MULTI-OBJECTIVE ROBUST CONTROL OF AN EVAPORATION PROCESS

Wenjun Yan [*,1] **Yi Cao** [**]

[*] *Department of Systems Science and Systems Engineering, Zhejiang University, Hangzhou, 310027, China, email: yanwenjun@zju.edu.cn*
[**] *Department of Process and Systems Engineering, Cranfield University, Bedford, MK43 0AL, U.K., email: y.cao@cranfield.ac.uk*

Abstract: In this paper, a new multi-objective robust control design approach is applied to an evaporation process. The new approach, proposed in (Yan and Cao, 2002) extends the standard generalized-l_2 (Gl_2) control problem based on a new Lyaponov stability condition to a set of new linear matrix inequality (LMI) constraints such that the multi-objective robust control problem associated with robust stability and robust performance objectives can be less conservatively solved using computationally tractable algorithms. A comparison of simulation results with controllers designed by different techniques demonstrates the superiority of the new method.

Keywords: Generalized l_2 synthesis, Multi-objective optimization, Robust Controller, Discrete linear time-invariant system, Evaporation process

1. INTRODUCTION

In the last two decades, many analysis and synthesis approaches in control theory have been developed. Among those control strategies, multi-objective control and robust control have obtained more and more attractions. The former is to resolve the inherent trade-offs among conflictive design specifications. Many control strategies have been studied in this aspect, such as H_∞, H_2, mixed H_2/H_∞ and mixed L_1/H_∞. More details can be found in (Scherer *et al.*, 1997) and reference therein. Nominal systems are mainly considered with these methodologies.

On the other hand, due to uncertainty in practical systems, robustness of control systems has to be taken into account. In H_∞ design, disturbances and uncertainties are lumped into a single norm rather than bounded separately. This certainly leads to some conservatism. In contrast, the μ-synthesis technique overcomes the conservatism by introducing structured uncertainty blocks. However, the optimization has to be solved via a so-called D-K iteration, in which the joint convexity is not guaranteed although individual step(K-step or D-step) is convex. Hence, it may become computationally intractable (Skogestad and Postlethwaite, 1996) to achieve a globally optimal performance. As it was pointed out by Toker and Ozbay (1995), most robust synthesis problems proposed so far are NP-hard and computationally intractable. However, by combining the concepts of H_∞ optimization, linear matrix inequalities (LMI) and integral quadratic constraints, a convex solution to a large class of robust and optimal control problems, the generalized-l_2 (Gl_2) formulation has been proposed by D'Andrea (1999)

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recently. The Gl_2 problem can be represented in LMI formulations (Gahinet *et al.*, 1995) and has been successfully applied to an active suspension system (Wang and Wilson, 2001). The Gl_2 approach has the potential to get solutions less conservative than those obtained via a H_∞ control synthesis approach whilst the computations involved in solving a Gl_2-optimization problem is more tractable than those required by the μ-optimization.

Like other LMI-based methodologies, the standard Gl_2 formulation relies on Lyapunov stability condition and the solution depends on the Lyapunov symmetric matrix P. Furthermore, in multi-objective synthesis, the Lyapunov matrices P in different LMI's are always assumed to be identical for the sake of solvability. Hence, it will inevitably introduce some new conservativeness to the whole optimal solution. To reduce such conservativeness in multi-objective problem, Geromel*et al.* (1998; 1999*b*; 1999*a*) extended the Lyapunov inequalities (called Lyapunov-shaping paradigm) by introducing a new stability condition. Using this new stability condition, the matrix G, possibly non-symmetric, decouples the Lyapunov matrices and the dynamical matrices of controller. Therefore, less conservative solutions are expected from the *G-shaping* paradigm (de Oliverira *et al.*, 1999*a*). This technique has been applied to many different control synthesis problems, such as H_2 and H_∞ optimization problems.

In this work, the *G-shaping* paradigm has been extended further to the Gl_2 optimization and pole placement problem. The later has been proven to be effective to improve the transient behavior of the closed-loop system.

The paper is organized as follows: In section 2, dynamics of an industrial nonlinear process, a evaporator system, is introduced. Then, section 3 discusses the extended Gl_2 controller design in details, which include performance weight selection and uncertainty modelling. In section 4, simulation results with the Gl_2 controller are compared with those obtained under other controllers. Finally, a brief summary of the proposed method is provided.

2. EVAPORATION PROCESS

A forced-circulation evaporator, described by Newell and Lee (1989), is shown in Figure 1. This is a typical nonlinear plant with potential model uncertainties. The system has been tested by many effective control strategies since it was published, such as PID, loop-shaping and model predictive Control (Newell and Lee, 1989; Samyudia *et al.*, 1995; Maciejowski, 2002).

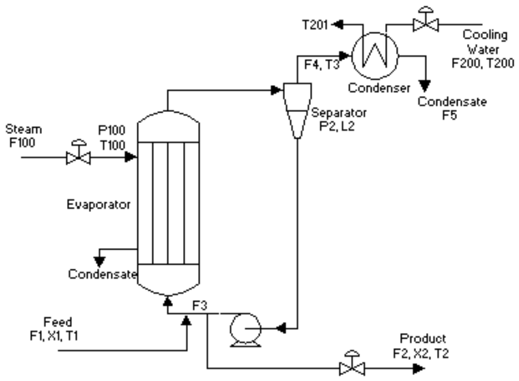

Fig. 1. Evaporator System

The nonlinear model of the plant is linearized at the nominal operating point as shown in Table 1, where S, M and D are variable abbreviations of State, Manipulated and Disturbance respectively. The corresponding linear state space representation is as follows:

$$\begin{bmatrix} \dot{L}_2 \\ \dot{X}_2 \\ \dot{P}_2 \end{bmatrix} = A \begin{bmatrix} L_2 \\ X_2 \\ P_2 \end{bmatrix} + B_1 \begin{bmatrix} P_{100} \\ F_2 \\ F_{200} \end{bmatrix} + B_2 \begin{bmatrix} F_1 \\ X_1 \\ T_1 \\ T_{200} \end{bmatrix}$$

where the matrices A, B_1 and B_2 are given as follows:

$$A = \begin{bmatrix} 0 & 0.0042 & 0.0075 \\ 0 & -0.1 & 0 \\ 0 & -0.0209 & -0.0558 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -0.0500 & -0.0019 & 0 \\ -1.2500 & 0 & 0 \\ 0 & 0.0096 & -0.0018 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.0467 & 0 & -0.0009 & 0 \\ 0.2500 & 0.5000 & 0 & 0 \\ 0.0164 & 0 & 0.0045 & 0.0360 \end{bmatrix}$$

Table 1. Description of Variables

Var	Description	Steady State	Type
L_2	Separator Level	1[m]	S
X_2	Product Composition	25%	S
P_2	Operating Pressure	50.5[kPa]	S
P_{100}	Steam Pressure	194.7[kPa]	M
F_2	Product Flow rate	2.0[kg/m]	M
F_{200}	Steam Flow rate	208.0[kg/m]	M
F_1	Feed Flow rate	10.0[kg/m]	D
X_1	Feed Composition	5.0%	D
T_1	Feed Temperature	40.0[^{o}C]	D
T_{200}	Cooling Water Temp	25.0[^{o}C]	D

The corresponding transfer functions of the system, G_p, from manipulated variables to outputs, and G_d, from disturbances to outputs, are denoted as:

$$G_p := \left[\begin{array}{c|c} A & B_1 \\ \hline I & 0 \end{array}\right], G_d := \left[\begin{array}{c|c} A & B_2 \\ \hline I & 0 \end{array}\right]$$

For the synthesis purpose, the input and output variables are scaled by divided by their steady-state values. Then, the scaled transfer functions are $G_P = M_y^{-1} G_p M_u$ and $G_D = M_y^{-1} G_d M_d$, where M_y, M_u and M_d are diagonal scaling matrices with corresponding values shown in Table 1.

The control design objective is to achieve the following design specifications under disturbances and uncertainty.

- All controlled variables should be within 2% of the desired final value within 20 min of an upset and their maximum variations should be less than 100%.
- Variations in the manipulated variables should be less than 100% of their steady- state values.
- The above specifications should be satisfied for nonlinear model with reasonable disturbances and measurement noises.

3. EXTENDED GL_2 CONTROL DESIGN

A multi-objective optimization problem based on *G-shaping* is considered in this section. The aim is to compute a dynamical output feedback controller to meet the performance requirements specified as above. More precisely, these specifications are defined as sensitivity functions and their corresponding performance subject to uncertainties. Three weighting functions are to be chosen for this problem.

3.1 Performance Weight Selection

Applying the procedure presented in (Skogestad and Postlethwaite, 1996), the following performance weight for SISO system is considered:

$$w_e(s) = \frac{s/M + \omega_b}{s + \omega_b \sigma}$$

It specifies a minimum bandwidth ω_b, a maximum peak of the sensitivity S less than M, a steady state error less than $\sigma < 1$, and that at frequencies lower than the bandwidth the sensitivity is required to reduce by at least $20dB/dec$. In order to reduce the steady-state error to zero, $\sigma = 0$ is selected. Hence, the weight is actually a PI controller. For the sake of simplicity and tractable computation, the final performance weight matrix is to take the diagonal form as follows:

$$W_e(s) = \mathbf{diag}\{\frac{25s + 25}{s + 10^{-6}}, \frac{25s + 25}{s + 10^{-6}}, \frac{40s + 10}{s + 10^{-6}}\}$$

In the meantime, as usual, the control input sensitivity weight, $W_u(s)$ is simply selected as a constant matrix:

$$W_u(s) = \mathbf{diag}\{0.1, 0.1, 0.3\}$$

3.2 Uncertainty

Due to the nonlinearity and complicity of the actual plant, control design based on a simplified linear model has to consider model uncertainties. Therefore, an uncertainty weight to meet the robust control requirement is to be selected.

In this case study, it is assumed that the actual plant, $\tilde{G}_P$ is subject to a multiplicative unstructured input uncertainty, $\triangle^u$, weighted by W_{rob}, i.e.:

$$\tilde{G}_P := (I + W_{rob} \triangle^u) G_P, \ \|\triangle^u\|_\infty \leq 1$$

Normally, the uncertainty weight for SISO systems can take a simple form as follows:

$$w_{rob}(s) = \frac{\tau s + r_0}{(\tau / r_\infty) s + 1}$$

where r_0 is the relative uncertainty at steady state, $1/\tau$ is approximately the frequency where the relative uncertainty reaches 100%, and r_∞ is the magnitude of the weight at higher frequencies.

In the evaporation process, the main uncertainty is caused by nonlinearity rather than the neglected dynamics. By performing simulation with different set points and disturbances, the corresponding parameters of the weight are determined as $\tau = 20, r_0 = 8, r_\infty = 1$. It implies that in this design, 800% uncertainty of static gain is permitted. The final robust weight matrix is chosen as follows:

$$W_{rob} = \frac{(s + 4)}{(s + 0.5)} \mathbf{diag}\{2, 8, 4\}$$

By combining the above three weights, an augmented plant (see Figure 2) is defined as follows:

$$\begin{bmatrix} z_\triangle \\ \begin{bmatrix} z_e \\ z_u \end{bmatrix} \\ e \end{bmatrix} := G \begin{bmatrix} u_\triangle \\ \begin{bmatrix} r \\ d \\ n \end{bmatrix} \\ u \end{bmatrix}$$

where r, y and e represent system input, output and error(measurement output), u, z_e and z_u represent control output and controlled outputs, d and n are disturbance and measurement noise, and $u_\triangle$ and $z_\triangle$ are the output and input of uncertainty respectively.

In order to get a low order controller, disturbance d and measurement noise n in the above formulation will be ignored in the design. It will be shown that this ignorance has no noticeable effect on the final results in this case study. Thus, the augmented plant, G can be derived as:

$$G := \begin{bmatrix} 0 & 0 & W_{rob} \\ \begin{bmatrix} -W_e G_P \\ 0 \end{bmatrix} & \begin{bmatrix} W_e \\ 0 \end{bmatrix} & \begin{bmatrix} -W_e G_P \\ W_u \end{bmatrix} \\ -G_P & I & -G_P \end{bmatrix} \quad (1)$$

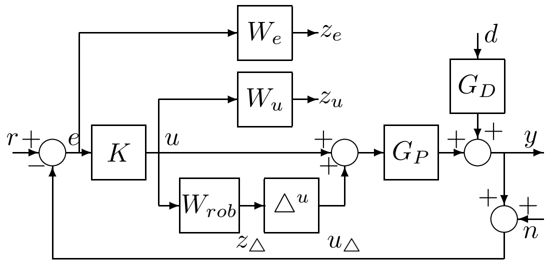

Fig. 2. Block Diagram Describing Weighted System with Multiplicative Uncertainty

3.3 Controller Synthesis

To use Gl_2 synthesis techniques, the augmented plant is represented in discrete state space form as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + B_w w(k) + B_u u(k) \\ z(k) &= C_z x(k) + D_{zw} w(k) + D_{zu} u(k) \\ y(k) &= C_y x(k) + D_{yw} w(k) \end{aligned} \tag{2}$$

where, $x(k) \in \mathbb{R}^n$, $w(k) \in \mathbb{R}^{n_w}$, $u(k) \in \mathbb{R}^{n_u}$,$z(k) \in \mathbb{R}^{n_z}$ and $y(k) \in \mathbb{R}^{n_y}$ are discrete state, exogenous input, control input, controlled output and measurement output respectively. An output feedback controller $K(\zeta)$ defined in the following state-space form is to be designed:

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c y(k) \\ u(k) &= C_c x_c(k) + D_c y(k) \end{aligned} \tag{3}$$

The closed-loop transfer function from w to z is defined as follows:

$$T_{zw}(\zeta) := \left[\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline \tilde{C} & \tilde{D} \end{array}\right]$$

where the closed-loop system matrices are as follows:

$$\begin{aligned} \tilde{A} &:= \begin{bmatrix} A + B_u D_c C_y & B_u C_c \\ B_c C_y & A_c \end{bmatrix} \\ \tilde{B} &:= \begin{bmatrix} B_w + B_u D_c D_{yw} \\ B_c D_{yw} \end{bmatrix} \\ \tilde{C} &:= \begin{bmatrix} C_z + D_{zu} D_c C_y & D_{zu} C_c \end{bmatrix} \\ \tilde{D} &:= \begin{bmatrix} D_{zw} + D_{zu} D_c D_{yw} \end{bmatrix} \end{aligned}$$

In order to extend the Gl_2 optimization problem in (D'Andrea, 1999) to multi-objective synthesis based on G-shaping paradigm (Geromel *et al.*, 1998), the disturbance set $\mathcal{D}$ and criterion set $\mathcal{E}$ are reconstructed as follows

$$\mathcal{D} := \{d_k \in l_2 : \|d_k\| \leq 1, k \in [1, m]\}$$

$$\mathcal{E} := \{z_l \in l_2 : \|z_l\| \leq 1, l \in [1, n]\}$$

These sets result in the following inequalities

$$X := x_1 I_{d_1} \oplus x_2 I_{d_2} \oplus \cdots \oplus x_m I_{d_m} > 0, \sum_{k=1}^{m} x_k \leq \mu \tag{4}$$

$$Y := y_1 I_{z_1} \oplus y_2 I_{z_2} \oplus \cdots \oplus y_n I_{z_n} > 0, \sum_{l=1}^{n} y_l \leq \mu \tag{5}$$

where, $\oplus$ stands for direct sum of matrices, i.e.

$$A \oplus B = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$$

I_{d_k} and I_{z_l} are the identity matrices with the same dimensions as $d_k d_k'$ and $z_l z_l'$ respectively. The Gl_2 optimization problem is to find a controller such that $\|T_{zw}(\zeta)\|_{Gl_2}^2 < \mu$, or equivalently, $\left\|X^{-1/2} T_{zw} Y^{-1/2}\right\|^2 < \mu$. Therefore, by new parameterization, the following central result to controller synthesis can be obtained (Yan and Cao, 2002).

Theorem 1. All controllers in the form (3) such that the inequality $\left\|X^{-1/2} T_{zw} Y^{-1/2}\right\|^2 < \mu$ holds are parameterized in (6) shown at the top of the next page.

In (6) the matrices $\Xi, L, \Pi, F, Q, R, S, J$ and the symmetric matrices P and H are the decision variables. The proofs of this and the coming theorem can be found in (Yan and Cao, 2002).

Remark 1. Given matrices $\Xi, L, \Pi, F, Q, R, S, J$ from the theorem, a feasible controller is available by choosing V and U nonsingular such that $VU = S - \Pi\Xi$ and calculating the following matrices in order

$$\begin{aligned} D_c &:= R \\ C_c &:= (L - RC_y\Xi)U^{-1} \\ B_c &:= V^{-1}(F - \Pi B_u R) \\ A_c &:= V^{-1}[Q - \Pi(A + B_u D_c C_y)\Xi]U^{-1} \\ &\quad - B_c C_y \Xi U^{-1} - V^{-1}\Pi B_u C_c \end{aligned} \tag{7}$$

Remark 2. The most important feature of this new framework is that the feasible controller (7) does not depend on any of the Lyapunov matrices P, J or H so that it can reduce the conservatism involved in standard multi-objective optimization problems and allow for more flexible and accurate specification of the closed loop behavior. Particularly, if $P = \Xi = \Xi'$, $J = S = I$, $H = \Pi = \Pi'$, it obviously encompasses the results obtained in (D'Andrea, 1999). Moreover, if the matrices X and Y in (6) are both set to μI, then the extended Gl_2 synthesis theorem is reduced to the standard H_∞ theorem. Therefore, the new framework is indeed a generalization of the standard Gl_2 framework.

$$\begin{pmatrix} P & J & A\Xi + B_uL & A + B_uRC_y & B_w + B_uRD_{yw} & 0 \\ (\cdot)' & H & Q & \Pi A + FC_y & \Pi B_w + FD_{yw} & 0 \\ (\cdot)' & (\cdot)' & \Xi + \Xi' - P & I + S' - J & 0 & \Xi' C_z' + L' D_{zu}' \\ (\cdot)' & (\cdot)' & (\cdot)' & \Pi + \Pi' - H & 0 & C_z' + C_y' R' D_{zu}' \\ (\cdot)' & (\cdot)' & (\cdot)' & (\cdot)' & X & D_{zw}' + D_{yw}' R' D_{zu}' \\ (\cdot)' & (\cdot)' & (\cdot)' & (\cdot)' & (\cdot)' & Y \end{pmatrix} > 0 \qquad (6)$$

Normally, in order to improve the transient performance, a suitable closed-loop poles assignment is also necessary. Some closed-loop system poles constraints have been addressed recently in LMI form, and a general description for the continuous time system is presented in (Chilali and Gahinet, 1996). For a discrete time system, the following simple and effective constraint is defined to reduce the time response overshoot:

$$\mathcal{C}_{\mathcal{D}}(z_0, \rho) := \{\lambda \in \mathbb{C}, |\lambda + z_0| < \rho\} \qquad (8)$$

Analogous to the above theorem, the extended poles placement LMI constraint for G-shaping is derived from (Yedavalli, 1993):

Theorem 2. (Poles Placement). All controllers in the form (3) such that poles of the closed-loop system satisfying (8) are parameterized by the following LMI

$$\begin{bmatrix} \rho P & \rho J & (A + z_0 I)\Xi + B_u L & A + B_u RC_y + z_0 I \\ (\cdot)' & \rho H & Q + z_0 S & \Pi(A + z_0 I) + FC_y \\ (\cdot)' & (\cdot)' & \rho\Xi + \rho\Xi' - \rho P & \rho I + \rho S' - \rho J \\ (\cdot)' & (\cdot)' & (\cdot)' & \rho\Pi + \rho\Pi' - \rho H \end{bmatrix} > 0$$

Combining these two theorems, the main results of the paper is stated as follows: The multi-objective Gl_2 synthesis problem is to find an optimal controller such that $\|T_{zw}(\zeta)\|^2_{Gl_2} < \mu$ and the closed loop poles located inside the sub-region $\mathcal{C}_{\mathcal{D}}$.

This synthesis framework achieves the optimal performance of $T_{zw}(\zeta)$ while guaranteeing a certain level of robust stability and satisfactory of the system transient behavior. If the performance level μ is given, then the optimization problem is only to find the feasible solution under constraints stated in Theorems 1 & 2; otherwise, by absorbing μ into the corresponding two inequalities and casting it as another decision variable, the problem becomes a minimization problem with constraints. The programs for H_∞ and Gl_2 synthesis based on *G-shaping* have been developed in MATLAB using LMI Toolbox (Gahinet *et al.*, 1995).

4. SIMULATION RESULT COMPARISON

Based on the augmented plant of the evaporation process, the uncertainty block $\triangle$ for Gl_2(including two fictitious block) can be simply partitioned as follows:

$$\triangle := \triangle_{Gl_2} = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \end{bmatrix}$$

where $\delta_{ij} \in \mathbb{C}$, and $|\delta_{ij}| \leq 1, i \in \{1,2\}, j \in \{1,2,3\}$. Then the sets $\mathcal{D}$ and $\mathcal{E}$, related to the uncertainty $\triangle_{Gl_2}$ are defined as:

$$\mathcal{D} := \{d_i \in L_2 : \|d_i\| \leq 1, i \in \{1,2\}\}$$

$$\mathcal{E} := \{z_j \in L_2 : \|z_j\| \leq 1, j \in \{1,2,3\}\}$$

Consequently, inequality (4) and (5) become

$$X = x_1 I_3 \oplus x_2 I_3, Y = y_1 I_3 \oplus y_2 I_3 \oplus y_3 I_3$$

$$x_1 + x_2 \leq \mu, y_1 + y_2 + y_3 \leq \mu$$

where $x_i, y_j \in \mathbb{R}^+, i \in \{1,2\}, j \in \{1,2,3\}$. Then Gl_2 synthesis problem is simplified as:

$$\begin{aligned} \|T_{zw}\| &= \sup_{z \in \mathcal{E}, d \in \mathcal{D}} \langle z, T_{zw} d \rangle \\ &= \sup_{u_\triangle, r \in \mathcal{D}} \{\|T_{zw} u_\triangle\| + \|T_{zw} r\|\} \end{aligned}$$

Here we lump the weighted uncertainty outputs into $\|z_\triangle\|$, the weighted errors into $\|z_e\|$, the weighted control inputs into $\|z_u\|$, and the uncertainty inputs into $\|u_\triangle\|$, system set-points of L_2, X_2, P_2 into $\|r\|$. In μ-tools, δ_{12}, δ_{13} and δ_{21} are set 0. Moreover, some other partitions are also allowable in this framework. In fact, the 9 criterion elements and 6 disturbance elements can all be individual blocks, i.e., 'element by element'. Therefore, Gl_2 has more flexibility than H_∞ or μ-tools in dealing with uncertainty.

For the purpose of comparison, the above design problem is re-treated as a standard H_∞ (Scherer *et al.*, 1997) problem and a G-shaping H_∞ (de Oliverira *et al.*, 1999*a*) problem by constructing some fictitious performance blocks between z_e, z_u and r. Using the weights defined in section 3, two feasible nominal controllers K_1, K_2 and a robust optimal controller K_3 are obtained for standard H_∞, G-shaping H_∞ (GH_∞) and G-shaping Gl_2 optimization problems respectively.

In simulation, it is assumed that the setpoints of X_2 and P_2 are required to change from 25 to 15 and from 50.5 to 70, respectively while preserving L_2 unchanged for practical reason. Simultaneously, sinusoid disturbances with 20% magnitude variation are applied to both F_1 and X_1 with frequencies varying from 1 to 60Hz, a 20% step increase is applied to T_1 and 20% random

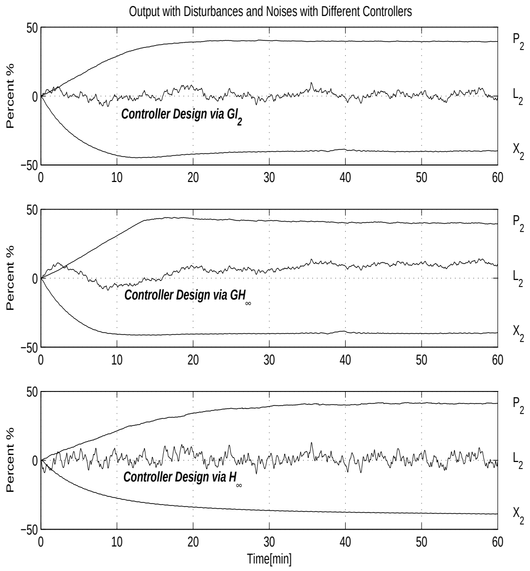

Fig. 3. Response to Step Changes in Setpoints

signal is applied to T_{200}. 20% random noises in all three measurement variables are also assumed.

Figure 3 illustrates the simulation results with different controllers. It demonstrates that the Gl_2 controller can produce better time performance than standard H_∞ and GH_∞ in this application. Under the same simulation condition, it is examined that PID control developed in (Newell and Lee, 1989) and Model predictive control developed in (Maciejowski, 2002) even cannot stabilize the system under such severe disturbance and noise conditions. It should be noted that if the μ-tools is applied to design this controller, it is quite difficult to choose properly scaled weight to get reasonable iterative results.

5. CONCLUSION

A new Gl_2 multiobjective robust control synthesis approach has been presented. It is jointly based on the work of Gl_2 by D'Andrea (1999) and G-shaping paradigm by Geromel *et al.* (1998). The new method can deal with disturbance and uncertainty simultaneously in a unified LMI form rather than $D - K$ iteration in μ synthesis. Based on G-shaping paradigm, the controller parameterization does not depend on the Lyapunov symmetric matrix P. It remains in the inequality just as an extra optimization variable. This results in a reduction of the conservativeness involved in a standard design framework. The freedom of design is improved as well. Although the approach presented in this paper is for discrete time system, it is easy to extend these concepts to continuous time system. Application to the evaporator system demonstrates that it can effectively improve the robustness and performance of a control system.

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