AN LMI-BASED CONSTRAINED MPC SCHEME WITH TIME-VARYING TERMINAL COST

B. Pluymers, L. Roobrouck, J. Buijs
J.A.K. Suykens, B. De Moor

Katholieke Universiteit Leuven
Department of Electrical Engineering, ESAT-SCD-SISTA
Kasteelpark Arenberg 10, B-3001 Heverlee (Leuven), Belgium
E-Mail : {bert.pluymers, luc.roobrouck, jeroen.buijs, johan.suykens, bart.demoor}@esat.kuleuven.ac.be

Abstract: Modelbased Predictive Control (MPC) is a control technique that is widely used in chemical process industry. In the past decade, stability of MPC has been an intensive research area, resulting in the general acceptance of a theoretical MPC stability framework introducing a terminal cost and terminal constraint to the classic MPC formulation. Although guaranteeing stability, issues regarding optimality and feasibility remain. In this paper, an LMI-based constrained MPC scheme for linear systems is introduced which guarantees stability by use of a time-varying terminal cost and terminal constraint. The online calculation of the terminal cost results in improved performance and feasibility compared to MPC schemes with fixed terminal cost. Finally, the technique is illustrated on a copolymerization reactor.

Keywords: modelbased predictive control, linear matrix inequalities, feasibility, stability, optimality, constraints, LQR

1. INTRODUCTION

In the past decade, wide consensus has been reached with respect to achieving stability of MPC controllers (Mayne et al. (2000)). The common denominator of stabilizing MPC schemes has been recognized to be the addition of a terminal cost and terminal constraint to the classic MPC formulation. It has also been recognized, however, that the choice of these two additions influences feasibility and local optimality of the resulting controller.

In this paper a new MPC scheme with time-varying terminal cost and constraint will be introduced which results in a controller that is locally optimal while conserving feasibility over a wide range of states. The approach taken here is to use a special case of the LMI formulations introduced in Kothare et al. (1996) and later used in Lee et al. (1998) to ensure stability for linear, time-varying systems. It is then shown that these LMI's can be merged with the classic MPC optimization problem, which results in an LMI-based MPC scheme that calculates in each time step both the optimal inputs and an optimal, stabilizing terminal cost and constraint.

This paper is organized as follows. In a first part a general introduction to linear MPC will be given, after which, more specifically, the established stability theory of MPC will be discussed. The third part explains how a stabilizing terminal cost and terminal constraint can be calculated using LMI's and how these stabilizing ingredients influence optimality. In a fifth part, the MPC scheme with time-varying terminal cost will be introduced, which is then illustrated on a copolymerization reactor in the final section.

2. MODELBASED PREDICTIVE CONTROL

Modelbased Predictive Control (MPC) is a control scheme that calculates in each time step an optimal future input sequence $u_{k,i}$, $i = 0 \dots P-1$ by solving an optimization problem. In this paper only linear MPC (i.e. using linear models) will be discussed, in which case the problem reduces to a Quadratic Program (QP) :

$$\min_{x,u} \; J_k = \min_{x,u} \left(\sum_{i=0}^{M-1} u_{k,i}^{\mathrm{T}} R u_{k,i} + \right.$$
$$\left. \sum_{i=1}^{P} (x_{k,i} - x_{k,i}^{\star})^{\mathrm{T}} Q (x_{k,i} - x_{k,i}^{\star}) \right) \quad \text{(1a)}$$

subject to

$$x_{k,i+1} = A x_{k,i} \; + \; B u_{k,i}, \quad i = 0, \dots P-1. \quad \text{(1b)}$$

$x_k \equiv x_{k,0}$ denotes the states measured at time k. $x_{k,i}, i > 0$ denote the states at time $k+i$ as predicted at time k, $x_{k,i}^{\star}, i \geq 0$ denote the future states at time $k+i$ as desired at time k and $u_{k,i}, i \geq 0$ denotes the input sequence as calculated at time k. Q and R are positive definite weighting matrices indicating the relative importance of the states and inputs in the control problem. M and P denote respectively the number of future inputs and states that are calculated in each time step and are called the *control horizon* and the *input horizon*. M and P are generally given equal values, which is what will be assumed in this paper. Equality constraints (1b) represent the system behavior, with given system matrices A and B.

From the calculated input sequence only the first input $u_k \equiv u_{k,0}$ is applied to the system, after which the calculation restarts in the next time step with a new measured state $x_{k+1,0}$.

Typically additional linear inequality constraints are incorporated in the QP, representing either physical system limitations, safety margins or economical constraints. Depending on the variables involved these are called *input constraints* or *state constraints* and will be denoted by $u_k \in \mathbb{U}$ and $x_k \in \mathbb{X}$ respectively.

3. STABILITY THEORY

Although MPC calculates in each time step an *optimal* input sequence, there is no guarantee that the controller is stable. In each time step only the first input of the calculated sequence is applied to the system and there is no guarantee that this sequence of first inputs is optimal in any way. The fundamental cause of this problem is the fact that the horizon P is finite, and the behavior of the system for $i \geq P$ is not taken into account in the optimization problem. Several MPC variants were developed to address this issue (Mayne et al. (2000); Rawlings and Muske (1993); Lee et al. (1998)). Most of these methods fit into the following stability framework.

Assuming $x_{k,i}^{\star} \equiv 0$ (or a previous transformation accomplishing this), a stabilizing MPC controller can be obtained by modifying the weight matrix of the terminal state $x_{k,P}$ into Q_P (called the *terminal cost*), the total control cost of a locally stabilizing feedback controller $\kappa(x) = Kx$ for $i \geq P$, and adding a terminal state constraint $x_{k,P} \in \mathbb{X}_P$ (also called *terminal constraint*) corresponding to the region in which the terminal cost is valid. More formally, asymptotic stability is achieved when following conditions are satisfied :

a. $\mathbb{X}_P \subset \mathbb{X}$ (2a)

b. $\kappa_N(x) \in \mathbb{U}, \forall x \in \mathbb{X}_P$ (2b)

c. $f(x, \kappa(x)) \in \mathbb{X}_P, \forall x \in \mathbb{X}_P$ (2c)

d. $F(x) - F(f(x, \kappa(x))) - l(x, \kappa(x)) \geq 0, \quad \forall x \in \mathbb{X}_P$ (2d)

given there exists a feasible solution to (1) supplemented with the terminal constraint and terminal cost. $f(x,u) \equiv Ax + Bu$ denotes the system state transition function, $l(x,u)$ denotes the cost function $x^{\mathrm{T}} Q x + u^{\mathrm{T}} R u$ for all $i = 1, 2, \dots, P-1$ and $F(x)$ the cost term $x^{\mathrm{T}} Q_P x$ for the terminal state.

Note that these conditions are sufficient but not necessary. Most stabilizing MPC schemes make different choices for $\mathbb{X}_P$ and K to satisfy these conditions. When they are satisfied $J_k^o = \min J_k$ can be proven to be monotonically descending to 0, so it can be used as a Lyapunov function to prove asymptotic stability.

In the unconstrained case (no input or state constraints), the above conditions can be easily satisfied by choosing $\mathbb{X}_P = \mathbb{R}$ and $K = K_{\mathrm{lqr}}$. The solution S to the corresponding Riccatti equation (Kalman (1960)) can then be used as terminal cost (Bitmead et al. (1990)).

4. DETERMINING TERMINAL COST USING LMI'S

The forementioned method to calculate a terminal cost cannot be used in the constrained case, in which case the constraints have to be accounted for explicitly. As proposed in Boyd et al. (1994) and Kothare et al. (1996), a linear, stabilizing feedback controller, which is optimal in an LQR

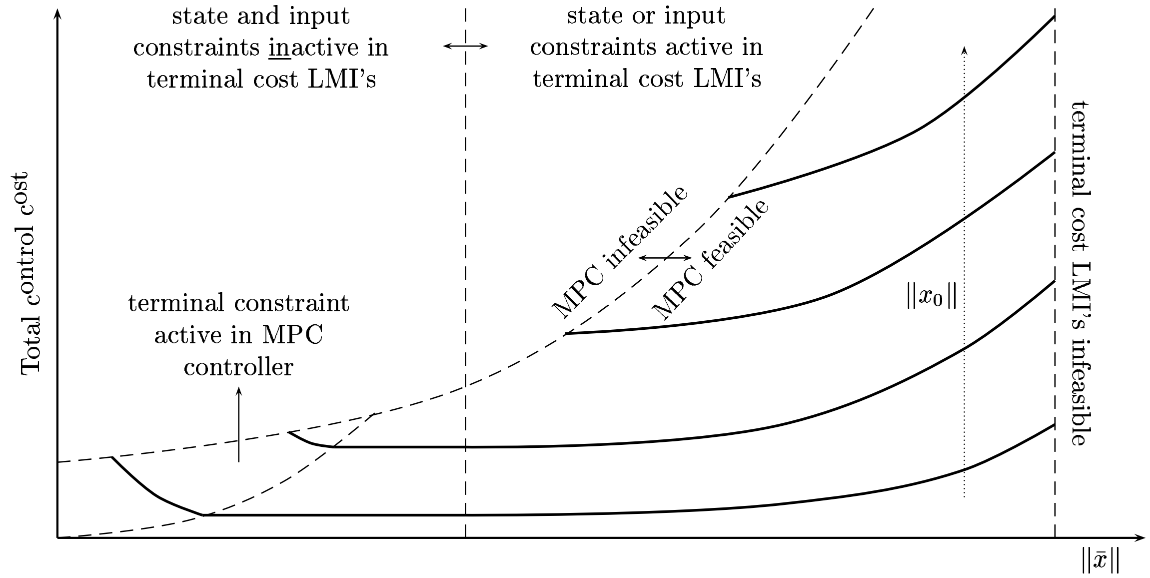

Fig. 1. Total control cost (full line) of recovering from an initial disturbance x_0 for a typical system, in function of $\bar{x}$ and for different disturbance sizes. Different regions of operation are delimited by dashed lines. It is clear that performance is very dependent on the choice of $\bar{x}$. A compromise has to be made between feasibility (larger $\bar{x}$) and optimality (smaller $\bar{x}$).

sense and which respects input and state constraints, can be found by solving a linear optimization problem with LMI constraints :

$$\min_{\gamma,Z,Y,X} \gamma \tag{3a}$$

subject to stability constraints

$$\begin{bmatrix} 1 & \bar{x}^{\mathrm{T}} \\ \bar{x} & Z \end{bmatrix} \geq 0 \tag{3b}$$

$$\begin{bmatrix} Z & (AZ+BY)^{\mathrm{T}} & ZQ^{\frac{1}{2}} & Y^{\mathrm{T}}R^{\frac{1}{2}} \\ AZ+BY & Z & 0 & 0 \\ Q^{\frac{1}{2}}Z & 0 & \gamma I & 0 \\ R^{\frac{1}{2}}Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \tag{3c}$$

$$Z > 0, \tag{3d}$$

input constraints

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0 \tag{3e}$$

$$\begin{bmatrix} u_{1,\max}^2 & & & \\ & u_{2,\max}^2 & & \\ & & \ddots & \\ & & & u_{n_u,\max}^2 \end{bmatrix} \geq X \tag{3f}$$

and state constraints

$$\begin{bmatrix} Z & (AZ+BY)^{\mathrm{T}}C_j^{\mathrm{T}} \\ C_j(AZ+BY) & y_{j,\max}^2 \end{bmatrix} \geq 0$$

$$j = 1 \ldots n_{\mathrm{sc}} \tag{3g}$$

after which the feedback matrix and terminal cost can be calculated as

$$K = YZ^{-1} \tag{4a}$$

$$Q_P = \gamma Z^{-1}. \tag{4b}$$

$\bar{x}$ is an arbitrary state, representing typical excitations, to be chosen by the user.

Remark 1. The resulting linear feedback controller minimizes $\bar{x}^{\mathrm{T}}Q_P\bar{x}$, representing the total control cost of the controller to achieve equilibrium from initial condition $\bar{x}$, while respecting input and state constraints

$$|[u]_i| \leq u_{i,\max} \quad i = 1 \ldots n_u$$
$$|C_j x| \leq y_{j,\max} \quad j = 1 \ldots n_{\mathrm{sc}}$$

for all initial states x satisfying

$$\|x\|_{Q_P} \leq \|\bar{x}\|_{Q_P} \tag{5}$$

with n_u denoting the number of inputs and n_{sc} denoting the number of state constraints.

Remark 2. For sufficiently small values of $\bar{x}$, where input and state constraints are not active, the resulting K and Q_P can be shown to be identical to those obtained by calculating an LQR controller with the help of Riccatti equations. In this case, the terminal cost exactly represents the remaining control cost beyond the horizon which leads to a locally optimal controller.

Remark 3. For sufficiently large values of $\bar{x}$, it will not be possible to find a stabilizing linear feedback controller which still respects the input and state constraints. In this case optimization problem (3) won't have a feasible solution.

Remark 4. (5) can be shown to satisfy conditions (2a), (2b) and (2c) for $\mathbb{X}_P$, so it can be used as terminal constraint. A more thorough proof is given in Pluymers et al. (2003). This constraint is not linear but quadratic, which can complicate the optimization problem. Two arguments can be given however to relativate this. First of all, the constraint can be approximated by a set of linear constraints, which again reduces the problem to a QP. Secondly, because of the fact that Q_P is a positive definite matrix, the terminal constraint is elliptic, thus convex, so the use of efficient convex optimization algorithms is still possible.

5. OPTIMALITY

In the previous section an approach is explained to calculate a stabilizing terminal cost and terminal constraint for linear MPC with input and state constraints using LMI's. One aspect that has not been clarified, however, is the choice of $\bar{x}$. It is clear that, on the one hand, $\bar{x}$ should be chosen small enough to make sure (3) has a feasible solution, while, on the other hand, $\bar{x}$ should still be chosen large enough to make sure (5) isn't overrestrictive which can result in an infeasible MPC optimization problem.

Apart from feasibility, which is a necessary condition for stability, another important feature which is influenced by the choice of $\bar{x}$ is the performance of the controller. This is shown in figure 1. It can be observed that feasibility (large $\bar{x}$) and optimality (small $\bar{x}$) cannot be achieved simultaneously for all $||x_0||$. One should thus choose the smallest $\bar{x}$ that still results in a feasible MPC controller for all disturbances that can be expected.

It is clear that the use of a time-varying terminal cost and terminal constraint could result in improved performance, while preservering feasibility of the controller. A technique accomplishing exactly this is proposed in the next section.

6. MPC WITH TIME-VARYING TERMINAL COST

As shown in the previous section, significant performance improvements can potentially be achieved by adaptively choosing $\bar{x}$ depending on the current state of the system. The strategy chosen in this paper, is to incorporate the choice of $\bar{x}$ directly into the MPC optimization problem

$$\min_{x,u,\bar{x}} J_k = \min_{x,u,\bar{x}} \left(\sum_{i=0}^{P-1} u_{k,i}^{\mathrm{T}} R u_{k,i} + \sum_{i=1}^{P-1} x_{k,i}^{\mathrm{T}} Q x_{k,i} + x_{k,P}^{\mathrm{T}} Q_P(\bar{x}) x_{k,P} \right) \quad (6a)$$

subject to

$$x_{k,i+1} = Ax_{k,i} + Bu_{k,i} \quad (6b)$$

$$u_{k,i} \in \mathbb{U} \quad (6c)$$

$$x_{k,i} \in \mathbb{X} \quad (6d)$$

$$x_{k,P}^{\mathrm{T}}\ Q_P(\bar{x})\ x_{k,P} \leq \bar{x}^{\mathrm{T}}\ Q_P(\bar{x})\ \bar{x} \quad (6e)$$

where $i = 0, \ldots P-1$ and $Q_P(\bar{x})$ explicitly denotes the dependence of the terminal cost and terminal constraint on $\bar{x}$. This way in each time step $\bar{x}$ is chosen to minimize J_k.

The above optimization problem cannot be implemented as such, due to the fact that evaluating $Q_P(\bar{x})$ in turn requires the solution of (3), which is not efficient. The approach taken here is to convert the above optimization problem into LMI's, after which these can be merged with (3), resulting in a unified (and convex) optimization problem.

Before doing this, another simplification can be made. It can be rigourously proven that in the optimum, the equality $Q(\bar{x}) = Q(x_{k,P})$ holds. In this way $\bar{x}$ can be eliminated from the optimization problem by replacing $Q_P(\bar{x})$ with $Q_P(x_{k,P})$. Consequently, the terminal constraint can be removed because it is trivially satisfied. The validity of this elimination is proven in Pluymers et al. (2003).

To convert the MPC optimization problem into an LMI problem, we first eliminate the equality constraints, because these cannot be efficiently converted to LMI's. After elimination, the MPC problem can be written as

$$\min_u u^{\mathrm{T}} K_{\mathrm{quad}} u + k_{\mathrm{lin}} u + (C_P u + D_P)^{\mathrm{T}} Q_P (C_P u + D_P)(C_P u + D_P) \quad (7a)$$

subject to

$$A_{\mathrm{ineq}} u \leq B_{\mathrm{ineq}} \quad (7b)$$

where $C_P u + D_P$ is an expression for the terminal state $x_{k,P}$, so $Q_P(C_P u + D_P)$ again expresses the dependency of Q_P on this state. This QP, without equality constraints, can be converted to a linear optimization problem with LMI constraints

$$\min_{u,\gamma_1,\gamma_2} \gamma_1 + \gamma_2 \quad (8a)$$

subject to

$$\begin{bmatrix} \gamma_1 - k_{\mathrm{lin}} u & u^{\mathrm{T}} \\ u & K_{\mathrm{quad}}^{-1} \end{bmatrix} \geq 0 \quad (8b)$$

$$\begin{bmatrix} \gamma_2 & (C_P u + D_P)^{\mathrm{T}} \\ C_P u + D_P & Q_P (C_P u + D_P)^{-1} \end{bmatrix} \geq 0 \quad (8c)$$

$$B_{\mathrm{ineq}} - A_{\mathrm{ineq}} u \geq 0. \quad (8d)$$

It can be easily seen that γ_1 represents the cost of states and input up to $i = P - 1$, while γ_2 represents the cost of the terminal state $x_{k,P}\ Q_P(x_{k,P})\ x_{k,P}$. Because of the equivalence of $Q_P(x_{k,P})$ and $Q_P(\bar{x})$ this is exactly the same expression as the objective function of (3) as mentioned in remark 1. This is illustrated by the fact that the objective function of (3) is represented by (3a), which can be made equivalent with (8c) by applying (4b). The dependence of Q_P on $x_{k,P}$ can thus be explicitly incorporated into (8) by adding constraints (3b)-(3g). This results in the following optimization problem :

$$\min_{u,\gamma_1,\gamma_2,Z,Y,X} \gamma_1 + \gamma_2 \quad (9a)$$

subject to

$$\begin{bmatrix} \gamma_1 - k_{\text{lin}} u & u^{\text{T}} \\ u & K_{\text{quad}}^{-1} \end{bmatrix} \geq 0 \tag{9b}$$

$$\begin{bmatrix} 1 & (C_P u + D_P)^{\text{T}} \\ C_P u + D_P & Z \end{bmatrix} \geq 0 \tag{9c}$$

$$B_{\text{ineq}} - A_{\text{ineq}} u \geq 0. \tag{9d}$$

and (3c)-(3g).

As will be shown in the next section, this MPC scheme with time-varying terminal cost achieves better performance than the traditional MPC scheme with fixed terminal cost and terminal constraint, while preserving feasibility of the controller. The optimization problem is converted from a QP into a linear problem with LMI contraints. This is still a convex optimization problem, for which efficient algorithms exist. The disadvantage, however, is the increased number of optimization variables, which causes a significantly higher computational complexity. A more detailed analysis of the computational complexity and a way to largely eliminate this disadvantage is given in Pluymers et al. (2003).

7. EXAMPLE

To illustrate the concepts introduced in the previous sections, control of a continuously stirred copolymerization reactor is considered. The model used in this paper has already been discussed in Congalidis et al. (1989, 1986). Although the model is stable at the operating point used in this paper, the introduction of an terminal cost and the adaptive, online calculation hereof results in improved performance.

The reactor consists of a continuously stirred tank to which the reagents are fed with a feed rate chosen by the controller. The reaction product (polymers), solvent and residual reagents are simultaneously drained from the tank, after which the first is separated from the latter. From an engineering point of view, the most important measured variables are the reactor temperature $T_{\text{r}}(K)$, the polymer production rate, $G_{\text{p}}(kg/h)$, the mass fraction Y_{ap} of monomer A in the polymer and the average molar mass $M_{\text{p}}(g/mole)$ of the polymer. See table 1 for an overview of inputs and outputs and their steady state values.

The differential equations of the nonlinear reactor model as derived in Congalidis et al. (1989) were implemented in MATLAB. The model was discretized in time ($T_{\text{s}} = 300\text{s}$)by using a MATLAB differential equation solver and linearized around the operating point described in the same reference. Resulting in a linear state-space model with 6 inputs and 15 states, among which the 4 forementioned variables. This model was then

input	s.s.-value
monomer A feed rate (G_{af})	18.00 kg/h
monomer B feed rate (G_{bf})	89.99 kg/h
initiator feed rate (G_{if})	0.18 kg/h
solvent feed rate (G_{sf})	36.02 kg/h
chain transfer agent feed rate (G_{tf})	2.70 kg/h
inhibitor mass feed rate (G_{zf})	0.0003 kg/h
output	**s.s.-value**
polymer production rate (G_{p})	23.31 kg/h
monom. A mass fract. in polym. (Y_{ap})	0.56
polymer molar mass (M_{p})	35003.48 g/mole
reactor temperature (T_{r})	353.00 K

Table 1. Overview of the input and output variables of the reactor model and their steady state values.

normalized with respect to the steady state values ($x_{\text{n}} = (x - x_{\text{ss}})/x_{\text{ss}}$ and $u_{\text{n}} = (u - u_{\text{ss}})/u_{\text{ss}}$) to avoid numerical problems. All states were assumed to be measured.

To compare the performance of the different MPC schemes discussed in this paper, the recovery of the system from an initial disturbance in the reactor temperature was investigated. More specificly, initial disturbances of the reactor temperature of respectively 1%, 2%, 3% and 4% were applied and the behaviour of the system was observed.

The following observations can be made. As shown in table 2, standard MPC is feasible for all disturbances, but has a worse control cost, compared to some of the controllers with fixed terminal cost. The controllers with fixed terminal cost, however, impose a trade-off between local optimality and feasibility. This is overcome by the controller with time-varying terminal cost which is feasible for all disturbances and has superior control cost. The different input and output trajectories for the 3% disturbance can be observed in fig. 2.

8. CONCLUSION

In this paper a linear LMI-based MPC scheme using a time-varying terminal cost was introduced and asymptotic stability was proven. Existing LMI-formulations for calculating a stabilizing terminal cost were combined with the classical MPC formulation to obtain a single optimization problem, leading to this scheme. The advantages are improved feasibility and conservation of local optimality, which are illustrated using a model of a continuously stirred copolymerization reactor.

The disadvantage of the proposed scheme is the increase in computational complexity. A further analysis and a new scheme with time-varying terminal cost with reduced computational complexity is discused in Pluymers et al. (2003), as well as a generalization towards nonlinear systems, using linear, time-varying models.

$b \setminus a$	/	0.01	0.02	0.03	0.04	time-varying $\bar{x}$
0.01	5.46	4.83	4.83	5.57	6.71	4.83
0.02	21.84	/	19.32	22.29	26.86	19.31
0.03	49.15	/	/	50.16	60.43	43.78
0.04	87.37	/	/	/	107.32	79.97

Table 2. Control cost of MPC without terminal cost (column 1), MPC with fixed terminal cost (column 2 to 5) for different $\bar{x}$ (with $T_{\mathrm{r,n}} = a$) and MPC with time-varying terminal cost (column 6) for different initial disturbances x_0 (with $T_{\mathrm{r,n}} = b$) on the copolymerization reactor model. A slash (/) means the controller resulted in an infeasible optimization problem. The parameters used were $Q = I$, except $Q_{(7,7)} = Q_{(13,13)} = Q_{(14,14)} = Q_{(15,15)} = 10$, $R = I$ and $P = 3$. Component-wise bounds $[-1, 1]$ were applied to the (normalized) inputs and states.

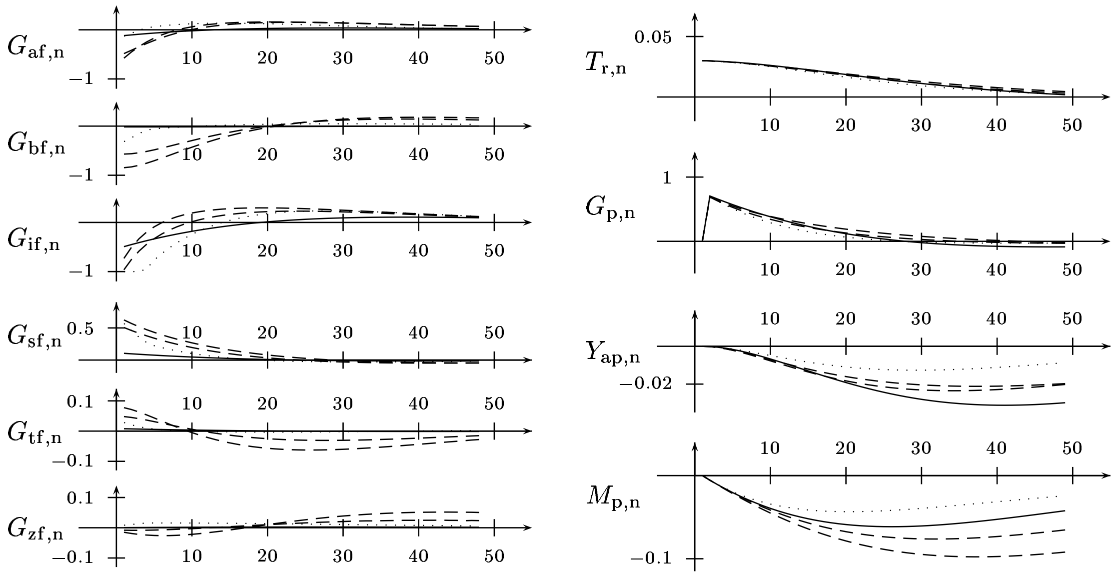

Fig. 2. Input(left) and output sequences (right) for the simulations with MPC without terminal cost (solid), MPC with fixed terminal cost (dashed) and MPC with time-varying terminal cost (dotted) for an initial disturbance of 3% in the reactor temperature. The same parameters as in table 2 were used. The method with time-varying terminal cost clearly performs better than the other methods.

9. ACKNOWLEDGEMENTS

Bert Pluymers is a research assistant with the I.W.T. (Flemish Institute for Scientific and Technological Research in Industry).

Jeroen Buijs is a research assistant with the Katholieke Universiteit Leuven.

Johan Suykens is a postdoctoral researcher with the FWO Flanders and a professor at the Katholieke Universiteit Leuven.

Dr. Bart De Moor is a full professor at the Katholieke Universiteit Leuven, Belgium.

Research supported by Research Council KUL: GOA-Mefisto 666, several PhD/postdoc & fellow grants;

Flemish Government: FWO: PhD/postdoc grants, projects, G.0240.99 (multilinear algebra), G.0407.02 (support vector machines), G.0197.02 (power islands), G.0141.03 (Identification and cryptography), G.0491.03 (control for intensive care glycemia), G.0120.03 (QIT), research communities (ICCoS, ANMMM); AWI: Bil. Int. Collaboration Hungary/ Poland; IWT: PhD Grants, Soft4s (softsensors);

Belgian Federal Government: DWTC (IUAP IV-02 (1996-2001) and IUAP V-22 (2002-2006)), PODO-II (CP/40: TMS and Sustainibility); EU: CAGE; ERNSI; Eureka 2063-IMPACT; Eureka 2419-FliTE;

Contract Research/agreements: Data4s, Electrabel, Elia, LMS, IPCOS, VIB;

REFERENCES

R. R. Bitmead, M. Gevers, and V. Wertz. *Adaptive Optimal Control - The Thinking Man's GPC.* Englewood Cliffs, NJ: Prentice Hall, 1990.

S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory.* SIAM Publications, 1994.

J.P. Congalidis, J.R. Richards, and W.H. Ray. Modeling and control of a copolymerization reactor. *Proceedings of ACC*, 3:1779–1793, 1986.

J.P. Congalidis, J.R. Richards, and W.H. Ray. Feedforward and feedback controler of a solution copolymerization reactor. *AIChE Journal*, 35 (6), 1989.

R.E. Kalman. Contributions to the theory of optimal control. *Bull. Soc. Lath. Mex.*, pages 102–119, 1960.

M.V. Kothare, V. Balakrishnan, and M. Morari. Robust constrained model predictive control using linear matrix inequalities. *Automatica*, 32:1361–1379, 1996.

J.-W. Lee, W. H. Kwon, and J. Choi. On stability of constrained receding horizon control with finite terminal weighting matrix. *Automatica*, 34:1607–1612, 1998.

D.Q. Mayne, J.B. Rawlings, C.V. Rao, and P.O.M. Scokaert. Constrained model predictive control: Stability and optimality. *Automatica*, 36:789–814, 2000.

B. Pluymers, L. Roobrouck, J. Buijs, J. Suykens, and B. De Moor. An LMI-based constrained MPC-scheme with time-varying terminal cost for nonlinear systems. *SISTA Internal Report 03-53*, 2003.

S.J. Qin and T.A. Badgwell. An overview of industrial model predictive control technology. *AIChE Symposium Series 316*, 93:232–256, 1996.

J. B. Rawlings and K. R. Muske. Stability of constrained receding horizon control. *IEEE Transactions on Automatic Control*, 38:1512–1516, 1993.