VARIANCE-CONSTRAINED FILTERING FOR UNCERTAIN STOCHASTIC SYSTEMS WITH MISSING MEASUREMENTS

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Abstract: In this paper, we consider a new filtering problem for linear uncertain discrete-time stochastic systems with missing measurements. The parameter uncertainties are allowed to be norm-bounded and enter into the state matrix. The system measurements may be unavailable (i.e., missing data) at any sample time, and the probability of the occurrence of missing data is assumed to be known. The purpose of this problem is to design a linear filter such that, for all admissible parameter uncertainties and all possible incomplete observations, the error state of the filtering process is mean square bounded, and the steady-state variance of the estimation error of each state is not more than the individual prescribed upper bound. It is shown that, the addressed filtering problem can effectively be solved in terms of the solutions of a couple of algebraic Riccati-like inequalities or linear matrix inequalities. The explicit expression of the desired robust filters is parameterized, and an illustrative numerical example is provided to demonstrate the usefulness and flexibility of the proposed design approach.

Keywords: Kalman filtering; robust filtering; incomplete observation; missing signal; linear matrix inequality.

1. INTRODUCTION

The well-known Kalman filtering is one of the most successful H_2 filtering approaches widely used in various fields of signal processing and control. However, it has now been recognized that the standard Kalman filtering algorithm will generally not guarantee satisfactory performance when there exist parameter uncertainties in the system model. To improve the robustness, in recent years, many alternative design methods have been developed, among them we just mention the H_∞ filtering and robust filtering approaches, see for example Fu, *et al.*, 2001, Palhares, *et al.*, 2001, Shaked, *et al.*, 2001, and references therein.

In practical engineering, however, it is often the case that, for a class of filtering problems such as the tracking of a maneuvering target, the performance objectives are naturally described as the upper bounds on the error variances of estimation, see *e.g.* Skelton and Iwasaki (1993) and Yaz and Skelton (1991). Unfortunately, it is usually difficult to utilize traditional methods to deal with this class of *constrained variance* filtering problems. For instance, the theory of weighted least-squares estimation minimizes a weighted scalar sum of the error variances of the state estimation, but minimizing a scalar sum does not ensure

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that the multiple variance requirements will be satisfied (Stengel, 1986). Motivated by this fact, a novel filtering method, namely, error covariance assignment (ECA) theory (see *e.g.* NaNacara and Yaz, 1997 and Yaz and Skelton, 1991), was developed to provide a closed form solution for directly assigning the specified steady-state estimation error covariance. Subsequently, the idea of ECA theory has been applied in investigating the so-called variance-constrained filtering problems for parameter uncertain systems (Wang and Huang, 2000), sampled-data systems (Wang, *et al.* 2001), and bilinear systems (Wang and Qiao, 2001), where a prespecified upper bound is placed onto the steady-state estimation error variance.

So far, in the literature mentioned above, it is assumed that the measurements always contain the signal. However, in practical applications such as target tracking, there may be a nonzero probability that any observation consists of noise alone if the target is absent, *i.e.*, the measurements are not consecutive but contain missing observations. The missing observations are caused by a variety of reasons, *e.g.*, the high maneuverability of the tracked target, a certain failure in the measurement, intermittent sensor failures, accidental loss of some collected data, or some of the data may be jammed or coming from a high noise environment, *etc.*, see Rosen and Porat (1989).

Basically, the standard definition of covariance in the data statistical analysis does not directly apply if some of the measurements are unavailable. Thus, the popular robust and/or H_∞ filtering approaches, which are dependent on the system output covariance, do not suit the case when there are missing measurements. For filtering problem, only a very limited number of filter design methods for system output signals with missing measurements have been developed. In Kassel and Baxa (1988), the effect of missing data on the steady-state performance of a tracking filter was shown to be crucial. Chen (1990) proposed a suboptimal Kalman filtering method to cope with the case of measurement data missing. A measurement model with a binary multiplicative noise was employed in NaNacara and Yaz (1997) to study the filter design problem with error covariance assignment. Some more relevant references can also be found in Chow and Birkemeier (1990) and NaNacara and Yaz (1997). Up to now, to the best of the authors' knowledge, the issue of *variance-constrained* filtering on *parameter uncertain* systems with *missing measurements* has not been fully investigated and remains to be important and challenging.

In this paper, we are concerned with the variance-constrained filtering problem for uncertain discrete-time stochastic systems with probabilistic missing measurements. We aim at designing a linear filter such that, for all admissible parameter uncertainties and all possible incomplete observations, 1) the error state of the filtering process is mean square bounded; and 2) the steady-state variance of the estimation error of each state is not more than the individual prescribed upper bound. It is shown that, the solution to the addressed filtering problem is related to a couple of algebraic Riccati-like inequalities or linear matrix inequalities. The explicit expression of the desired robust filters is derived, and a numerical example is offered to illustrate the usefulness of the proposed design approach.

Notation. The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (*i.e.*, the filtration contains all P-null sets and is right continuous). $\mathcal{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. Prob$\{\cdot\}$ means the occurrence probability of the event "$\cdot$".

2. PROBLEM FORMULATION AND ASSUMPTIONS

Consider the following linear uncertain discrete-time stochastic system

$$x(k+1) = (A + \Delta A)x(k) + w(k), \qquad (1)$$

and the measurement equation

$$y(k) = \gamma(k)Cx(k) + v(k) \qquad (2)$$

where $x \in \mathbb{R}^n$ is a state vector, $y \in \mathbb{R}^p$ is a measured output vector, and A and C are known constant matrices. $w(k) \in \mathbb{R}^n$ and $v(k) \in \mathbb{R}^p$ are mutually uncorrelated zero mean Gaussian white noise sequences with respective covariances $W > 0$ and $V > 0$. The initial state $x(0)$ has the mean $\bar{x}(0)$ and covariance $P(0)$, and is uncorrelated with both $w(k)$ and $v(k)$. ΔA is a real-valued perturbation matrix being of the following form

$$\Delta A = MFN, \quad FF^T \leq I \qquad (3)$$

and M and N are known constant matrices of appropriate dimensions which specify how the elements of the nominal matrix A are affected by the uncertain parameters in F. The uncertainties in ΔA are said to be admissible if (3) holds. The stochastic variable $\gamma(k) \in \mathbb{R}$ is a Bernoulli distributed white sequence taking values on 0 and 1 with

$$\text{Prob}\{\gamma(k) = 1\} = \mathcal{E}\{\gamma(k)\} := \bar{\gamma} \quad (4)$$

where $\bar{\gamma}$ is a known positive constant, and $\gamma(k) \in \mathbb{R}$ is assumed to be independent of $w(k)$, $v(k)$, and $x(0)$. Therefore, we have

$$\text{Prob}\{\gamma(k) = 0\} = 1 - \bar{\gamma} \quad (5)$$

$$\sigma_{\gamma}^2 := \mathcal{E}\{(\gamma(k) - \bar{\gamma})^2\} = (1 - \bar{\gamma})\bar{\gamma} \quad (6)$$

Remark 1. The system measurement mode (2) has subsequently been used in many papers (see *e.g.* NaNacara and Yaz, 1997) to account for the probabilistic measurement missing.

Assumption 1. The matrix A is nonsingular and Schur stable (*i.e.*, all eigenvalues of A are located within the unit circle in the complex plane).

Introducing now a new stochastic sequence

$$\tilde{\gamma}(k) := \gamma(k) - \bar{\gamma}, \quad (7)$$

we can see that $\tilde{\gamma}(k)$ is a scalar zero mean stochastic sequence with variance

$$\sigma_{\tilde{\gamma}}^2 = (1 - \bar{\gamma})\bar{\gamma}. \quad (8)$$

The linear full-order filter considered in this paper is of the following structure

$$\hat{x}(k+1) = G\hat{x}(k) + K(y(k) - \bar{\gamma}C\hat{x}(k)) \quad (9)$$

where $\hat{x}(k)$ stands for the state estimate, and G and K are the filter parameters to be scheduled.

The steady-state estimation error covariance is defined by

$$P := \lim_{k\to\infty} P(k) := \lim_{k\to\infty} \mathcal{E}[e(k)e^T(k)], \quad (10)$$

where $e(k) = x(k) - \hat{x}(k)$.

From (1)-(2), (7) and (9), we have $y(k) - \bar{\gamma}C\hat{x}(k) = \tilde{\gamma}(k)Cx(k) + \bar{\gamma}Ce(k) + v(k)$, and subsequently

$$\begin{aligned} e(k+1) =& (A + \Delta A - G - \tilde{\gamma}(k)KC)x(k) \\ &+ (G - \bar{\gamma}KC)e(k) + w(k) - Kv(k). \end{aligned} \quad (11)$$

Define $x_f(k) := \begin{bmatrix} x^T(k) & e^T(k) \end{bmatrix}^T$, and

$$A_f := \begin{bmatrix} A & 0 \\ A - G - \tilde{\gamma}(k)KC & G - \bar{\gamma}KC \end{bmatrix}, \quad (12)$$

$$A_n := \begin{bmatrix} A & 0 \\ A - G & G - \bar{\gamma}KC \end{bmatrix}, \quad J := \begin{bmatrix} 0 & 0 \\ \sigma_{\tilde{\gamma}}KC & 0 \end{bmatrix} \quad (13)$$

$$M_f := \begin{bmatrix} M \\ M \end{bmatrix}, \quad N_f := \begin{bmatrix} N & 0 \end{bmatrix}, \quad \Delta A_f = M_f F N_f, \quad (14)$$

$$W_f := B_f B_f^T := \begin{bmatrix} W & W \\ W & W + KVK^T \end{bmatrix}, \quad (15)$$

$$X(k) := \mathcal{E}[x_f(k)x_f^T(k)] := \begin{bmatrix} X_{xx}(k) & X_{xe}(k) \\ X_{xe}^T(k) & X_{ee}(k) \end{bmatrix}. \quad (16)$$

Considering (1) and (11), we obtain the following augmented system

$$x_f(k+1) = (A_f + \Delta A_f)x_f(k) + B_f w_f(k), \quad (17)$$

where $w_f(k)$ denotes a zero mean Gaussian white noise sequence with unity intensity $I > 0$.

Remark 2. It is mentionable that there is a stochastic variable $\tilde{\gamma}(k)$ involved in A_f, which reflects the characteristic of the missing measurement for the addressed filtering problem, and the augmented system (17) is therefore essentially a stochastic parameter system. Note that robust filtering problem for stochastic parameter systems has not gain much attention in the literature.

Using the statistics of the noises $w(k)$, $v(k)$ and, in particular, $\tilde{\gamma}(k)$, the state covariance $X(k)$ defined in (16) is found to satisfy

$$\begin{aligned} X(k+1) =& (A_n + \Delta A_f)X(k)(A_n + \Delta A_f)^T \\ &+ JX(k)J^T + W_f \end{aligned} \quad (18)$$

We know from Agniel and Jury (1971) and DeKoning (1984) that, if the state of the system (17) is mean square bounded, the steady-state covariance X of the system (17) defined by

$$X := \lim_{k\to\infty} X(k) \quad (19)$$

exists and satisfies the following discrete-time modified Lyapunov equation

$$\begin{aligned} X =& (A_n + \Delta A_f)X(A_n + \Delta A_f)^T \\ &+ JXJ^T + W_f. \end{aligned} \quad (20)$$

Remark 3. It follows from Agniel and Jury (1971) and DeKoning (1984) that, there exists a unique symmetric positive semi-definite solution to (20) if and only if

$$\rho\big\{(A_n + \Delta A_f) \otimes (A_n + \Delta A_f) + J \otimes J\big\} < 1 \quad (21)$$

where ρ is the spectral radius and $\otimes$ is the Kronecker product. Furthermore, we also know from Agniel and Jury (1971) and DeKoning (1984) that the condition (21) is equivalent to the mean square boundedness of the state of the system (17). Hence, we conclude here that, if there exists a positive definite solution to the equation (20), then (21) holds, and the convergence of $X(k)$ in (16) will be guaranteed to a constant value X.

The purpose of this paper is to design the filter parameters, G and K, such that for all admissible perturbations ΔA, 1) the state of the augmented system (17) is mean square bounded, *i.e.*, (21) holds; and 2) the steady-state error covariance X_{ee} satisfies

$$[X_{ee}]_{ii} \le \alpha_i^2, \quad i = 1, 2, \cdots, n. \quad (22)$$

where $[X_{ee}]_{ii}$ means the steady-state variance of the ith error state, and α_i^2 $(i = 1, 2, \cdots, n)$ denotes the prespecified steady-state error estimation variance constraint on the ith state.

In the next section, we will first characterize an upper bound on the steady-state error covariance X satisfying (20) in terms of some free parameters, and let this upper bound meet the prespecified variance constraints, and then we will parameterize all desired filter gains with which the resulting steady-state error covariance is not more than the obtained upper bound.

3. MAIN RESULTS AND PROOFS

Lemma 1. Let a positive scalar $\varepsilon > 0$ and a positive definite matrix $Q_f > 0$ be such that $N_f Q_f N_f^T < \varepsilon I$, and $\Delta A_f = M_f F N_f$ with $FF^T \le I$. Then

$$(A_n + \Delta A_f) Q_f (A_n + \Delta A_f)^T \le A_n (Q_f^{-1} - \varepsilon^{-1} N_f^T N_f)^{-1} A_n^T + \varepsilon M_f M_f^T \quad (23)$$

holds for all admissible perturbations ΔA.

Lemma 2. (Wang and Huang, 2000) For a given negative definite matrix $\Pi < 0$ $(\Pi \in \mathbb{R}^{n\times n})$, there always exists a matrix $L \in \mathbb{R}^{n\times p}$ $(p \le n)$ such that $\Pi + LL^T < 0$.

Lemma 3. (Matrix Inverse Lemma) Let A, B, C and D be given matrices of appropriate dimension with A, D, and $D^{-1} + CA^{-1}B$ being invertible, then $(A + BDC)^{-1} = A^{-1} - A^{-1}B(D^{-1} + CA^{-1}B)^{-1}CA^{-1}$.

Lemma 4. (Schur complement) Given constant matrices Ω_1, Ω_2, Ω_3 where $\Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0.$$

For presentation convenience, we denote:

$$\Phi := (A - G)(P_1^{-1} - \varepsilon^{-1} N^T N)^{-1} (A - G)^T + \varepsilon M M^T + W, \quad (24)$$

$$R := (\bar{\gamma}^2 + \sigma_{\bar{\gamma}}^2) C P_2 C^T + V, \quad (25)$$

$$\Pi := \Phi + G P_2 G^T - P_2 - \bar{\gamma}^2 G P_2 C^T R^{-1} C P_2 G^T, \quad (26)$$

where $\bar{\gamma}$ and $\sigma_{\bar{\gamma}}$ are defined in (4) and (8), respectively.

Theorem 1. Assume that there exists a positive scalar $\varepsilon > 0$ such that the following two quadratic matrix inequalities

$$A P_1 A^T - P_1 + A P_1 N^T (\varepsilon I - N P_1 N^T)^{-1} N P_1 A^T + \varepsilon M M^T + W < 0 \quad (27)$$

$$\Pi = \Phi + G P_2 G^T - P_2 - \bar{\gamma}^2 G P_2 C^T R^{-1} C P_2 G^T < 0 \quad (28)$$

respectively have positive definite solutions $P_1 > 0$ $(N P_1 N^T \le \varepsilon I)$ and $P_2 > 0$, where

$$G = A + (\varepsilon M M^T + W)(A^{-1})^T \cdot (P_1^{-1} - \varepsilon^{-1} N^T N). \quad (29)$$

Moreover, let $L \in \mathbb{R}^{n\times p}$ $(p \le n)$ be an arbitrary matrix satisfying $\Pi + LL^T < 0$ (see Lemma 2), and $U \in \mathbb{R}^{p\times p}$ be an arbitrary orthogonal matrix (*i.e.*, $UU^T = I$). Then, the filter (9) with the parameters determined by (29) and

$$K = \bar{\gamma} G P_2 C^T R^{-1} + L U R^{-1/2}, \quad (30)$$

will be such that, for all admissible perturbations ΔA, 1) the state of the augmented system (17) is mean square bounded; 2) the steady-state error covariance X_{ee} meets $X_{ee} < P_2$.

Proof: Define $P_f := \mathrm{diag}(P_1, P_2)$. Then, it follows directly from Lemma 1 and the definitions (24)-(26) that

$$\begin{aligned}&(A_n + \Delta A_f) P_f (A_n + \Delta A_f)^T - P_f + J P_f J^T \\ &+ W_f \le A_n (P_f^{-1} - \varepsilon^{-1} N_f^T N_f)^{-1} A_n^T + \varepsilon M_f M_f^T \\ &- P_f + J P_f J^T + W_f := \Psi := \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{12}^T & \Psi_{22} \end{bmatrix} \quad (31)\end{aligned}$$

where

$$\Psi_{11} = A(P_1^{-1} - \varepsilon^{-1} N^T N)^{-1} A^T - P_1 + \varepsilon M M^T + W, \quad (32)$$

$$\Psi_{12} = A(P_1^{-1} - \varepsilon^{-1} N^T N)^{-1} (A - G)^T + \varepsilon M M^T + W, \quad (33)$$

$$\begin{aligned}\Psi_{22} = &(A - G)(P_1^{-1} - \varepsilon^{-1} N^T N)^{-1} (A - G)^T \\ &+ (G - \bar{\gamma} K C) P_2 (G - \bar{\gamma} K C)^T \\ &+ \varepsilon M M^T - P_2 + \sigma_{\bar{\gamma}}^2 K C P_2 C^T K^T \\ &+ W + K V K^T. \quad (34)\end{aligned}$$

It follows immediately from Lemma 3 that

$$(P_1^{-1} - \varepsilon^{-1} N^T N)^{-1} = P_1 + P_1 N^T (\varepsilon I - N P_1 N^T)^{-1} N P_1$$

and therefore the inequality (27) implies that $\Psi_{11} < 0$. Moreover, substituting the expression of G in (29) into (33) leads to $\Psi_{12} = 0$ easily.

Next, we shall consider Ψ_{22}. By using the definitions (24)-(26), we can rearrange (34) as follows

$$\begin{aligned}\Psi_{22} &= \Phi + (G - \bar{\gamma}KC)P_2(G - \bar{\gamma}KC)^T - P_2 \\ &\quad + \sigma_{\bar{\gamma}}^2 KCP_2C^TK^T + KVK^T \\ &= \Phi + GP_2G^T - P_2 + K[(\bar{\gamma}^2 + \sigma_{\bar{\gamma}}^2)CP_2C^T \\ &\quad + V]K^T - \bar{\gamma}GP_2C^TK^T - \bar{\gamma}KCP_2G^T \\ &= \Phi + GP_2G^T - P_2 - \bar{\gamma}^2GP_2C^TR^{-1}CP_2G^T \\ &\quad + (KR^{1/2} - \bar{\gamma}GP_2C^TR^{-1/2}) \\ &\quad \cdot (KR^{1/2} - \bar{\gamma}GP_2C^TR^{-1/2})^T \\ &= \Pi + (KR^{1/2} - \bar{\gamma}GP_2C^TR^{-1/2}) \\ &\quad \cdot (KR^{1/2} - \bar{\gamma}GP_2C^TR^{-1/2})^T. \end{aligned} \tag{35}$$

Noticing the expression of $K = \bar{\gamma}GP_2C^TR^{-1} + LUR^{-1/2}$ in (30) and the fact that $UU^T = I$, we have

$$\begin{aligned}&(KR^{1/2} - \bar{\gamma}GP_2C^TR^{-1/2}) \\ &\cdot(KR^{1/2} - \bar{\gamma}GP_2C^TR^{-1/2})^T = LL^T.\end{aligned}$$

Thus, it follows from (35), the definition of the matrix L ($L \in \mathbb{R}^{n\times p}$) and the inequality (28) that $\Psi_{22} = \Pi + LL^T < 0$.

To this end, we can conclude that $\Psi < 0$. Therefore, it follows from (31) that

$$\begin{aligned}(A_n + \Delta A_f)P_f(A_n + \Delta A_f)^T - P_f + JP_fJ^T \\ \le -W_f + \Psi < 0\end{aligned} \tag{36}$$

which leads to (21). As discussed earlier Remark 3, we know that the state of the augmented system (17) is mean square bounded, and there exists a symmetric positive semi-definite solution to (20). The first claim of this theorem is then proved.

Furthermore, subtract (20) from (36) to give

$$\begin{aligned}(A_n + \Delta A_f)(P_f - X)(A_n + \Delta A_f)^T - (P_f - X) \\ +J(P_f - X)J^T \le \Psi < 0\end{aligned} \tag{37}$$

which indicates again from Remark 3 that $P_f - X \ge 0$ and therefore

$$X_{ee} = [X]_{22} \le [P_f]_{22} = P_2$$

This completes the proof of this theorem.

Remark 4. It is clear from Theorem 1 that, if the quadratic matrix inequalities (27)(28) respectively have positive definite solutions $P_1 > 0$, $P_2 > 0$, and $P_2 > 0$ satisfies

$$[P_2]_{ii} \le \alpha_i^2, \qquad i = 1, 2, \cdots, n, \tag{38}$$

then the filter (9) determined by (29)-(30) will be such that: 1) the state of the augmented system (17) is mean square bounded; and 2) $[X_{ee}]_{ii} < [P_2]_{ii} \le \alpha_i^2$, $i = 1, 2, \cdots, n$. Hence, the design objective of variance-constrained robust filter with missing measurements will be accomplished. Note that the existence of a positive definite solution to (27) implies the asymptotical Schur stability of system matrix A, and the nonsingularity of A is required in the expression (29). This means, the Assumption 1 should hold.

We now briefly discuss the solvability of the quadratic matrix inequalities (27)-(28). By using the Schur Lemma (Lemma 4), we can transform (27) into the following linear matrix inequality (LMI):

$$\begin{bmatrix} AP_1A^T - P_1 + \varepsilon MM^T + W & AP_1N^T \\ NP_1A^T & -\varepsilon I + NP_1N^T \end{bmatrix} < 0 \tag{39}$$

The inequality (39), together with the inequality constraint

$$-\varepsilon I + NP_1N^T < 0, \tag{40}$$

are both linear on $\varepsilon > 0$ and $P_1 > 0$. Therefore, we can employ the standard LMI techniques in Gahinet *et al.* (1995) to check the solvability of the original matrix inequality (27). After P_1 is obtained, the inequality (28) becomes a standard Riccati-like matrix inequality, which is easy to solve. It is mentionable that, in the past decade, linear matrix inequalities (LMIs) have gained much attention for their computational tractability and usefulness in signal processing and control engineering Gahinet *et al.* (1995).

Remark 5. A typical feature of the present parameterization design approach is that, there exists much *explicit* freedom, such as the choices of the free parameters L ($L \in \mathbb{R}^{n\times p}$ satisfies $\Pi + LL^T < 0$), the orthogonal matrix $U \in \mathbb{R}^{p\times p}$, *etc.* This makes it possible that more performance constraints (*e.g.*, the transient requirement and reliability behavior on the filtering process) could be taken into account within the same framework.

As a summary, we give our main results as follows.

Corollary 1. If there exist a positive scalar $\varepsilon > 0$ and two positive definite matrices $P_1 > 0$, $P_2 > 0$ such that the LMIs (39)(40) and the matrix Riccati inequality (28) hold, and $P_2 > 0$ satisfies $[P_2]_{ii} \le \alpha_i^2$ ($i = 1, 2, \cdots, n.$), then the filter (9) determined by (29)-(30) will achieve the desired robust filtering performance for uncertain systems with missing measurements.

4. A NUMERICAL EXAMPLE

Consider the linear uncertain discrete-time stochastic system (1)-(2) with parameters given by

$$A = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & -0.5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.1 & 0.05 \\ -0.02 & 0.8 \end{bmatrix}, \quad N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad V = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

and the probability for complete observation is assumed to be 0.9.

The purpose of this example is to design the filter parameters, G and K, such that for all admissible perturbations ΔA, the augmented system (17) is mean square bounded, and the steady-state error covariance X_{ee} satisfies

$$[X_{ee}]_{11} \leq 0.8, \quad [X_{ee}]_{22} \leq 4.$$

Solving the LMIs (39)-(40) for ε, P_1, and then the Riccati-like matrix inequality (28) for P_2, we obtain

$$\varepsilon = 1.8286, \quad P_1 = \begin{bmatrix} 5.8346 & 0.0064 \\ 0.0064 & 3.6628 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 0.7765 & 0.0052 \\ 0.0052 & 3.6983 \end{bmatrix},$$

One of the filter parameters, G, is calculated from (29) as follows:

$$G = \begin{bmatrix} 0.5437 & 0.0768 \\ 0.2040 & -1.1470 \end{bmatrix}.$$

To obtain another parameter, K, we choose $L = 0.5I_2$ such that $\Pi + LL^T < 0$ and select the orthogonal matrix U as I_2. Then, it follows from (30) that

$$K = \begin{bmatrix} 0.8040 & 0.0725 \\ 0.1246 & -0.8165 \end{bmatrix}.$$

Alternatively, to show the design flexibility, we choose U as $-I_2$, and subsequently have

$$K = \begin{bmatrix} -0.1346 & 0.0732 \\ 0.1253 & -1.3490 \end{bmatrix}.$$

5. CONCLUSIONS

In this paper, the linear filtering problem has been considered for parameter uncertain discrete-time stochastic systems where there is a nonzero probability of signal being absent in the measurement. This problem has been approached by assigning an upper bound to the steady-state error covariance, and by parameterizing the set of all filter gains that could achieve such an upper bound. It has been shown that, the problem is solvable if several linear matrix inequalities or Riccati-like matrix inequalities have positive definite solutions. In particular, the characterization of the desired filter gains has been given in terms of some 'free' parameters, and much design flexibility have been offered, which could be utilized to achieve more expected performance requirements. An numerical example has been provided to illustrate the effectiveness of the proposed design approach.

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