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Optimizing crude oil operations scheduling considering blending in tanks

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Abstract:
This paper focuses on the optimization of crude oil operations scheduling in a refinery that is supplied with crude oil by ship. One of the main challenges associated with the crude oil operations scheduling problem is the management of crude storage in tanks. Since storage capacity is limited and there are several types of crude oil depending on their composition, it is necessary to store mixtures of crude oil in tanks. This feature makes necessary the inclusion of nonlinear, non-convex constraints, which complicates the resolution of mathematical programming models. To address this problem, we have developed a mathematical programming model based on a continuous-time formulation using time slots, along with a strategy based on piecewise McCormick relaxation that allows us to efficiently handle the nonlinear constraints generated by blending crude oils in tanks.

Keywords: Modelling and decision making in complex systems, Efficient strategies for large scale complex systems, Production planning and control, Optimization and control of large-scale network systems, Complex logistic systems.

1. INTRODUCTION

In this paper, we address the problem of optimizing the scheduling of crude oil operations in a refinery supplied with crude oil by ship. We analyze a system composed of a marine terminal and an oil storage and processing unit connected by a pipeline. The terminal serves as the location for unloading the crude oil transported by the ships. We consider a single dock terminal, which allows the unloading of one ship at a time.

Concerning the storage and processing section, it is divided into two areas: the storage tank area and the crude distillation unit area. The first is connected to the marine terminal by a pipeline and, as the name suggests, consists of tanks for storing crude oil received from the terminal. Most refineries have two types of tanks: storage tanks, which receive and store crude oil from ships, and charging tanks, used for creating blends to feed distillation units, meeting certain quality specifications. Due to the traditional operation of refineries utilizing both types of tanks, a wide variety of articles addressing the optimization of crude oil operations scheduling in such refineries have been published in recent decades (Lee et al. (1996), Mouret et al. (2009), Castro and Grossmann (2014), Yang et al. (2020)). However, some refineries opt to eliminate charging tanks to save space and reduce immobilized capital. Instead, they implement online mixing in the pipelines feeding the crude distillation units (CDUs) using a suitable control system. While researchers have studied this case, there is a smaller number of published works focusing on this type of refinery (Cerdá et al. (2015), García-Verdier et al. (2022)).

An important characteristic present in both cases is that the concentration of crude oil in the outflow of a tank must be equal to the concentration inside the tank. This behavior is represented by a set of nonlinear non-convex constraints that give rise to mixed-integer nonlinear programming (MINLP) models that are difficult to solve. In the literature, we can find works proposing different strategies to address this problem. For example, in de Assis et al. (2017), the authors propose a two-step MILP-NLP decomposition algorithm, where the mixed-integer linear programming (MILP) model is obtained by replacing each side of the nonlinear constraint with piecewise McCormick envelopes. Also, the work presented in Castro and Grossmann (2014) introduces a two-step MILP-NLP algorithm, where the bilinear blending constraints are relaxed using multiparametric disaggregation. This technique involves discretizing one of the variables of the bilinear term over a set of powers. In Garcia-Verdier et al. (2024), the authors propose a two-step MILP-NLP algorithm, where the approximate MILP formulation is obtained by replacing the nonlinear constraint with linear constraints, which determine that a tank maintains the initial crude con-

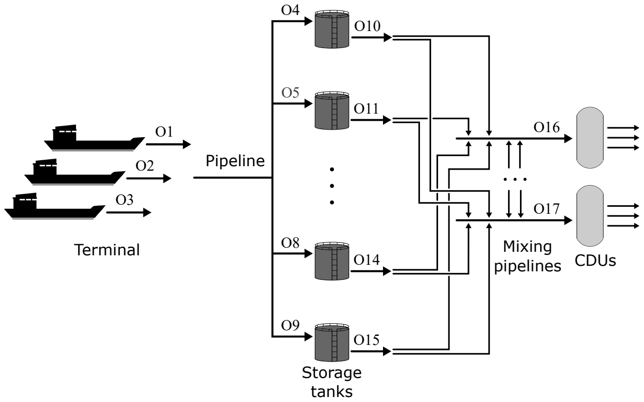

Fig. 1. Schematic of refinery

centration until the moment it receives crude oil from a ship. It is worth noting that in none of the cases is a procedure proposed in case the nonlinear programming (NLP) solution is infeasible.

Continuing with the description of the system, the tanks are connected to the crude distillation unit area through a piping system (mixing pipelines), where the final mixtures of crudes take place to achieve the desired flows and properties required by the different crude distillation units (CDUs). All operations are subjected to multiple rules and constraints, among them, the arrival over time of different types and amounts of crude, and the fulfillment of the company production plan. Figure 1 shows a schematic of the refinery under study, which has only storage tanks.

The aim of this work is to develop a novel model consisting of a network of interconnected resources (ships, tanks, and units) to solve the crude oil operations scheduling problem in a refinery that has only storage tanks that store crude oil blends. In this model, each ship has one associated outbound operation, each tank has one inbound and one outbound operation, and each unit has one inbound operation.

Furthermore, the model is based on a continuous-time formulation using time slots (Méndez et al. (2006)), and each operation has its own time grid. Although the set of postulated slots is unique and equal for each operation, the optimization process will assign the necessary slots to each operation to obtain the optimal solution.

To establish the connections between resources, pairs of operations (o, o') are defined to indicate that the execution of operation o can trigger the execution of operation o'.

For example, suppose we have operation O1 representing the unloading of ship 1, and operation O4 representing the loading of tank 1, which may receive crude oil from that ship. In this case, the pair (O1, O4) exists because the unloading of ship 1 may lead to the loading of tank 1.

To address the nonlinear non-convex constraints, we present a new approach that consists of expressing these constraints as functions of a variable that represents the proportion of each crude oil in the tanks. Then, an approximate MILP model is obtained based on McCormick envelopes. From the solution of this MILP, we fix the binary variables of the original MINLP and solve the resulting NLP. Finally, if a feasible solution is not obtained, a cut is added to the MILP model, and the procedure is repeated.

The rest of the paper is structured as follows. The proposed mathematical formulation is described in Section 2. The solution procedure for the MINLP model is discussed in Section 3. Next, a problem instance and computational results are reported in Section 4. Finally, conclusions are given in Section 5.

2. MODEL FORMULATION

In this section, we present the MINLP model, which is formulated based on a continuous-time approach using time slots. The following assumptions have been considered in formulating the mathematical programming model:

- Only one vessel can unload at a time.
- A vessel that starts unloading crude oil can only leave the terminal when it is completely empty.
- Each ship is dedicated to carrying a single type of crude, and the volume of the pipeline is considered negligible compared to the volume being unloaded.
- A tank cannot simultaneously receive crude oil from a vessel and feed a CDU.
- The simultaneous loading of tanks from each vessel is limited to a maximum number, and no transfer between tanks is allowed.
- Each tank can feed only a limited number of CDUs at any one time.
- There is a maximum number of tanks that can feed a CDU at one time, with negligible time required for tank changeovers.
- Perfect mixing of crudes is assumed in the mixing pipelines.
- Continuous feeding of crude distillation units is mandatory.

2.1 Notation

Sets

- C = types of crude oils
- K = key properties
- M = mixtures
- N = time slots
- O = operations
- IN_o = {O4, O5, O6, O7, O8, O9, O16, O17} are the input operations.
- OUT_o = {O1, O2, O3, O10, O11, O12, O13, O14, O15} are the output operations.
- OPO = pair of operations (o, o'), where operation o can activate operation o'
- R = resources
- RS = tanks
- RU = crude distillation units
- RV = vessels
- INR_r = pair (o, r), o is an input operation of r
- OUTR_r = pair (o, r), o is an output operation of r
- V = partitions of piecewise McCormick relaxation

Parameters

- AT_r = arrival time of ship r
- $CDMG_b$ = demurrage or sea waiting cost

- $CTDN_b$ = departure tardiness cost
- DEM_m = total demand of mixture m
- ED_r = expected departure time of ship r
- GM_c = gross margin
- H = horizon length
- $I0_r$ = initial inventory in tank r
- $KP_{c,k}$ = property value k in crude of type c
- $\underline{KP}_{m,k}$ = lower bound of property k in mixture m
- $\overline{KP}_{m,k}$ = upper bound of property k in mixture m
- TS = settling time for tanks
- $VC_{r,c}$ = volume of crude c transported by ship r
- $\underline{VP}_{o,n,o',n'}$ = lower bound of volume transferred by operation o during slot n to operation o' assigned to slot n'
- $\overline{VP}_{o,n,o',n'}$ = upper bound of $VP_{o,n,o',n'}$
- $\underline{VP^{MC1}}_{o,n,o',n',c,v}$ = lower bound of $VP^{MC1}{}_{o,n,o',n',c,v}$ in partition v
- $\overline{VP^{MC1}}_{o,n,o',n',c,v}$ = upper bound of $VP^{MC1}{}_{o,n,o',n',c,v}$ in partition v
- $\underline{VT}_{o,r}$ = minimum rate of operation o in resource r
- $\overline{VT}_{o,r}$ = maximum rate of operation o in resource r
- $\underline{\alpha^{MC1}}_{o,c,n,v}$ = lower bound of $\alpha^{MC1}_{o,c,n,v}$ in partition v
- $\overline{\alpha^{MC1}}_{o,c,n,v}$ = upper bound of $\alpha^{MC1}_{o,c,n,v}$ in partition v

Continuous variables

- DMG_r = demurrage of vessel r
- $DOP_{o,n}$ = duration of operation o in slot n
- $FO_{o,n}$ = end time of operation o in slot n
- $I_{r,n}$ = inventory level in tank r at the end of slot n
- $IIN_{r,n}$ = inventory level in tank r after receiving a load during slot n
- $IINC_{r,n,c}$ = inventory level of crude c in tank r after receiving a load during slot n
- $IO_{o,n}$ = start time of operation o in slot n
- TAR_r = departure tardiness of vessel r
- $VP_{o,n,o',n'}$ = volume transferred by operation o during slot n to operation o' assigned to slot n'
- $VPC_{o,n,o',n',c}$ = volume transferred of crude c by operation o during n to operation o' assigned to n'
- $VP^{MC1}{}_{o,n,o',n',c,v}$ = value of $VP_{o,n,o',n'}$ in v
- $VT_{o,n}$ = total volume transferred by operation o during slot n
- $VTC_{o,n,c}$ = total volume transferred of crude c by operation o during slot n
- $VTCM_{o,n,c,m}$ = total volume transferred of crude c in mixture m by operation o (input to CDU) during slot n
- $VTM_{o,n,m}$ = total volume transferred of mixture m by operation o (input to CDU) during slot n
- $\alpha_{o,c,n}$ = auxiliary variable representing ratio of inventory level variables
- $\alpha^{MC1}{}_{o,c,n,v}$ = value of $\alpha_{o,c,n}$ in partition v

Binary variables

- $W_{o,n}$ = indicates assignment of operation o to slot n
- $WM_{o,n,m}$ = indicates if operation o assigned to slot n transfers mixture m
- $X_{o,n,o',n'}$ = indicates if (o,n) produces (o',n')
- $Z_{o,c,n,v}$ = indicates activation of partition v

2.2 Constraints

In order not to exceed the established page limit, certain constraints have been omitted. The most important constraints are listed below.

The pair (o,n) can produce the pair (o',n'), as long as operations o and o' have been assigned to slots n and n', respectively.

$$X_{o,n,o',n'} \leq W_{o,n} \quad o,o' \in OPO \quad n,n' \in N \tag{1}$$

$$X_{o,n,o',n'} \leq W_{o',n'} \quad o,o' \in OPO \quad n,n' \in N \tag{2}$$

The volume transferred by an operation o assigned to slot n to an operation o' assigned to slot n' must be equal to the sum of the volumes transferred for each crude oil.

$$\begin{aligned} &VP_{o,n,o',n'} = \sum_{c\in C} VPC_{o,n,o',n',c} \\ &o,o' \in OPO,\ n,n' \in N \end{aligned} \tag{3}$$

The lower and upper bound for $VP_{o,n,o',n'}$ are set by (4) and (5), respectively.

$$\begin{aligned} &VP_{o,n,o',n'} \geq \underline{VP}_{o,n,o',n'} * X_{o,n,o',n'} \\ &o,o' \in OPO,\ n,n' \in N \end{aligned} \tag{4}$$

$$\begin{aligned} &VP_{o,n,o',n'} \leq \overline{VP}_{o,n,o',n'} * X_{o,n,o',n'} \\ &o,o' \in OPO,\ n,n' \in N \end{aligned} \tag{5}$$

The total volume transferred by operation o must be equal to the sum of the volumes transferred to each operation o'.

$$VT_{o,n} = \sum_{o'\in OPO} \sum_{n'\in N} VP_{o,n,o',n'} \quad o \in IN_o,\ n \in N \tag{6}$$

The total volume of operation o' must be equal to the sum of the volumes received from each operation o.

$$\begin{aligned} &VT_{o',n'} = \sum_{o\in OPO} \sum_{n\in N} VP_{o,n,o',n'} \\ &o' \in OUT_o,\ n' \in N \end{aligned} \tag{7}$$

The total volume transferred by operation o assigned to slot n must be equal to the sum of the total volumes of each crude oil.

$$VT_{o,n} = \sum_{c\in C} VTC_{o,n,c} \quad o \in O,\ n \in N \tag{8}$$

The lower and upper bound for $VT_{o,n}$ are set by (9) and (10), respectively.

$$\begin{aligned} &VT_{o,n} \geq \underline{VT}_{o,r} * DOP_{o,n} \\ &o \in INR_r \cup OUTR_r,\ n \in N,\ r \in R \end{aligned} \tag{9}$$

$$\begin{aligned} &VT_{o,n} \leq \overline{VT}_{o,r} * DOP_{o,n} \\ &o \in INR_r \cup OUTR_r,\ n \in N,\ r \in R \end{aligned} \tag{10}$$

The start time, end time, and duration of operations are given by (11), (12), and (13).

$$IO_{o,n+1} \geq FO_{o,n} \quad o \in O,\ n \in N \setminus \{n_{|N|}\} \tag{11}$$

$$FO_{o,n} = IO_{o,n} + DOP_{o,n} \quad o \in O,\ n \in N \tag{12}$$

$$DOP_{o,n} \leq H * W_{o,n} \quad o \in O,\ n \in N \tag{13}$$

Operations cannot finish after the horizon end date.

$$FO_{o,n} \leq H \quad o \in O,\ n \in N \tag{14}$$

Constraint (15) states that the start of the unloading of a tank (i.e., operation o') must be later than the end of the loading of the same tank (i.e., operation o), plus the settling time TS, as long as both operations were assigned to the same slot n, or if the unloading operation was assigned to a slot n' after n.

$$\begin{gathered} IO_{o',n'} \geq FO_{o,n} + TS - H * (2 - W_{o,n} - W_{o',n'}) \\ o \in INR_r,\ o' \in OUTR_r,\ n, n' \in N,\ n' \geq n,\ r \in RS \end{gathered} \tag{15}$$

Constraint (16) states that the start of the loading operation of a tank must be later than the end of the unloading operation, as long as the unloading operation has been assigned to a previous slot.

$$\begin{gathered} IO_{o,n} \geq FO_{o',n'} - H * (2 - W_{o,n} - W_{o',n'}) \\ o \in INR_r,\ o' \in OUTR_r,\ n, n' \in N,\ n' < n,\ r \in RS \end{gathered} \tag{16}$$

The inventory level of tank r at the end of slot n is equal to the initial inventory, plus the volumes of all loadings received up to that time, minus the volumes of all discharges performed up to that time.

$$\begin{gathered} I_{r,n} = I0_r + \sum_{n' \in N, n' \leq n} VT_{o,n'} - \sum_{n' \in N, n' \leq n} VT_{o',n'} \\ o \in INR_r,\ o' \in OUTR_r,\ n \in N,\ r \in RS \end{gathered} \tag{17}$$

Constraint (18) establishes the inventory level of tank r during slot n, after receiving a charge and before discharging. This takes into account the initial inventory, plus the volumes of all loadings received up to that time, minus the volumes of all unloadings performed up to the previous slot.

$$\begin{gathered} IIN_{r,n} = I0_r + \sum_{n' \in N, n' \leq n} VT_{o,n'} - \sum_{n' \in N, n' < n} VT_{o',n'} \\ o \in INR_r,\ o' \in OUTR_r,\ n \in N,\ r \in RS \end{gathered} \tag{18}$$

The CDUs must operate continuously.

$$\sum_{n} DOP_{o,n} = H \quad o \in INR_r,\ r \in RU \tag{19}$$

A CDU is fed with only one type of mixture at a time.

$$\sum_{m \in M} WM_{o,n,m} = 1 \quad o \in INR_r,\ n \in N,\ r \in RU \tag{20}$$

In case mixture m is not selected, then the volumes transferred must be null. To this purpose, we use the big-M method, where the value of $BIGM$ is determined based on physical limits.

$$\begin{gathered} VTCM_{o,n,c,m} \leq BIGM * WM_{o,n,m} \\ o \in INR_r,\ n \in N,\ c \in C,\ r \in RU \end{gathered} \tag{21}$$

$$\begin{gathered} VTM_{o,n,m} \leq BIGM * WM_{o,n,m} \\ o \in INR_r,\ n \in N,\ r \in RU \end{gathered} \tag{22}$$

Only one term of the sum will be positive and will have the same value as the aggregate variable.

$$\begin{gathered} VTC_{o,n,c} = \sum_{m \in M} VTCM_{o,n,c,m} \\ o \in INR_r,\ n \in N,\ c \in C,\ r \in RU \end{gathered} \tag{23}$$

$$VT_{o,n} = \sum_{m \in M} VTM_{o,n,m} \quad o \in INR_r,\ n \in N,\ r \in RU \tag{24}$$

The total demand for each mixture must be met.

$$\sum_{o \in INR_r} \sum_{n \in N} \sum_{r \in RU} VTM_{o,n,m} \geq DEM_m \quad m \in M \tag{25}$$

The concentrations of properties in the volumes of mixtures transferred to each CDU must be within the established range.

$$\begin{gathered} \sum_{c \in C} VTCM_{o,n,c,m} * KP_{c.k} \geq \underline{KP}_{m,k} * VTM_{o,n,m} \\ o \in INR_r,\ n \in N,\ m \in M,\ k \in K,\ r \in RU \end{gathered} \tag{26}$$

$$\begin{gathered} \sum_{c \in C} VTCM_{o,n,c,m} * KP_{c.k} \leq \overline{KP}_{m,k} * VTM_{o,n,m} \\ o \in INR_r,\ n \in N,\ m \in M,\ k \in K,\ r \in RU \end{gathered} \tag{27}$$

A ship can start unloading only after arrival.

$$IO_{o,n} \geq AT_r * W_{o,n} \quad o \in OUTR_r,\ r \in RV,\ n \in N \tag{28}$$

The demurrage is calculated as the time elapsed between the arrival of a ship and the start of its unloading.

$$\begin{gathered} DMG_r \geq IO_{o,n} - AT_r \\ - (1 - W_{o,n} + \sum_{n' \in N, n' < n} W_{o,n'}) * H \\ o \in OUTR_r,\ r \in RV,\ n \in N \end{gathered} \tag{29}$$

Vessel r will incur tardiness if it leaves the terminal after its expected departure time.

$$\begin{gathered} TAR_r \geq FO_{o,n} - ED_r - (1 - W_{o,n}) * H \\ o \in OUTR_r,\ r \in RV,\ n \in N \end{gathered} \tag{30}$$

Vessels must be completely emptied.

$$\sum_{n \in N} VTC_{o,n,c} = VC_{r,c} \quad o \in OUTR_r,\ r \in RV,\ c \in C \tag{31}$$

The start time of operation o', that is triggered by operation o, must be greater than or equal to the start time of this operation o.

$$\begin{gathered} IO_{o',n'} \geq IO_{o,n} - H * (1 - X_{o,n,o',n'}) \\ o, o' \in OPO,\ n, n' \in N \end{gathered} \tag{32}$$

The end time of operation o', that is triggered by operation o, must be less than or equal to the end time of this operation o.

$$\begin{gathered} FO_{o',n'} \leq FO_{o,n} + H * (1 - X_{o,n,o',n'}) \\ o, o' \in OPO,\ n, n' \in N \end{gathered} \tag{33}$$

If a tank is being discharged, then the crude oil concentration in the outflow must be equal to the concentration inside the tank. Note that the variables $IIN_{r,n}$ and $IINC_{r,n,c}$ are used because they refer to the total inventory level and the level of each crude oil before unloading.

$$IIN_{r,n} * VPC_{o,n,o',n',c} = IINC_{r,n,c} * VP_{o,n,o',n'} \quad o,o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n,n' \in N,\ c \in C \tag{34}$$

2.3 Objective function

The objective function is to minimize vessel demurrage and tardiness costs while maximizing the profit from the processed crude.

$$MIN \sum_{r \in RV} (CDMG_r * DMG_r + CTAR_r * TAR_r) - \sum_{o \in INR_r} \sum_{n \in N} \sum_{c \in C} \sum_{r \in RU} GM_c * VTC_{o,n,c} \tag{35}$$

3. MINLP SOLUTION PROCEDURE

Given the nonconvex nature of the nonlinear constraint (34), it is essential to explore strategies to effectively address this challenge. In this paper, we have developed a strategy that consists of two steps: first solving a MILP model and then an NLP model. Moreover, we iteratively add cuts to eliminate infeasible solutions. Initially, we rearrange the nonlinear equation (34) as follows:

$$VPC_{o,n,o',n',c} = \alpha_{o,c,n} * VP_{o,n,o',n'} \quad o,o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n,n' \in N,\ c \in C \tag{36}$$

$$IINC_{r,n,c} = \alpha_{o,c,n} * IIN_{r,n} \quad o \in OUTR_r,\ r \in RS,\ n \in N,\ c \in C \tag{37}$$

Then, we apply piecewise McCormick relaxation to both equations. Only the set of constraints obtained by relaxing (36) is shown below; similarly, we can obtain the relaxation of (37).

$$VPC_{o,n,o',n',c} \geq \sum_{v \in V} (VP^{MC1}{}_{o,n,o',n',c,v} * \underline{\alpha^{MC1}}_{o,c,n,v} + \underline{VP^{MC1}}_{o,n,o',n',c,v} * \alpha^{MC1}{}_{o,c,n,v} - \underline{VP^{MC1}}_{o,n,o',n',c,v} * \underline{\alpha^{MC1}}_{o,c,n,v} * Z_{o,c,n,v}) \quad o,o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n,n' \in N,\ c \in C \tag{38}$$

$$VPC_{o,n,o',n',c} \geq \sum_{v \in V} (VP^{MC1}{}_{o,n,o',n',c,v} * \overline{\alpha^{MC1}}_{o,c,n,v} + \overline{VP^{MC1}}_{o,n,o',n',c,v} * \alpha^{MC1}{}_{o,c,n,v} - \overline{VP^{MC1}}_{o,n,o',n',c,v} * \overline{\alpha^{MC1}}_{o,c,n,v} * Z_{o,c,n,v}) \quad o,o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n,n' \in N,\ c \in C \tag{39}$$

$$VPC_{o,n,o',n',c} \leq \sum_{v \in V} (VP^{MC1}{}_{o,n,o',n',c,v} * \underline{\alpha^{MC1}}_{o,c,n,v} + \overline{VP^{MC1}}_{o,n,o',n',c,v} * \alpha^{MC1}{}_{o,c,n,v} - \overline{VP^{MC1}}_{o,n,o',n',c,v} * \underline{\alpha^{MC1}}_{o,c,n,v} * Z_{o,c,n,v}) \quad o,o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n,n' \in N,\ c \in C \tag{40}$$

$$VPC_{o,n,o',n',c} \leq \sum_{v \in V} (VP^{MC1}{}_{o,n,o',n',c,v} * \overline{\alpha^{MC1}}_{o,c,n,v} + \underline{VP^{MC1}}_{o,n,o',n',c,v} * \alpha^{MC1}{}_{o,c,n,v} - \underline{VP^{MC1}}_{o,n,o',n',c,v} * \overline{\alpha^{MC1}}_{o,c,n,v} * Z_{o,c,n,v}) \quad o,o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n,n' \in N,\ c \in C \tag{41}$$

The lower and upper bounds for $\alpha^{MC1}{}_{o,c,n,v}$ and $VP^{MC1}{}_{o,n,o',n',c,v}$ are given by (42)-(45).

$$\alpha^{MC1}{}_{o,c,n,v} \geq \underline{\alpha^{MC1}}_{o,c,n,v} * Z_{o,c,n,v} \quad o \in OUTR_r,\ r \in RS,\ n \in N,\ c \in C,\ v \in V \tag{42}$$

$$\alpha^{MC1}{}_{o,c,n,v} \leq \overline{\alpha^{MC1}}_{o,c,n,v} * Z_{o,c,n,v} \quad o \in OUTR_r,\ r \in RS,\ n \in N,\ c \in C,\ v \in V \tag{43}$$

$$VP^{MC1}{}_{o,n,o',n',c,v} \geq \underline{VP^{MC1}}_{o,n,o',n',c,v} * Z_{o,c,n,v} \quad o,o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n,n' \in N,\ c \in C,\ v \in V \tag{44}$$

$$VP^{MC1}{}_{o,n,o',n',c,v} \leq \overline{VP^{MC1}}_{o,n,o',n',c,v} * Z_{o,c,n,v} \quad o,o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n,n' \in N,\ c \in C,\ v \in V \tag{45}$$

Only one partition must be selected.

$$\sum_{v \in V} Z_{o,c,n,v} = 1 \quad o \in OUTR_r,\ r \in RS,\ n \in N,\ c \in C \tag{46}$$

Finally, only one term of the sum will be positive and must have the same value as the aggregate variable.

$$VP_{o,n,o',n'} = \sum_{v \in V} VP^{MC1}{}_{o,n,o',n',c,v} \quad o,o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n,n' \in N,\ c \in C \tag{47}$$

$$\alpha_{o,c,n} = \sum_{v \in V} \alpha^{MC1}{}_{o,c,n,v} \quad o \in OUTR_r,\ r \in RS,\ n \in N,\ c \in C \tag{48}$$

By replacing the nonlinear equation with this set of constraints, a MILP model is obtained and solved. Subsequently, the values of the binary variables in the original MINLP are set according to the solution found for the MILP, and the resulting NLP model is solved.

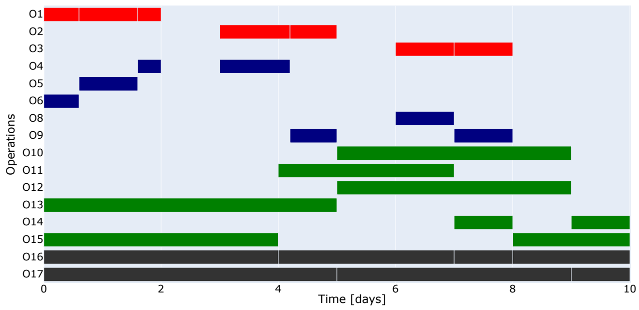

Fig. 2. Gantt chart of the solution.

If the solution is not feasible, we add the following "no good" constraint to the MILP model and solve it again, forcing at least one of the variables to change value.

$$\sum_{(o,n,o',n'):\hat{X_{o,n,o',n'}}=0} X_{o,n,o',n'} + \sum_{(o,n,o',n'):\hat{X_{o,n,o',n'}}=1} (1 - X_{o,n,o',n'}) \geq 1 \tag{49}$$

This process is repeated until a feasible solution for the NLP is obtained or a certain number of iterations is exceeded.

4. RESULTS

To assess the performance of the formulated model and the proposed strategy, a case study is conducted using the data from problem 2 presented in Mouret et al. (2009), and a configuration corresponding to Figure 1. Additionally, the suggested number of time slots is six.

For this problem (6834 binary variables, 22889 real variables, and 52471 constraints), a solution with a profit of \$17500000 and a relative gap of less than 2% was obtained in approximately 50 seconds using GAMS 43.2 software, Gurobi 9.5.2 for MILPs, and CONOPT 4.29 for NLPs on a computer equipped with an Intel Core i9-13900K 3.00 GHz processor and 128 GB of RAM.

It should be noted that the same problem was solved using DICOPT, but no feasible solution was obtained after 50 iterations, while using the presented strategy, a feasible solution was obtained in the first iteration.

Figure 2 depicts a Gantt chart of the solution. As shown, the ships start and finish their unloading operations on schedule. Specifically, the unloading of ship 1 leads to the loading of tanks 1, 2 and 3 (operations O4, O5, and O6, respectively). Ship 2 unloads into tanks 1 and 6, while ship 3 unloads into tanks 5 and 6. Finally, it should be noted that the CDUs operate continuously (operations O16 and O17).

5. CONCLUSION

A model based on continuous time representation using time slots was developed, together with an iterative strategy to cope with the nonlinearities caused by the blending of crude oils in storage tanks. The performance of both, the model and the strategy, was evaluated by solving a case study from the literature, obtaining a very good and feasible solution in a sensible short time. Future work will focus on extending the scope of the system under study to include downstream processing units for more comprehensive solutions.

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