PERFORMANCE ASSESSMENT OF CONSTRAINED CONTROLLERS

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Abstract: The paper presents a new deterministic framework for assessing constrained control loop performance. The proposed dynamic performance index is based on the dynamic operating work of Uzturk and Georgakis (2002). It focuses on the time related characteristics of controllers' response to set-point changes or step-type disturbances. It explicitly takes into account the existence of constraints on manipulated variables. These constraints include minimum and maximum values as well as an upper limit on the rate of change of the input variables.

Keywords: Constrained Controller, Performance Assessment, Time Optimal control, Minimum Settling Time, Deterministic Disturbances.

1. INTRODUCTION

The controller design task aims to find a suitable controller given a model of the system to be controlled and a set of design goals. A well designed control system should satisfy both performance and robustness specifications. Performance specifications include stability, disturbance regulation, set-point tracking, transient response, and constraints (e.g. Boyd and Barratt, 1991). In practice, unfortunately, controllers seldom work as initially designed due to inapt assumptions and compromises made during design, improper controller tunings, and unaccounted model-plant mismatch, etc.

For reliable and profitable process control applications, the chemical industry is in need of effective controller performance monitoring and diagnosis technology. Harris (1989) reported the estimation of the minimum achievable variance of SISO controlled variable from 'normal' closed-loop data. Since then, *Minimum variance control* (Åström, 1970) has been widely used as a benchmark for assessing control loop performance (e.g. Desborough and Harris, 1992). Extensions of SISO control loop performance assessment techniques to MIMO cases were first addressed by Huang *et al.* (1997) and Harris *et al.* (1996).

Minimum variance control provides a lower stochastic bound on the achievable performance of any feedback controller if (i) the control objective is to minimize the steady-state output variance, (ii) the time delay is the leading performance limiting factor, (iii) and the disturbance acting on the process can be reduced to a filtered white noise. However, in most practical applications, it is the constraints on the manipulated variables, along with the time

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delay and the inverse response dynamics, that place an upper limit on the achievable performance. With the wide availability of controllers that explicitly take into account constraints, the assessment of control loop performance needs to consider the effects of constraints as well. Ko and Edgar's (2001) approach deserves notice as it is the first attempt to explicitly account for constraints in control performance assessment.

The present paper describes a new deterministic framework for assessing constrained control loop performance. The proposed dynamic performance index focuses on the time related characteristics of the controller's response to set-point changes or step disturbances. It explicitly accounts for the constraints on manipulated variables, including magnitude and rate of change limits. The paper is organized as follows. In section 2, we review the minimum time-optimal control benchmark. It is used as the basis for the proposed performance index described in section 3. The demonstration examples are given in section 4, and section 5 provides the conclusions.

2. TIME DOMAIN CONSTRAINED APPROACH

The objective of this paper is to develop a performance index to assess controller performance of constrained systems with respect to deterministic disturbances. The motivations are as follows. As pointed out by MacGregor *et al.* (1984), in most chemical engineering processes, the major disturbances are not stochastic disturbances, but deterministic disturbances such as sudden loads on the system and set-point changes made by operators. Furthermore, Eriksson and Isaksson (1994) have shown, through a very interesting example, that the minimum variance control based performance assessment technique (e.g. Desborough and Harris, 1992) gives an inadequate measure of performance if the control objective is set-point tracking.

To develop a time-domain controller performance criterion, the minimum time optimal control is adopted as the benchmark to evaluate control loop performance. Minimum time optimal control explicitly takes into account the input constraints and provides a time domain upper bound of the achievable control performance. It is independent of the feedback control structure one might use on-line and reflects the inherent performance limitations of the process.

2.1 Minimum-time Optimal Control

Minimum-time optimal control aims to drive the process to the desired set-point within minimum settling time t_s, given process dynamics and constraints on controlled variables and manipulated variables. The final time constraints incorporated in the formulation of minimum-time optimal control problem ensure that the system reaches the set-point at the minimum settling time and stays there afterwards. In addition, set-points and disturbances entering the process are assumed to be in step form. The solution of the minimum time control problem can be computationally intensive. Linear programming (LP) technique is commonly used to solve the problem in the case of linear systems. (Refer to Uzturk and Georgakis (2002) for more details.)

2.2 Approximate Equivalence between Minimum Time Optimal Control and Minimum IAE or ISE Control

Simulations of several model SISO systems have shown the set-point response under minimum time optimal control to be overdamped. Very interestingly, minimum integral absolute error (IAE) control or minimum integral square error (ISE) control, if modified to only allow overdamped responses, perform almost identically to minimum time optimal control. This equivalence in performance is evaluated under any of the three criteria (settling time, IAE, and ISE) [2]. In the following sections, we refer as optimal control to any of these three control schemes.

The above equivalence among minimum time optimal control, modified minimum IAE control, and modified minimum ISE control shows that settling time, IAE, and ISE are similar performance measures. Hence, any of them can be applied as a candidate measure for performance assessment. In addition, for system models of order higher than two, we can estimate the minimum settling time by solving a modified minimum IAE or ISE control problem, which is computationally easier and involves one pass LP or QP optimization.

3. CONSTRAINED PERFORMANCE INDEX

The performance index can be defined as the ratio of the performance measure (settling time, IAE, ISE) under optimal control to that under present control. However, as reported by Uzturk and Georgakis (2002), for constrained controllers, the optimal achievable performance measure depends on operating conditions such as the initial operating points, desired set-points, and disturbances (see Figure 2 in Uzturk and Georgakis (2002)). As a result, the performance index used

[2] Due to space limitation, demonstration plots are not presented.

might vary with operating conditions. The major issue is whether the settling time, IAE, or ISE alone is a sufficient measure of controller performance. An additional concern comes when a process model is required for the estimation of the optimal performance criteria.

3.1 Proposed Performance Index

A new constrained performance index is proposed in this section which consists of three components related to integral absolute error, overshoot, and response time:

$$\eta \equiv \frac{1}{3}\left(\frac{\mathrm{IAE}^{\mathrm{ref}}}{\mathrm{IAE}} + \frac{r^{\mathrm{d}}}{r^{\mathrm{d}} + y^{\mathrm{os}}} + \frac{t_{\mathrm{s}}^{\mathrm{ref}}}{t_{\mathrm{s}}}\right) \qquad (1)$$

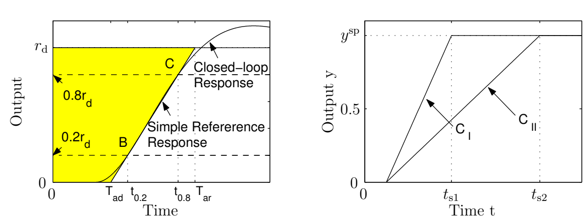
The first term is defined as the ratio of a reference integral absolute error (IAE) to that of the set-point response. The second term penalizes the overshoot, where r_{d} is set-point change and y^{os} is output overshoot (absolute value). The third term evaluates the ratio of a reference settling time to the actual settling time t_{s} of the set-point response. In the above definition, the settling time t_{s} is defined as the time when the set-point response reaches and remains inside a band which is equal to $\pm 3\%$ of the set-point change r_{d}.

If a process model is available, the reference IAE and settling time can be calculated by solving a constrained optimization problem.

Without a process model at hand, the reference settling time and reference IAE are estimated directly from the set-point response:

$$t_{\mathrm{s}}^{\mathrm{ref}} = t_{\mathrm{r}}; \qquad \mathrm{IAE}^{\mathrm{ref}} = \frac{1}{2} r_{\mathrm{d}}(T_{\mathrm{ad}} + T_{\mathrm{ar}}) \qquad (2)$$

The rise time t_{r}, defined as the time when the set-point response reaches the value $0.97 r_{\mathrm{d}}$, is adopted as the reference settling time. The reference IAE is the integral absolute error of a simple reference response, which is characterized by three parameters: the set-point change r_{d}, the apparent rise time T_{ar}, and the apparent dead time T_{ad} (see Fig. 1(a)). The main advantage with this choice of reference settling time and reference IAE is that there is no requirement for a process model to estimate the performance index η. In addition, the first and third terms in Eq. (1) implicitly evaluate the contribution of the oscillating part of the set-point response to the IAE and the settling time, respectively.



(a) Definition of η (b) Drawback of η

Fig. 1. Demonstration diagram for definition & drawback of proposed performance index η

The parameters (r_{d}, T_{ar}, and T_{ad}) can be determined graphically from the set-point response (see Fig. 1(a)). The set-point change (r_{d}) can be determined from the final steady-state level of the process output. $t_{0.2}$ is the time when the set-point response reaches the value $0.2 r_{\mathrm{d}}$ (point B), and $t_{0.8}$ is the time when the set-point response reaches the value $0.8 r_{\mathrm{d}}$ (point C). The intercept of line (BC) with the horizontal axes gives the dead time T_{ad}. The line BC intersects the line $y(t) = y^{\mathrm{sp}}$ and provides the response time T_{ar}. Furthermore, from Fig. 1(a), we can see that the following equality holds: $T_{\mathrm{ar}} + T_{\mathrm{ad}} = t_{0.8} + t_{0.2}$. Note, however, the rise time t_{r} and the apparent rise time T_{ar} are two different concepts.

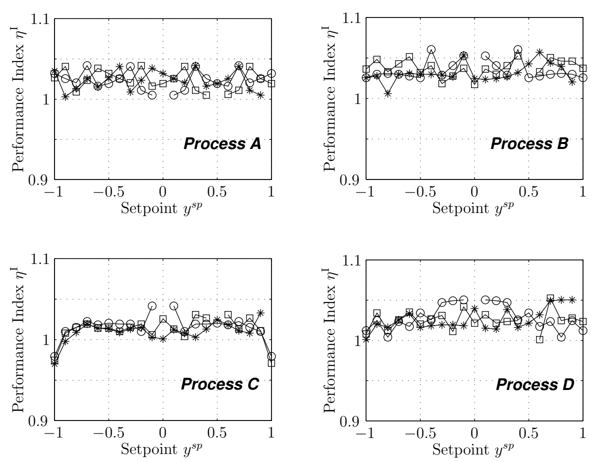


Fig. 2. Characteristics of proposed performance index under optimal control: curve with ($\circ$) corresponds to initial condition $u_0 = 0$, $y_0 = 0$; curve with ($\square$) corresponds to initial condition $u_0 = 0.5$, $y_0 = 0.5$; curve with ($*$) corresponds to initial condition $u_0 = 1$, $y_0 = 1$.

Simulation results have shown that, under optimal control, the first term ($\eta^{\mathrm{I}} = \mathrm{IAE}^{\mathrm{ref}}/\mathrm{IAE}$) defined in (Eq. 1) is very close to unity 1 (see Fig. 2) [3]. This is true for any initial conditions, final conditions, constraints, or process dynamics. In addition, the second term ($\eta^{\mathrm{II}} = r^{\mathrm{d}}/(r^{\mathrm{d}} + y^{\mathrm{os}})$) in Eq. (1) is equal to 1 as the set-point response under optimal control is overdamped, monotonic, and without steady-state offset. With the rise time and settling time defined above, the third term ($\eta^{\mathrm{III}} = t_{\mathrm{r}}/t_{\mathrm{s}}$) is also unity for optimal

[3] In the simulation, all the processes considered (see Table 1) are normalized with unit gain. In addition, the input values are constrained in $|u(k)| \leq 1$, $\Delta u(k)| \leq 0.2$.

Table 1. Processes considered

Process A	$G(s) = 1 \times e^{-8s}/(232s^3 + 37s^2 + s)$
Process B	$G(s) = 1 \times e^{-3s}/(s^2 + s + 1)$
Process C	$G(s) = 1/(2s + 1)(4s + 1)(6s + 1)$
Process D	$G(s) = 1e^{-0.8s}/(2s + 1)^8$

controllers. Therefore, the performance index η defined in Eq. (1) will be close to unity for optimal controllers w.r.t. any operating conditions.

3.2 Drawbacks

However, some drawbacks exist for the proposed performance index η (Eq. 1). For simplicity, consider a hypothetical process which provides two set-point response curves in Fig. 1(b) corresponding to two different controllers C_{I} and C_{II}. For both controllers, the performance indices are close to unity, which means both controllers are almost optimal. However, this conclusion is not true because the closed-loop process with controller C_{I} responds faster than that with controller C_{II}.

To accommodate this drawback, it is desirable to utilize the minimum settling time and minimum IAE as the reference settling time and reference IAE in the definition of performance index η rather than their approximation estimated from the closed-loop set-point response (Eq. 2). Certainly, this requires the availability of a process model. This issue will be discussed in the next section.

3.3 Performance Assessment Framework

We propose our controller performance assessment framework for constrained systems as follows:

Step 1: Estimate the steady state output deviation from the desired reference set-point. If this offset is beyond certain tolerance, either the controller needs improvement or there exists operability issue. Else, go to the next step.

Step 2: From closed-loop set-point response, estimate $t_{0.2}$ and $t_{0.8}$. Then calculate the performance index η. No model is necessary at this stage. If $\eta \ll 1$, it means the current controller does not perform well. On the other hand, if $\eta \simeq 1$, it does not necessarily mean that the current controller is almost optimal (see Section 3.2). In this case, we do need to move to a third step for further evaluation.

Step 3: In this step, different from step 2, minimum settling time and minimum IAE are calculated and employed as the reference settling time and reference IAE in the estimation of the performance index η. If the newly calculated performance index η is also close to 1, it confirms that the controller under evaluation performs well. However, the calculation of the above minimum settling time and minimum IAE demands the knowledge of an approximate process model, which is discussed in the next section.

3.4 Estimation of Minimum Settling Time and Minimum IAE

We assume that the process can be approximated as a first order plus time delay model: $G(s) = K_{\mathrm{p}}e^{-\mathrm{ds}}/(\tau s + 1)$. The parameters can be easily estimated from closed-loop data using strategies such as linear regression.

The advantage of this approximation lies in the fact that the minimum settling time of a system with delay is equal to the time delay plus the minimum settling time corresponding to the delay free system. In addition, an analytical solution of optimal settling time exists for a first order system if there are magnitude constraints on manipulated variables (Uzturk and Georgakis, 2002). If, on the other hand, there are constraints on the rate of change of the manipulated variables as well, then we need to estimate the minimum settling time numerically.

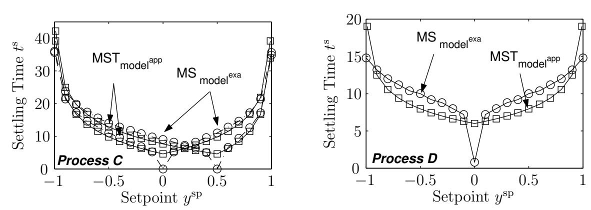


Fig. 3. Comparison of minimum settling time (MST) obtained via approximate model and exact model

Simulation results (see Fig. 3) have shown that the minimum settling time obtained from approximate first order plus time delay models is quite close to that obtained from the exact models. Processes considered are process C and D in Table 1. This is true for different initial conditions, set-point changes, and for different process dynamics. The same results also hold for the estimation of the minimum IAE, calculated in "open-loop". However, this approximation is not very satisfactory if the operating conditions are far from the initial operating point. To handle this inefficiency, one could use more accurate models, for example, a second order plus time delay model.

4. EXAMPLES

In this section, we will use two examples to illustrate the proposed method.

4.1 Example I

For simplicity, consider the integral process ($G(s) = 0.104 \times e^{-3s}/s$). PI controllers with different tunings (Ziegler-Nichols (ZN) and Tyreus-Luyben (TL) tunings), both with and without anti-reset windup (ARWU) schemes, are selected for comparison (see Table 2). τ_t in Table (2) is the tracking time constant (Ogunaike and Ray, 1994, p. 585). Only magnitude constraints on manipulated variables are considered: $|u| \leq 1$.

Table 2. PI tunings for example I

Tuning	K_c	τ_I	τ_t
TL	1.58	26.4	3.5
ZN	2.30	10.0	1.7

Closed-loop set-point step responses corresponding to different controllers are shown in Fig. (4).

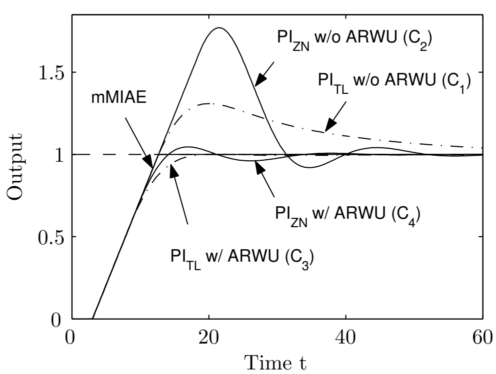


Fig. 4. Comparison of closed-loop response under different controllers

Set-point responses show that controller C_2 performs the worst because it gives the largest overshoot and longest settling time. Controller C_1 is a bit better as it has smaller overshoot. However, both controllers have comparable performances because they have much the same IAE and settling time values. Controllers C_3 and C_4 have performances very close to that of the modified minimum IAE (mMIAE) controller. However, controller C_3 is a bit better than controller C_4 as it has fewer oscillations and shorter settling time.

Based on the above qualitative analysis of the controller responses, we expect that $\eta_{C_2} <\approx \eta_{C_1} \ll 1$ and $\eta_{C_4} < \eta_{C_3} \simeq 1$. The performance index η given in Table (3), which is calculated without a process model, agrees very well with this expectation.

We can easily determine the poor performance of both controllers C_2 and C_1 with the index estimated without the knowledge of a process model. As performance index η is very close to 1 for controllers C_3 and C_4, further evaluation is necessary. The minimum settling time and minimum IAE are employed as the reference values in the calculation of performance index η. For simplicity, minimum settling time and minimum IAE are determined using the exact model in this example. The newly calculated performance index, given in Table (3), confirms that both controllers have very good performances. Controller C_3 has higher index than C_4 because it allows shorter settling time.

Table 3. Results and Comparison

	C_1	C_2	C_3	C_4
η w/o model	0.48	0.43	0.97	0.77
η w model	0.48	0.43	0.89	0.76

4.2 Example II

In this example, we consider the third order process (process C given in Table 1). The following unconstrained PI tunings are chosen for comparison purpose.

Table 4. PI tunings for example II

	PI_1	PI_2	PI_3
K_c	0.99	2.48	0.25
τ_I	9.54	8.03	5.52

Closed-loop set-point responses under different PI tunings are shown in Fig. (5). The constraints on manipulated variables considered in this example are $|u| \leq 1$, $|\triangle u| \leq 0.2$. Fig. 6(a) and (b) present the performance index calculated without and with an approximate model (1st order plus delay), respectively.

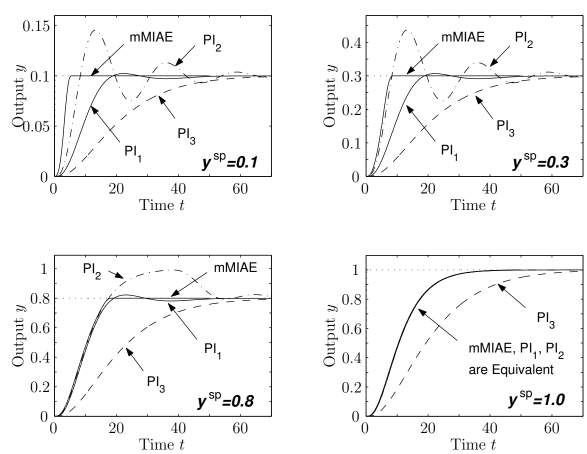
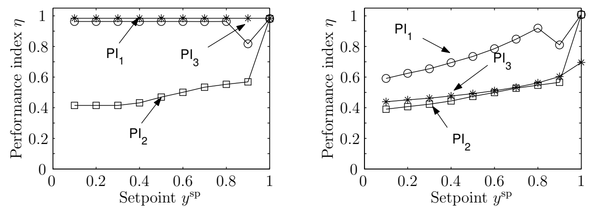


Fig. 5. Comparison of closed-loop set-point responses under different controllers

Fig. 6(a) indicates that the controller PI_2 is inadequate for $y^{sp} \leq 0.9$ as its performance index is very small. This is in accordance with the fact that PI_2 results in significantly oscillating set-point response (Fig. 5) with very large overshoot and long settling time.



(a) w/o model (b) w/ approx. model

Fig. 6. Comparison of performance index η of three different controllers (see Table 4)

Fig. 6(b) confirms that the controller PI_1, which has high performance index value in Fig. 6(a), is an acceptable one. This can be explained by the rather satisfactory set-point responses (Fig. 5) achieved by controller PI_1. On the other hand, Fig. 6(b) tells that controller PI_3 has very poor performance, although it leads to comparably high performance index as controller PI_1 given in Fig. 6(a). This is in agreement with the very sluggish set-point responses (Fig. 5) caused by PI_3. Moreover, Fig. 6(b) shows and verifies that, for maximum available set-point change, controllers PI_1 and PI_2 perform similarly as the optimal controller. We see from Fig. (5) that the set-point responses realized with controllers PI_1 and PI_2 coincide with that under optimal control for $y^{sp} = 1$.

This example clearly illustrates the dependence of controller's performance upon the set-point changes. By comparison with the optimal controller, constrained PI controller's performance turns to be better with the increase of set-point changes. This is due to the fact that more of the available forcing power of the manipulated variables are being exploited. Secondly, the performance index does not change much with set-point magnitudes when the set-point changes are relatively small. This is because the constraints are almost inactive in such cases, and, therefore, the set-point responses are similar.

5. CONCLUSIONS

In this paper, we introduced a new performance index for constrained controller performance assessment w.r.t. deterministic disturbances. It involves integral absolute error (IAE), overshoot, and response time.

A three-step framework is proposed. Steady-state offset is the concern of the first step. In the second step, the performance index is calculated directly from the closed-loop set-point response. No process model is required. If the performance index η is far less than 1, it indicates the controller has poor performance. However, if the index η is close to one, minimum settling time and minimum IAE benchmarks are necessary for further evaluation. In such cases, an approximate process model is required. We have shown that a simple model such as first order plus time delay provides adequate estimate for the needed minimum IAE and minimum settling time.

Remark: Current research is focused on set-point step responses only. We need and will deal with disturbance response in the future.

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