s_0 - s_e) - S(s_e) \Big) \quad (11)$$

and

$$\frac{\partial^2 y_e}{\partial s_e^2} = \frac{k_2}{k_1} W^{*T} \Big( d^2S(s_e)(s_0 - s_e) - 2dS(s_e) \Big) (12)$$

where $dS = \frac{\partial S}{\partial s}$ and $d^2S = \frac{\partial^2 S}{\partial s^2}$. Assuming that the parameter vector $W^*$ is such that $\frac{\partial^2 y_e}{\partial s_e^2} > 0, \forall s_e \geq 0$ then $y_e(s)$ has a maximum

$$y^* = y_e(s^*) = \frac{k_2}{k_1} W^{*T} S(s^*) x^* \quad (13)$$

with $x^* = \frac{s_0 - s^*}{k_1}$ at the system equilibrium.

The objective of this study is to develop a controller that maximizes the steady-state value of the production rate, $y^*$. However, since the exact values of the ideal weights, $W^*$, are not known *a priori* , they must be estimated. In the next section, we propose an adaptive extremum seeking algorithm is developed to search the unknown process set-point where the production rate, y, is optimized. The strategy attempts to estimate the gradient of the production rate with respect to the substrate concentration, $s$. A controller is then designed to bring the process to points where the gradient vanishes and where the second order derivatives of the production rate with respect to the substrate is negative. The resulting technique provides a real-time optimization techniques that can be used to a large class of bioreactors and chemical reactors.

## 3. CONTROLLER DESIGN

In this section, we design a control strategy that tracts the unknown optimum production rate. We first develop the parameter estimation algorithm for the unknown parameter vector $W^*$. Equations (1)-(2) can be re-expressed as

$$\dot{x} = (W^{*T} S(s) + \mu_l(t))x - ux \quad (14)$$

$$\dot{s} = -k_1(W^{*T} S(s) + \mu_l(t))x + u(s_0 - s) \quad (15)$$

We assume that the biomass and the substrate concentration are available for measurement.

Let $\hat{W}$ denote the estimate of the true parameter $W^*$ and let $\hat{s}$ and $\hat{x}$ be the predictions of $s$ and $y$. The predicted states $\hat{s}$ and $\hat{x}$ are generated by

$$\dot{\hat{x}} = \hat{W}^T Sx - ux + k_x e_x + c_1(t)^T \dot{\hat{W}} \quad (16)$$

$$\dot{\hat{s}} = -k_1 \hat{W}^T Sx + u(s_0 - s) + k_s e_s + c_2(t)^T \dot{\hat{W}} (17)$$

with gain functions $k_s, k_y > 0$, prediction errors $e_s = s - \hat{s}$ and $e_x = x - \hat{x}$ and $c_1(t)$, $c_2(t)$ time-varying functions to be assigned later. It follows from (14)-(17) that

$$\dot{e}_x = \tilde{W}^T Sx + \mu_l(t)x - k_x e_x - c_1(t)^T \dot{\hat{W}} \quad (18)$$

$$\dot{e}_s = -k_1 \tilde{W}^T Sx - k_1 \mu_l(t) - k_s e_s - c_2(t)^T \dot{\hat{W}} \ (19)$$

where $\tilde{W} = W^* - \hat{W}$.

The objective of the extremum-seeking control is stabilize the closed-loop system around a point where the gradient of the production y with respect to s given in eq.(11) vanishes while attenuating the effect of the modelling uncertainty $\mu_l(t)$. Since the parameter vector $W^*$ is unknown, we first design a controller to make the system states track points where the estimated gradient

$$z = \frac{k_2}{k_1} \hat{W}^T \Big( dS(s)(s_0 - s) - S(s) \Big) \quad (20)$$

vanishes. In order to ensure that the estimated gradient approaches the true gradient asymptotically, we have to ensure that the parameter estimates approach the optimal weight vector $W^*$. To achieve this objective, an excitation signal is designed and injected into the adaptive system to ensure convergence of the estimated parameters to their true value. The extremum seeking control objective is achieved when the system systems are stabilized at the optimal operating point $x^*,\ s^*$.

Define

$$z_s = \hat{W}^T \Big( dS(s)(s_0 - s) - S(s) \Big) - d(t) \quad (21)$$

where $\frac{k_2}{k_1} > 0$ has been removed for simplicity and $d(t) \in C^1$ is an excitation signal that will be assigned later. In the remainder, the dependence of the radial basis functions $S$ on the substrate concentration $s$ is implied and we write $S$, $dS$ and $d^2S$.

Next we define the variables,

$$\begin{aligned} \eta_1 &= e_x - c_1(t)^T \tilde{W} \\ \eta_2 &= e_s - c_2(t)^T \tilde{W} \\ \eta_3 &= z_s - c_3(t)^T \tilde{W} \end{aligned} \quad (22)$$

where $c_3(t)$ is a vector of time-varying functions to be defined in the design procedure. We propose the Lyapunov function candidate

$$V = \frac{\eta_1^2}{2} + \frac{\eta_2^2}{2} + \frac{\eta_3^2}{2}. \quad (23)$$

We pose the following equation for the dither signal, $d(t)$,

$$\begin{aligned} \dot{d}(t) = c_3(t)^T \dot{\hat{W}} + \Gamma_1^T \dot{\hat{W}} - (\hat{W}^T \Gamma_2)^2 d(t) \\ + \hat{W}^T \Gamma_2 a(t) + k_z z_s \end{aligned} \quad (24)$$

where $a(t)$ is an external signal providing excitation to the process and $k_z > 0$ is a positive gain function to be assigned. We then assign $\dot{c}_1$, $\dot{c}_2$ and $\dot{c}_3$ as

$$\begin{aligned}\dot{c}_1^T &= -k_x c_1^T + xS^T \\ \dot{c}_1^T &= -k_s c_2^T - k_1 xS^T \\ \dot{c}_3^T &= -k_z c_3^T - k_1 x\hat{W}^T\Gamma_2 S^T\end{aligned} \tag{25}$$

and we let the control be given by

$$u = \frac{1}{(s_0 - s)}\Big(k_1\hat{W}^T Sx + a(t) - \hat{W}^T\Gamma_2\big). \tag{26}$$

Taking the time derivative of $V$, we substitute eqs.(24)-(26) and we substitute $e_x$, $e_s$ and $z_s$ using eq.(22) to obtain

$$\begin{aligned}\dot{V} = \mu_l(t)x\eta_1 - k_x\eta_1^2 - k_1 x\mu_l(t)\eta_2 - k_s\eta_2^2 \\ -k_1 x\mu_l(t)\hat{W}^T\Gamma_2\eta_3 - k_z\eta_3^2.\end{aligned} \tag{27}$$

where $\Gamma_1 = dS(s_0 - s) - S$ and $\Gamma_2 = d^2S(s_0 - s) - 2dS$. Next, we complete the squares and assign the gain functions

$$\begin{aligned}k_x &= k_{x0} + \frac{k_4}{2}x^2 \\ k_s &= k_{s0} + \frac{k_3 k_5}{2}x^2 \\ k_z &= k_{z0} + \frac{k_3 k_6}{2}x^2(\hat{W}^T\Gamma_2)^2\end{aligned} \tag{28}$$

where $k_4 > 0$, $k_5 > 0$, $k_6 > 0$, $k_{x0} > 0$, $k_{s0} > 0$ and $k_{z0} > 0$ are positive constants. We finally obtain the inequality

$$\begin{aligned}\dot{V} \leq -k_{x0}\eta_1^2 - k_{s0}\eta_2^2 - k_{z0}\eta_3^2 \\ + \left(\frac{1}{2k_4} + \frac{1}{2k_5} + \frac{1}{2k_6}\right)\mu_l(t)^2\end{aligned} \tag{29}$$

Eq.29) establishes that the state, $\eta$, converges to a small neighborhood of the origin. It remains to show that the original state variables, $e_x$, $e_s$ and $z_s$ and the parameter estimation errors $\tilde{W}$ converge to a small neighborhood of the origin. Note that it is not sufficient to check that $e_x$, $e_s$ and $z_s$ can be made small since the value of $z_s$ depends on the parameter estimates, $\hat{W}$. To this end, we derive a persistency of excitation condition that guarantees the convergence of the parameter estimates to the ideal weights, $W^*$.

Consider the following matrix,

$$\Upsilon(t) = \begin{bmatrix} c_1(t)^T \\ c_2(t)^T \\ c_3(t)^T \end{bmatrix}$$

By construction, this matrix solves the matrix differential equation

$$\dot{\Upsilon}(t) = -K(t)\Upsilon(t) + B(t) \tag{30}$$

where

$$K(t) = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_z \end{bmatrix}$$

and

$$B(t) = \begin{bmatrix} xS^T \\ -k_1 xS^T \\ -k_1 x\hat{W}^T\Gamma_2 S^T \end{bmatrix}.$$

A bound on the parameter estimates $\hat{W}$ can be ensured by choosing the following parameter update law.

$$\dot{\hat{W}} = \begin{cases} \gamma_w\Gamma & \text{if } \|\hat{W}\| \leq w_m \text{ or} \\ & \text{if} \|\hat{W}\| = w_m \text{ and } \hat{W}^T\Gamma \leq 0 \\ \gamma_w\left(I - \dfrac{\hat{W}\hat{W}^T}{\hat{W}^T\hat{W}}\right)\Gamma & \text{otherwise}\end{cases} \tag{31}$$

where $\Gamma = \Upsilon(t)^T e$. Eq.(31) is a projection algorithm which ensures that $\|\hat{W}\| \leq w_m$. The convergence of the parameter estimation scheme is considered in the sequel.

By the property of the projection algorithm and for the specific choice of basis function it is possible to show that the norm of $B(t)$ is bounded. Using the exponential stability of system eq.(30)and the bound on $B(t)$, an explicit bound for the solution of eq.(30) can be obtained as follows,

$$\|\Upsilon(t)\| \leq C_2 e^{-\lambda_2(t-t_0)} + C_2\frac{B_M}{\lambda_2}. \tag{32}$$

where $C_2 = \|\Upsilon(t_0)\| > 0$ and $\lambda_2 > 0$ is a positive constant. Next, we want to show that the parameter estimation error $\tilde{W}$ converges to a neighborhood of the origin.

Substituting for $e = \eta + \Upsilon(t)\tilde{W}$ we obtain the perturbed dynamics

$$\begin{aligned}\dot{\tilde{W}} &= -\gamma_w\Upsilon(t)^T\Upsilon(t)\tilde{W} - \gamma_w\Upsilon(t)^T\eta \\ &+ \begin{cases} 0 \quad \text{if } \|\hat{W}\| \leq w_m \text{ or} \\ \quad \text{if} \|\hat{W}\| = w_m \text{ and } \hat{W}^T\Upsilon(t)^T e \leq 0 \\ \gamma_w\dfrac{\hat{W}\hat{W}^T}{\hat{W}^T\hat{W}}\left(\Upsilon(t)^T\Upsilon(t)\tilde{W} + \Upsilon(t)^T\eta\right) \text{ otherwise}\end{cases}\end{aligned} \tag{33}$$

To establish the convergence of the parameter estimation, we make the following persistency of excitation assumption.

*Assumption 3.1.* The solution of eq.(30) is such that there exists positive constants $T > 0$ and $k_N > 0$ such that

$$\int_{t}^{t+T} \Upsilon(\tau)^T \Upsilon(\tau) d\tau \geq k_N I_N \qquad (34)$$

where $I_N$ is the N-dimensional identity matrix.

By a standard adaptive control argument, the persistency of excitation condition guarantees that the origin of the differential equation

$$\dot{\tilde{W}} = -\gamma_w \Upsilon(t)^T \Upsilon(t) \tilde{W} \qquad (35)$$

is an exponentially stable equilibrium. Since $B(t)$ is a bounded function, it is shown that the parameter estimation error is guaranteed to decay exponentially as

$$\|\tilde{W}\| \leq \alpha_4 e^{-\lambda_4 (t-t_0)} + \frac{|\bar{\mu}_l|}{\sqrt{2kmc_3}} \qquad (36)$$

Hence the parameter estimation error and the redefined state variables, $\eta$, converge exponentially fast to an adjustable neighbourhood of the origin. By definition, convergence of $\eta$ and $\tilde{W}$ to a neighbourhood of the origin implies that $\|e\| \leq \|\eta\| + \|\Upsilon(t)\| \|\tilde{W}\|$. Substituting for $\|\eta\|$, $\|\Upsilon(t)\|$ and $\tilde{W}$, we obtain

$$\|e\| \leq \alpha_5 e^{-\lambda_5 (t-t_0)} + \beta_5 \qquad (37)$$

where $\alpha_5 > 0$ and $\beta)5 > 0$ are computable positive constants.

The convergence of the error vector, $e$, implies that the convergence of the prediction errors, $e_x$ and $e_s$ and the exponential convergence of the closed-loop system to an adjustable neighbourhood of the unknown steady-state optimum. We summarize the result of the above analysis as follows.

*Theorem 3.1.* Consider the two-state bioreactor model eqs.(1)-(2) with production rate, eq.(3) in closed-loop with the state-observer eqs.(16)-(17), the controller eq.(26), the dither signal eq.(24) and the adaptive learning law eq.(31). Assume that the signal $a(t)$ is such that

$$\int_{t}^{t+T} \Upsilon(\tau)^T \Upsilon(\tau) d\tau \geq k_N I_N \qquad (38)$$

for positive constants $T > 0$ and $k_N > 0$ where $\Upsilon(t)$ is the solution of eq.(30). Then

- the error dynamics eqs.(18)-(19) converge exponentially to a small neighbourhood of the origin
- the parameter estimation errors $\tilde{W}$ converge exponentially to a small neighbourhood of the origin
- the tracking error from the unknown steady-state, $z_s$, converges exponentially to a small neighbourhood of the origin.

## 4. SIMULATION RESULTS

To show the effectiveness of the proposed design, a simulation study is performed on three models.

In the first example, we consider a bioreactor with Haldane kinetics,

$$\mu(s) = \left( \frac{\mu_m s}{K_s + s + K_I s^2} \right)^{1.5}$$

The following parameters and initial states are used in the simulation experiment.

$$K_s = 0.2, \ \ \mu_m = 1.0, \ \ Y = 0.5, \ \ k_1 = 2.0,$$
$$k_2 = 1.0, \ \ K_I = 0.1, \ \ s_0 = 10.0, \ \ x(0) = 1.0$$
$$s(0) = 0.1, \ \ \hat{x}(0) = 0.5, \ \ \hat{s}(0) = 0.5$$

The design parameters in the adaptive controller (26) and the adaptive law eq.(31) are

$$\gamma_w = 100.0, \quad k_z 0 = k_{x0} = k_{s0} = k_4 = k_5 = k_6 = 2.0$$

The NN radial basis function approximation is of dimension 6 with parameters $\varphi_i = i$ and $\sigma_i = 1$ for $1 \leq i \leq 6$. The initial conditions for the adaptive learning weights are

$$\hat{W}_i(0) = 0, 1 \leq i \leq 6$$

The dither signal was set to

$$a(t) = exp(-0.1t) \sum_{i=1}^{6} (sin((0.5i)t) + cos((0.5i)t))$$

We let $d(0) = 0$ and $\Upsilon(0) = 0$.

Simulation results are shown in Figures 1-3. Figure 1 shows the value of the production rate $y$ and its estimated value. The closed-loop system converges quickly to a small neighbourhood of the origin. Moreover, the estimated production rate is shown to converge to the a small neighbourhood of the true production rate. In this case, the true optimum, 3.036, was recovered by the adaptive learning scheme. The required control action of the extremum-seeking control is shown in Figure 2. The biomass concentration and the substrate concentration are shown in Figure 3.

## 5. CONCLUSION

We have solved a class of extremum seeking control problems for continuous stirred tank bioreactors represented by an unknown growth kinetic model. An adaptive learning technique is used to derive an extremum seeking controller that drives the production rate to an adjustable neighbourhood of the unknown optimal production. It has been shown that when the external dither signal is designed such that the persistent excitation condition is satisfied, the proposed adaptive extremum seeking controller guarantees the exponential convergence of the production rate of the bioreactor to an adjustable neighborhood of its maximum.

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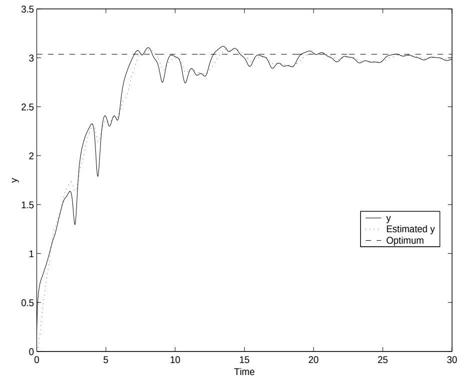


Fig. 1. Production rate $y$ ("—") and its maximum $y^*$ ("- -")

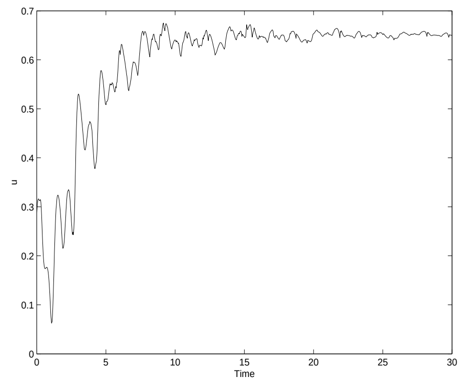


Fig. 2. Extremum-seeking dilution rate $u$

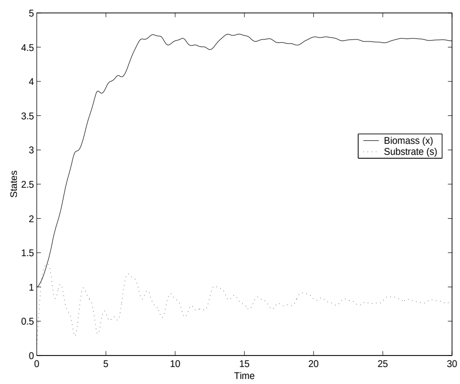


Fig. 3. Biomass $x$ ("—") and substrate $s$ ("- -") concentration