COMBINATIONS OF MEASUREMENTS AS CONTROLLED VARIABLES: APPLICATION TO A PETLYUK DISTILLATION COLUMN.

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Abstract: A new simple approach for selecting controlled variables, that give near-optimal operation with a constant set-point feedback structure in the presence of uncertainty, is presented. The controlled variables are linear combinations of a subset of the available measurements. A method for selecting the best sub-set and the required number of measurements is derived. The method is illustrated on a Petlyuk (Divided wall) distillation column.

Keywords: Self-optimizing control, Control structure design, Distillation columns

1. INTRODUCTION

Although not widely acknowledged, controlling the right variables is a key element in overcoming uncertainty in operation. Control systems often consist of several layers in a hierarchical structure, each operating on a different time scale. Typically, layers include scheduling (weeks), site-wide optimization (day), local optimization (hours), supervisory/predictive control (minutes) and regulatory control (seconds). The layers are interconnected through the controlled variables **c**. This paper focuses on the interaction between the local optimization layer and the feedback control layer, see Figure 1. The objective is to find good candidate controlled variables **c** with self-optimizing properties. Self-optimizing control follows the idea of Morari *et al.* (1980) where one want to find controlled variables that, when kept at constant set-points, operates near optimally under the influence of disturbances and implementation errors. The disturbances include both exogenous process disturbances and modeling errors. In typical plants the number of disturbances may be very large. In order to reduce the dimension of the problem, only slow varying disturbances that are economically important should be included in the analysis. Morari *et al.* (1980) propose to include disturbances that have a large effect on the objective, ($\frac{\partial J^{opt}}{\partial d_i}$), and let the remaining disturbances be handled by the regulatory layer

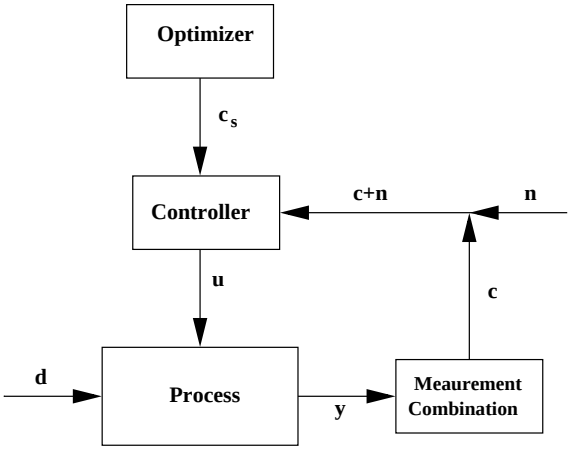

Fig. 1. Self-optimizing feedback control structure

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In plant operation the basic goal is to optimize an economic measure of the operation, while satisfying equality and inequality constraints (such as product specifications, safety constraints, environmental regulations etc.). Since plant economics is primarily decided by steady-state behavior, only steady-state information is used in the rest of this paper. For a given disturbance ($\mathbf{d}$), optimal operation is defined as the solution to the following problem:

$$\min_{u_0} J_0(\mathbf{x_0}, \mathbf{u_0}, \mathbf{d}) \quad (1)$$

$$\mathbf{f}(\mathbf{x_0}, \mathbf{u_0}, \mathbf{d}) = 0 \quad (2)$$

$$\mathbf{g}(\mathbf{x_0}, \mathbf{u_0}, \mathbf{d}) \leq 0 \quad (3)$$

$$\mathbf{x_0} \in \mathcal{R}^{n_{x_0}}, u_0 \in \mathcal{R}^{n_{u_0}}, d \in \mathcal{R}^{n_d}$$

where $\mathbf{f}$ are the equality constraints, $\mathbf{g}$ the inequality constraints, $\mathbf{u_0}$ the free independent variables (inputs), $\mathbf{d}$ the disturbances and $\mathbf{x_0}$ the states. At the nominal optimum a subset ($\mathbf{g'}$) of the inequality constraints will be active and for small changes in the disturbance from the nominal point, it is assumed that the active set does not change. Thus, the reduced space optimization problem is:

$$\min_{u} J_0(\mathbf{x}, \mathbf{u}, \mathbf{d}) \quad (4)$$

$$\mathbf{f'}(\mathbf{x}, \mathbf{u}, \mathbf{d}) = 0 \quad (5)$$

where $\mathbf{f'} = [\mathbf{f}\ \mathbf{g'}]^T$, $\mathbf{x} = [\mathbf{x_0}\ \mathbf{u'}]^T$ where $\mathbf{u'} \in \mathbf{u_0}$ is the subset used to fulfill the active constraints ($\mathbf{g'} = 0$) and $\mathbf{u} \in \mathbf{u_0}$ denotes the remaining unconstrained reduced space degrees of freedom. By formally eliminating the states ($\mathbf{x}$) by using the equality constraints ($\mathbf{f'} = 0$), the remaining unconstrained problem, which is the focus in the rest of this paper, becomes:

$$\min_{\mathbf{u}} J(\mathbf{u}, \mathbf{d}) \quad (6)$$

where $\mathbf{u} \in \mathcal{R}^{n_u}$. The solution of the resulting problem in (4) may be categorized into two classes. Let $n_{f'} = dim(\mathbf{f'})$. If $n_u = n_{u_0} + (n_{x_0} - n_{f'}) = 0$, all degrees of freedom must be used to fulfill the constraints and implementation is usually simple by using the ideas of active constraint control (Maarleveld and Rijnsdorp, 1970). In case of $n_u > 0$, implementing the remaining n_u unconstrained degrees of freedom is not straight forward and this will be the focus in the rest of this paper.

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Online information about the system behavior is available from the measurements in the plant:

$$\mathbf{y_0} = \mathbf{f}_{y_0}(\mathbf{u}, \mathbf{d}) \quad (7)$$

Based on the online information $\mathbf{y_0}$, the most obvious operational policy is to use some sort of optimizing controller with frequent model updates and re-optimization. A much simpler approach for practical applications, is to utilize the ideas of self-optimizing control.

Self-optimizing control (Skogestad, 2000) *is when an acceptable* ***loss*** *can be achieved using constant set-points* $\mathbf{c_s}$ *for the controlled variables* $\mathbf{c}$ *(without the need to re-optimize when disturbances occur).*

The loss is defined as the difference between the objective using the constant feedback structure and the true optimal objective

$$L = J(\mathbf{c_s} + \mathbf{n}, \mathbf{d}) - J_{opt}(\mathbf{d}) \quad (8)$$

where $\mathbf{n}$ is the implementation error (measurement and set-point error) in enforcing $\mathbf{c} = \mathbf{c_s}$. The central issue when searching for the self-optimizing control structure, is to decide how to best *implement* the optimal policy in the presence of uncertainty.

The optimal self-optimizing control structure may be formulated mathematically by:

$$\min_{\mathbf{h}} \int_{d \in \mathcal{D}} \int_{n \in \mathcal{N}} J(\mathbf{u}, \mathbf{d})\ \mathrm{d}n\ \mathrm{d}d \quad (9)$$

$$\mathbf{y} = \mathbf{f}_{\mathbf{y}}(\mathbf{u}, \mathbf{d}) \quad (10)$$

$$\mathbf{h}(\mathbf{y}) = \mathbf{c_s} + \mathbf{n} \quad (11)$$

where $\mathbf{y} \in \mathbf{y_0}$ and $\mathbf{u}$ is an implicit function of $\mathbf{h}$, $\mathbf{d}$, $\mathbf{c_s}$ and $\mathbf{n}$. The goal is to find the optimal function $\mathbf{h}$ interconnecting the measurements and the controlled variables. We assume in this paper that we use nominally optimal set-points, $\mathbf{c_s} = \mathbf{c_{opt}}(\mathbf{d}^*)$, but it is also possible to compute the "robust optimal set-points" by minimizing with respect to $\mathbf{c_s}$ in (9) (Govatsmark and Skogestad, 2002). In practice (9) may be solved by discretizing the disturbance and implementation error space and calculate some weighted average over all points. Clearly, this is a non-convex combinatorial optimization problem, that may be very difficult to solve in practice. A much simpler method for selecting the interconnecting structure $\mathbf{h}$ is needed.

2. PREVIOUS WORK ON SELECTION OF CONTROLLED VARIABLES

Skogestad *et al.* (2003) use a Taylor series expansion of the loss function around the nominal optimal point to develop two methods for selecting controlled variables, the "singular value rule" and the "exact local method". The exact local method, is based on the second order Taylor series expansion of the loss function $L = \frac{1}{2}\|\mathbf{z}\|_2^2$ with

$$\mathbf{z} = J_{uu}^{1/2}[(J_{uu}^{-1^*} J_{du}^* - G^{-1}G_d)(\mathbf{\Delta d}) + G^{-1}\mathbf{n}]$$

where $J_{uu}$and J_{ud} are the second derivatives of J and G and G_d are given by $\Delta\mathbf{c} = G\Delta\mathbf{u} + G_d\Delta\mathbf{d}$. By proper scaling and assuming that $\|[\mathbf{d}\ \mathbf{n}]^T\|_2 \leq 1$, the worst-case loss is:

$$L = \frac{\overline{\sigma}([M_d\ M_n])^2}{2} \quad (12)$$

$$M_d = J_{uu}^{1/2}(J_{uu}^{-1^*}J_{du}^* - G^{-1}G_d)W_d \quad (13)$$

$$M_n = J_{uu}^{1/2}G^{-1}W_n \quad (14)$$

where W_d and W_n are positive diagonal matrices representing the expected magnitudes of the disturbances and implementation errors respectively. This method require that, for each *candidate set* the singular value of the matrix M is calculated. The second method, the singular value rule, is based on scaling the candidate set of controlled variables, and select controlled variables that maximize the minimum singular value of the gain matrix G.

Mahajanam *et al.* (2001) propose a "short-cut" method to eliminate poor choices and to generate rank alternatives without solving the optimization problem. The method is based on scaling all candidate controlled variables so that they have similar effects on the steady-state profit.

3. PROPOSED METHOD FOR SELECTING CONTROLLED VARIABLES AS LINEAR COMBINATIONS OF THE MEASUREMENTS

We here consider the remaining unconstrained optimization problem in (6), and the objective is to find variables $\mathbf{c}$ to be kept at constant setpoints. In general, we have

$$\mathbf{c} = h(\mathbf{y}) \quad (15)$$

where $\mathbf{y} \in \mathbf{y_0}$ is the subset of all available measurements which we choose to make use of. Note that $\mathbf{y_0}$ generally also includes the input variables $\mathbf{u}$. Previous work (Skogestad, 2000) has mainly focused on using single measurements as controlled variables, ie. $\mathbf{c} = \mathbf{y}$. The generally nonlinear function $\mathbf{h}$ is free to choose, except that the controlled variables are assumed independent and that the number of controlled variables (c's) equals the number of remaining unconstrained degrees of freedom (u's). In this paper, we consider only linear combinations of the measurements

$$\Delta\mathbf{c} = H\Delta\mathbf{y} \quad (16)$$

where the matrix H is free to choose. Skogestad *et al.* (2003) use (12) to search for the optimal measurement combination (matrix H), taking into account both disturbances and implementation errors, but this is generally a very difficult problem. However, as shown below, it is actually trivial to find the optimal H for the case with *no implementation error* ($\mathbf{n} = 0$). We use the following insight: *With no implementation error, the constant set-point policy ($\mathbf{c} = \mathbf{c_s}$) is optimal if $\mathbf{c}_{\text{opt}}(\mathbf{d})$ is independent of d.* Of course, the optimal values of the individual measurements $\mathbf{y}$ depend on $\mathbf{d}$, which for a small disturbance change may be written

$$\Delta\mathbf{y_{opt}} = \mathbf{y_{opt}}(\mathbf{d}) - \mathbf{y_{opt}}(\mathbf{d}^*) = F(\mathbf{d} - \mathbf{d}^*) = F\Delta\mathbf{d} \quad (17)$$

where $F = \left(\frac{d\mathbf{y_{opt}}}{d\,\mathbf{d}}\right)^*$. For example, F may be obtained numerically by solving the optimization problem (4) for small changes in the disturbance, and from this obtaining $\mathbf{u_{opt}}(\mathbf{d})$ as well as $\mathbf{y_{opt}}(\mathbf{d})$. F must be understood as an constrained optimal linear mapping. Ganesh and Biegler (1987) give an efficient and rigorous strategy for finding the optimal sensitivity based on a reduced Hessian method. From (16) the corresponding change in the optimal value of $\mathbf{c}$ is

$$\Delta\mathbf{c_{opt}} = H\Delta\mathbf{y_{opt}} \quad (18)$$

Now require that

$$\Delta\mathbf{c_{opt}} = HF\Delta\mathbf{d} = 0 \quad (19)$$

This needs to be satisfied for any $\Delta\mathbf{d}$ so

$$HF = 0 \quad (20)$$

For this to hold, Hshould be in the left null space of F ($H \in \mathcal{N}(F^T)$). This requirement is always possible to fulfill, if there are enough measurements available in the plant. There are n_u unconstrained degrees of freedom (the length of vectors $\mathbf{u}$ and $\mathbf{c}$ are n_u), n_y independent measurements used when forming $\mathbf{c}$, and n_d independent disturbances. Then F is a $n_y \times n_d$ matrix and H a $n_u \times n_y$ matrix. The fundamental theorem of linear algebra (Strang, 1988) gives that $\mathcal{N}(F^T)$, the left null space of F has rank $n_y - r$, where $r = rank(F) = n_d$. Since $H \in \mathcal{N}(F^T)$ it follows that $rank(H) = n_y - n_d$ and by assuming that the number of controlled variables must be equal to the number of inputs, $rank(H) = n_u$.

$$n_y - n_d = n_u \Leftrightarrow n_y = n_u + n_d \quad (21)$$

so that the *minimum number of measurements needed, is equal to the number of inputs plus the number of disturbances.*

3.1 Comparison with the exact local method

The linearized models at the nominal point is

$$\Delta\mathbf{y} = G^y\Delta\mathbf{u} + G_d^y\Delta\mathbf{d} \quad (22)$$

where $G^y = (\partial \mathbf{f_y}/\partial \mathbf{u}^T)^*$ and $G_d^y = (\partial \mathbf{f_y}/\partial \mathbf{d}^T)^*$. For a disturbance change we have (Skogestad *et al.*, 2003).

$$\mathbf{u_{opt}}(\mathbf{d}) - \mathbf{u_{opt}}(\mathbf{d}^*) = -J_{uu}^{*-1} J_{du}^* (\mathbf{d} - \mathbf{d}^* \quad (23)$$

Thus

$$\Delta \mathbf{y_{opt}} = \left[-G^y J_{uu}^{*-1} J_{du}^* + G_d^y \right] (\mathbf{d} - \mathbf{d}^*) \quad (24)$$

By using $G = HG_y$ and $G_d = HG_d^y$ in (13), setting $M_d = 0$, assuming no implementation error and rearranging we get $GJ_{uu}^{-1^*} J_{du}^* - G_d = 0$ and inserting into (24) we re-derive $\Delta \mathbf{c_{opt}} = H \Delta \mathbf{y_{opt}} = 0$. Note that

$$F = -G^y J_{uu}^{-1} J_{du} + G_d^y \quad (25)$$

4. A TWO-STEP METHOD CONSIDERING DISTURBANCES AND IMPLEMENTATION ERRORS.

From the analysis in Section 3, see (12) to (14), it is evident that even if $M_d = 0$ the loss may still be large, since M_n is non-zero due to the implementation error. As stated, the selection matrix H is not unique, since there is freedom in selecting another sub-set of measurements. This may be utilized in order to reduce the effect of the implementation error, while still ensuring $M_d = 0$. The selection of a sub-set $\mathbf{y}$ of the available measurements $\mathbf{y_0}$, should reflect two goals. First, since the feedback structure must correct for disturbances in order to keep the plant optimal, the disturbances must be observable in the process (high gain in G_d). Second, in order to reduce the implementation error, it is evident from (14) that G^{-1} should be small in all directions (e.g $\underline{\sigma}(G)$ should be large). Based on these observations, it is proposed here to select measurements sequentially, that maximize the minimum singular value of the scaled augmented plant $\mathbf{\Delta y'} = \tilde{G}^y \tilde{\mathbf{\Delta u}} = [G^{y\prime} \; G_d^{y\prime}][\mathbf{\Delta u'} \; \mathbf{\Delta d'}]^T$.

The reason for using $\underline{\sigma}(\tilde{G}^y)$ rather than $\underline{\sigma}(G)$ and $\underline{\sigma}(G_d)$, is that H is not known *a priori*. To justify this, the following applies:

$$\underline{\sigma}(H)\underline{\sigma}(\tilde{G}^y) \leq \underline{\sigma}(H\tilde{G}^y) = \underline{\sigma}([G \; G_d]) \leq min(\underline{\sigma}(G), \underline{\sigma}(G_d)) \quad (26)$$

(Skogestad and Postlethwaite, 1996), (Horn and Johnson, 1991), where it is alway possible to select $\underline{\sigma}(H) = 1$. Thus, $\underline{\sigma}(\tilde{G}^y)$ provide a lower bound on $\underline{\sigma}(G)$ and $\underline{\sigma}(G_d)$. In addition, if $\underline{\sigma}(\tilde{G}^y)$ is nonzero this guarantees that F has full rank n_d, see (19), which is required to ensure that $F \neq 0$ for all $\mathbf{d}$. This follows from (25) since G^y and G_d^y has full rank n_u and n_d respectively, and $\tilde{G}^y$ has full rank $n_u + n_d$.

The proposed method of selecting controlled variables as linear combinations of the measurements is summarized in Section 4.1.

4.1 Details of procedure

Assume that $n_{y_0} \geq n_u + n_d$ and the nominal optimal point is $\mathbf{u_{opt}}(\mathbf{d}^*)$

(1) **Linearization**. Linearize the process model around the nominal optimal point. This give G^{y_0} and $G_d^{y_0}$ for all measurements $\mathbf{y_0}$.

(2) **Scaling** Scale each measurement $y_{0,i}$ with its corresponding implementation error ($|n_{y_{0,i}}|$), each input u_j with its corresponding allowable range ($\Delta u_{j,max}$) and each disturbance d_k by its corresponding expected disturbance. This give the scaling matrices $W_{n_{y_0}} = diag(|n_{0,i}|)$, $W_u = diag(\Delta u_{j,max})$ and $W_d = diag(|d_k|)$

(3) **Selection of measurements**.

(a) **Augmented process model**. Calculate the scaled process model $\mathbf{\Delta y_0'} = G^{y_0\prime}\mathbf{\Delta u'} + G_d^{y_0\prime}\mathbf{\Delta d'} = W_{n_{y_0}}^{-1} G^{y_0} W_u \mathbf{\Delta u'} + W_{n_{y_0}}^{-1} G_d^{y_0} W_d \mathbf{\Delta d'}$ and obtain a new process matrix

$$\mathbf{\Delta y_0'} = \tilde{G}^{y_0}\tilde{\mathbf{\Delta u}} = [G^{y_0\prime} \; G_d^{y_0\prime}][\mathbf{\Delta u'} \; \mathbf{\Delta d'}]^T$$

(b) **Selection of the first measurement**. Calculate the row norm $\|\tilde{G}_i^{\,y_0}\|_2$ for all rows i and sort by decreasing row norm. Select the row with highest norm and add the corresponding row of the process matrix to a selection process matrix $\tilde{G}_1^y = max_i \|\tilde{G}_i^{y_0}\|_2$

(c) **Selection of the additional measurements**. Until $n_y = n_u + n_d$ add measurements to the selection process matrix one-by-one

$$\tilde{G}_{j+1,i}^y = \begin{bmatrix} \tilde{G}_j^y \\ \tilde{G}_i^{y_0} \end{bmatrix}$$

for all i and calculate the minimum singular value for all the combinations. Select the new measurement which has the highest minimum singular value and add to the selection process matrix.

(4) **Null space of F and selection of controlled variables**.

(a) Obtain F, for example, numerically from the non-linear equations, $F = \left(\frac{d\mathbf{y_{opt}}}{d\,\mathbf{d}}\right)^*$, or from (24).

(b) Calculate the null space $\mathcal{N}(F^T)$.

(c) Select H such that $H \in \mathcal{N}(F^T)$ and the rows of H form a orthonormal basis. This ensure that $\Delta \mathbf{c_{opt}} = H \Delta \mathbf{y_{opt}} = 0$

5. EXAMPLE: "PETLYUK" (DIVIDING WALL) DISTILLATION COLUMN

The thermally integrated divided wall ("Petlyuk") arrangement has several advantages compared to the traditional arrangements. Smith and Triantafyllou (1992) report typical savings in the order of 30% in *both* energy and capital costs compared to traditional arrangements with two columns in series. The Petlyuk column shown

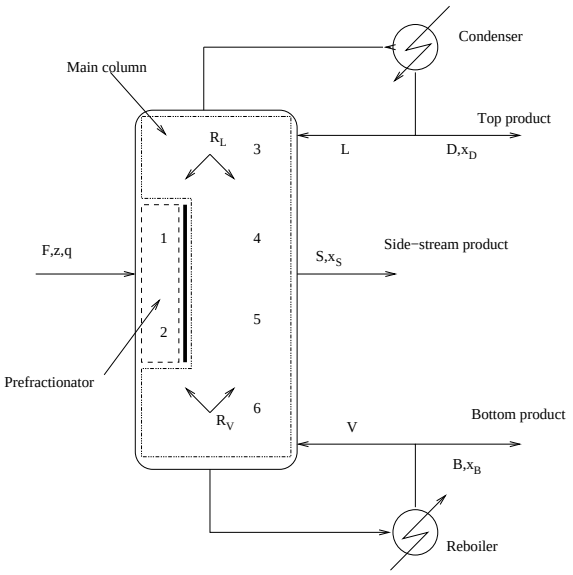

Fig. 2. The Petlyuk Distillation column implemented in a single column shell.

in Figure 2 has at steady state five degrees of freedom, which may be selected as the following inputs $u = [V\ L\ S\ R_l\ R_v]^T$ (boil-up, reflux, mid product side-stream flow, liquid split and vapor split). The product quality constraints correspond to the top purity ($x_{D,A}$), the bottom purity ($x_{B,C}$), the side-stream purity ($x_{S,B}$). The main contribution to the operational cost is the reboiler vapor flow, so the plant objective is to minimize:

$$J(x, u, d) = V \tag{27}$$

The model of the column is based on the assumption of constant relative volalities ($\alpha = [9\ 3\ 1]^T$), constant pressure, equilibrium on all stages, total condenser and constant molar flows. The tray temperatures are calculated based on the compositions using Antoine's equation.

Nominal data: Feed flow $F^* = 1$, liquid fraction $q^* = 0.477$, feed compositions $z_A^* = z_B^* = z_C^* = 1/3$ and product inequality constraints $[x_{D,A}\ x_{S,B}\ x_{B,C}]^T \geq 0.97$. The nominal optimal inputs are $u_{opt}(d*) = [L^*\ V^*\ S^*\ R_l^*\ R_v^*]^T = [0.718\ 0.5810\ 0.3227\ 0.3792\ 0.5123]^T$. Only economically important disturbances should be included in the analysis, which correspond to the disturbances z_A and q, the composition of component A the liquid fraction in the feed respectively, where $\mathbf{d} = [z_A\ q]^T = [z_A^* \pm 0.1\ q^* \pm 0.1]^T$. Since only intensive measurements are considered, feed flow rate is neglected as a disturbance. The implementation error is assumed to be $|\mathbf{n}| = [|n_{T_{j,i}}|\ |n_{R_l}|\ |n_{R_v}|]^T = [0.4\ 0.05\ 0.05]^T$.

For the disturbance range considered here, the optimally active constraints that need to be controlled, are the product compositions for the top, bottom and side-stream ($[x_{D,A}\ x_{S,B}\ x_{B,C}]^T = 0.97$), removing three degrees of freedom (L, V and S).

Based on the observation that the objective function has one "strong" and one "weak" direction, Halvorsen and Skogestad (1999) stated that in order to track the optimal trajectory only one of the remaining degrees of freedom need to be adjusted, so they propose to fix R_v. This is also reasonable from a practical point of view, due to the practical difficulties of implementing the vapor split. For the remaining degrees of freedom, only temperature measurements or combinations thereof are considered.

Halvorsen and Skogestad (1999) studied several candidate controlled variables for good self-optimizing properties, where the most promising control structure was to control DT_S, a measure of the temperature profile symmetry across the dividing wall, and R_v. DT_S is defined as $DT_S = \sum T_{1,i} - T_{4,i} - \sum T_{2,i} - T_{5,i}$, where $T_{j,i}$ is the temperature of tray i in section j.

For the remaining degree of freedom, only temperature measurements or combinations thereof are considered. In addition to the structure (R_v,DT_S) proposed by Halvorsen and Skogestad (1999), several other structures are considered; (R_v, $T_{1,7}$), (R_v, $T_{1,2}$) and the open loop structure (R_v, R_l).

In addition, two structures based on the methods proposed in this paper, are compared for self-optimizing properties.

1. (R_v,$c_{LC,3}$) with R_v fixed and the implementation error in R_v is added as a disturbance. Thus, $d = [z_A\ q\ n_{R_v}]$ and the number of measurements needed is $n_y = n_u + n_d = 1 + 3 = 4$. Maximizing the minimum singular value of the scaled augmented plant give that the subset $T_{5,5}, T_{2,2}, T_{4,2}, T_{2.1}$ of the temperature measurements should be combined. Selecting $\mathbf{c} = H\mathbf{y}$ such that H is in the left null space of F, result in $c_{LC,3} = -0.959T_{5,5} + 0.1969T_{2,2} + 0.00956T_{4,2} + 0.1770T_{6,3}$.

2. ($c_{LC,1}$,$c_{LC,2}$). This case was included in order to check if there is any additional economic advantage of using both degrees of freedom as inputs. Here we have two unconstrained degrees of freedom and the number of required measurements is

$n_y = n_u + n_d = 2 + 2 = 4$. The temperatures $T_{5,5}, T_{2,3}, T_{4,2}, T_{2,1}$ minimized the singular value of the augmented plant and the corresponding optimal measurement combinations are $c_{LC,1} = 0.222T_{5,5} - 0.7052T_{2,3} + 0.490T_{4,2} + 0.462T_{2,1}$ and $c_{LC,2} = -0.946T_{5,5} - 0.003T_{2,3} + 0.1592T_{4,2} + 0.2821T_{2,1}$. For the controlled variables that are linear combinations of the measurements it is assumed that the implementation error is $n_{c,i} = \|H_i W_y\|_2$.

In calculating the loss in Table 1, it is assumed that the combined implementation and disturbance vector is 2-norm bounded and that there is no implementation error in enforcing the active constraints. The average loss is calculated as a weighted sum of all combinations of the implementation and disturbance vector in which each disturbance and implementation error has a low, nominal and high value, with equal weighting.

Table 1. Loss for the different controlled variables in the Petlyuk Column case

c_1	c_2	Average loss (%)	Worst case loss (%)
$c_{LC,1}$	$c_{LC,2}$	0.01	0.02
R_v	$c_{LC,3}$	0.16	0.87
R_v	DT_s	2.40	11.9
R_v	R_l	18.0	123.0
R_v	$T_{1,7}$	22.7	118.4
R_v	$T_{5,2}$	infeasible	infeasible

As seen from Table 1, control structures ($c_{LC,1}$, $c_{LC,2}$) and (R_v, $c_{LC,3}$) track the optimal trajectory and give near-optimal operation. Controlling structure (R_v,DT_s) also give acceptable operation, while controlling the single temperatures ($T_{1,7}$ and $T_{5,2}$) give a very high loss or infeasible operation, which is expected since a change in the inflow composition is one of the disturbances. From Table 1 it is evident that fixing R_v gives only a small increase in the loss, but this is necessarily not true for all liquid fractions in the feed.

6. CONCLUDING REMARKS

Selecting the right variable to control is of great importance to overcome uncertainty in operation. A new method for selecting controlled variables as linear combinations of a subset of the available measurements has been proposed in addition to a method for selecting the subset of measurements. The idea is to find a linear combination of the measurements such that $\Delta c_{opt} = H\Delta y_{opt} = 0$ by using as many measurements as there are unconstrained inputs and disturbances. From a linear point of view, the proposed method guaranty perfect self-optimizing properties if we neglect implementation error. The proposed method has been illustrated on a simulated Petlyuk distillation column, which show that the proposed method give controlled variables with good self-optimizing properties.

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REFERENCES

Ganesh, N. and L. T. Biegler (1987). A reduced hessian strategy for sensitivity analysis of optimal flowsheets. *AIChE Journal* pp. 282–296.

Govatsmark, M. S. and S. Skogestad (2002). Selection of controlled variables and robust setpoints. In: *Proc. of IFAC World Congress*. IFAC. Paper T-Mo-M-11-4.

Halvorsen, I.J. and S. Skogestad (1999). Optimal operation of petlyuk distillation: steady-state behavior. *J. Proc. Control* pp. 407–424.

Horn, R. A. and C. R. Johnson (1991). *Topics in Matrix Analysis*. Cambridge.

Maarleveld, A. and J.E. Rijnsdorp (1970). Constraint control on distillation columns. *Automatica* **6**(1), 51–58.

Mahajanam, R.V., A. Zheng and J.M. Douglas (2001). A shortcut method for controlled variable selection and its application to the butane alkylation process. *Industrial and Engineering Chemistry Research* **40**(14), 3208–3216.

Morari, M., G. Stephanopoulos and Y. Arkun (1980). Studies in the synthesis of control structures for chemical processes. part i: Formulation of the problem. process decomposition and the classification of the controller task. analysis of the optimizing control structures. *AIChE Journal* **26**(2), 220–232.

Skogestad, S. (2000). Plantwide control: the search for the self-optimizing control structure. *J. Proc. Control* **10**, 487–507.

Skogestad, S. and I. Postlethwaite (1996). Multivariable feedback control. *John Wiley & Sons*.

Skogestad, S., I. Halvorsen, J.C. Morud and V.Alstad (2003). Self-optimizing control: Local taylor series analysis. *Ind. Eng. Chem. Res.* Submitted.

Smith, R. and C. Triantafyllou (1992). The design and operation of fully thermally coupled distillaton columns. *Trans. IChemE.* pp. 118–132.

Strang, G. (1988). *Linear Algebra and its Applications*. 3 ed.. Harcourt Brace & Company.